Baryon acoustic oscillations under the hood

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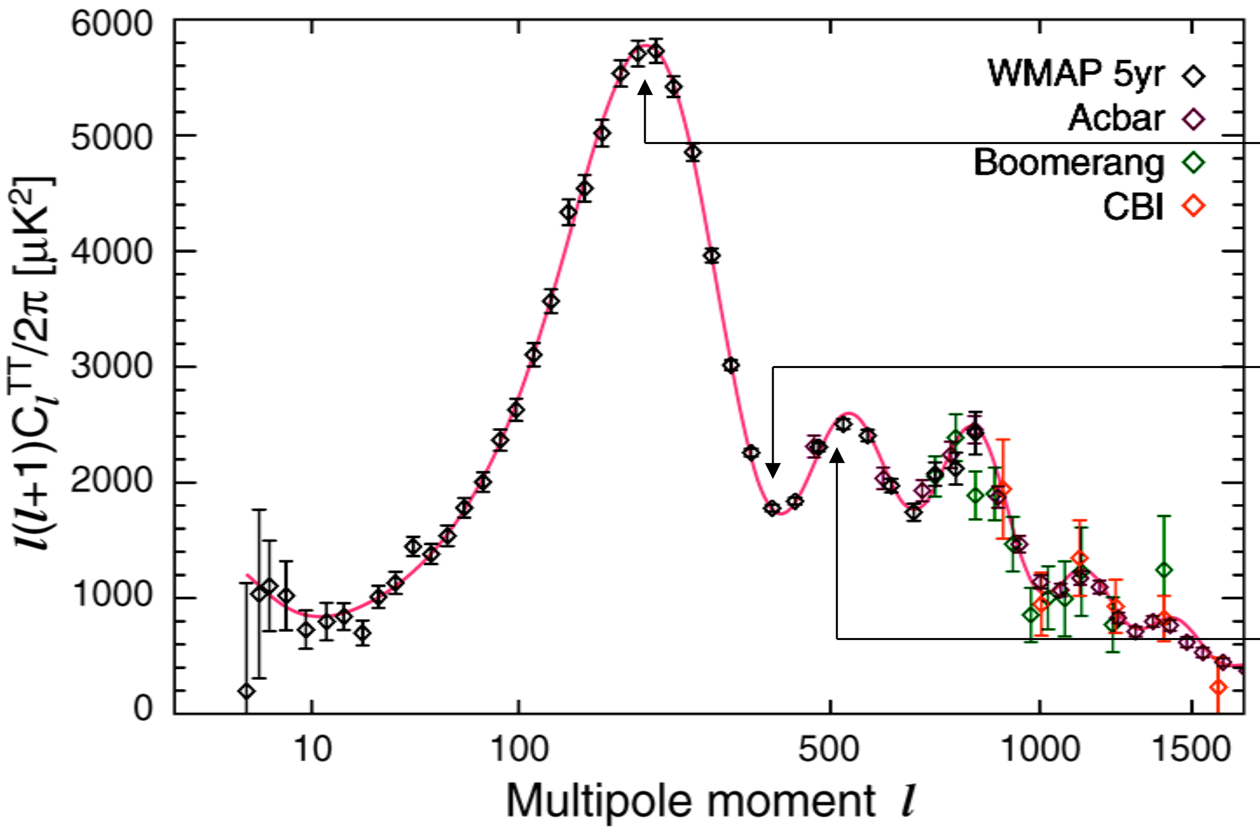
Acoustic oscillations seen!



First "compression", at $kc_st_{ls}=\pi$. Density maxm, velocity null.

Velocity maximum

First "rarefaction" peak at $kc_st_{ls}=2\pi$

Acoustic scale is set by the *sound horizon* at last scattering: $s = c_st_{ls}$

CMB calibration

- Not coincidentally the sound horizon is extremely well determined by the structure of the acoustic peaks in the CMB.

$$
\begin{aligned}
s &= 146.8 \pm 1.8 \text{ Mpc} \\
&= (4.53 \pm 0.06) \times 10^{24} \text{m}
\end{aligned}
$$

WMAP 5th yr data

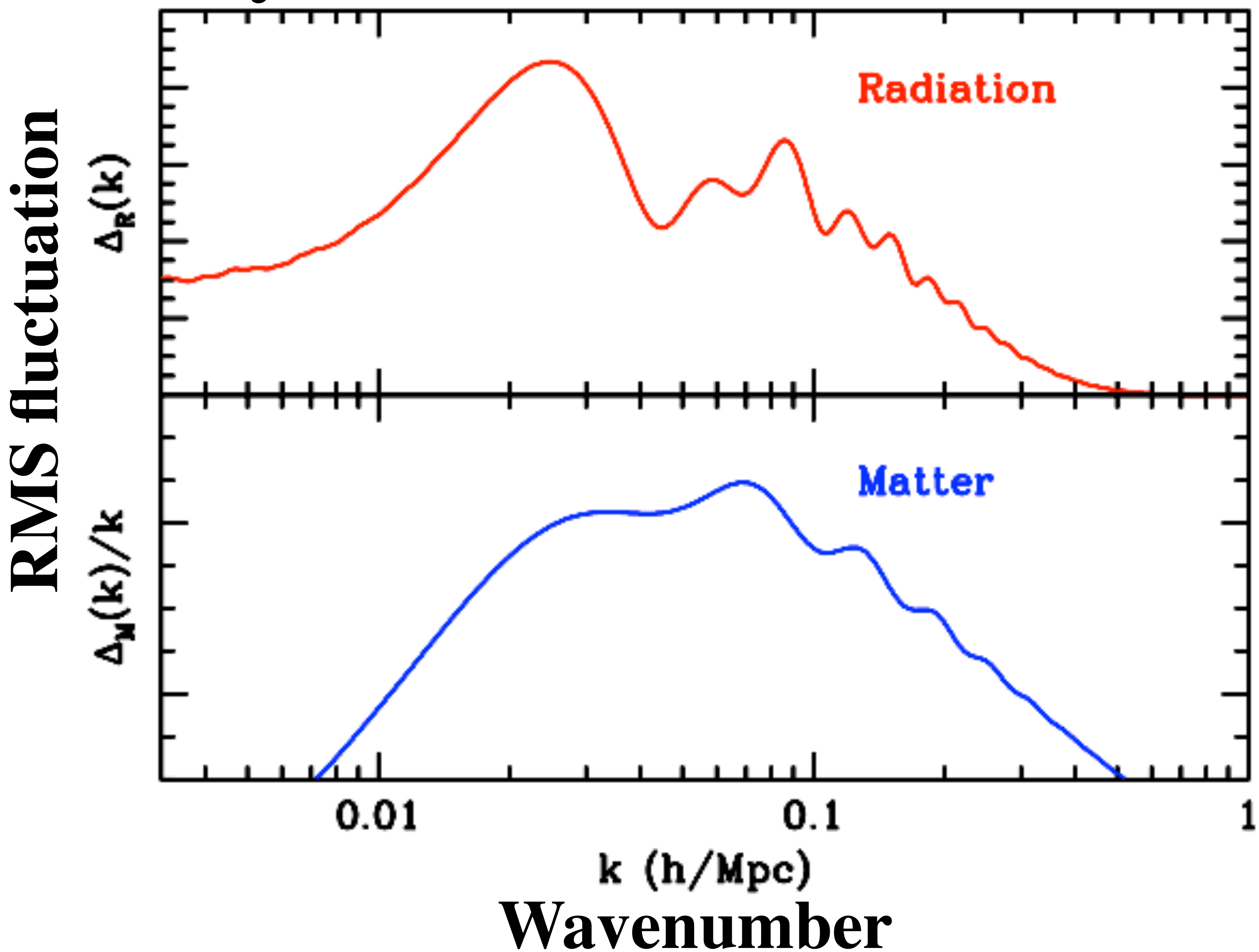
Dominated by uncertainty in ρ_m from poor constraints near 3rd peak in CMB spectrum. (Planck will nail this!)

Baryon oscillations in *P(k)*

- Since the baryons contribute ~15% of the total matter density, the total gravitational potential is affected by the acoustic oscillations with scale set by *s*.
- This leads to small oscillations in the matter power spectrum $P(k)$.
  - No longer order unity, like in the CMB
  - Now suppressed by $\Omega_b/\Omega_m \sim 0.1$

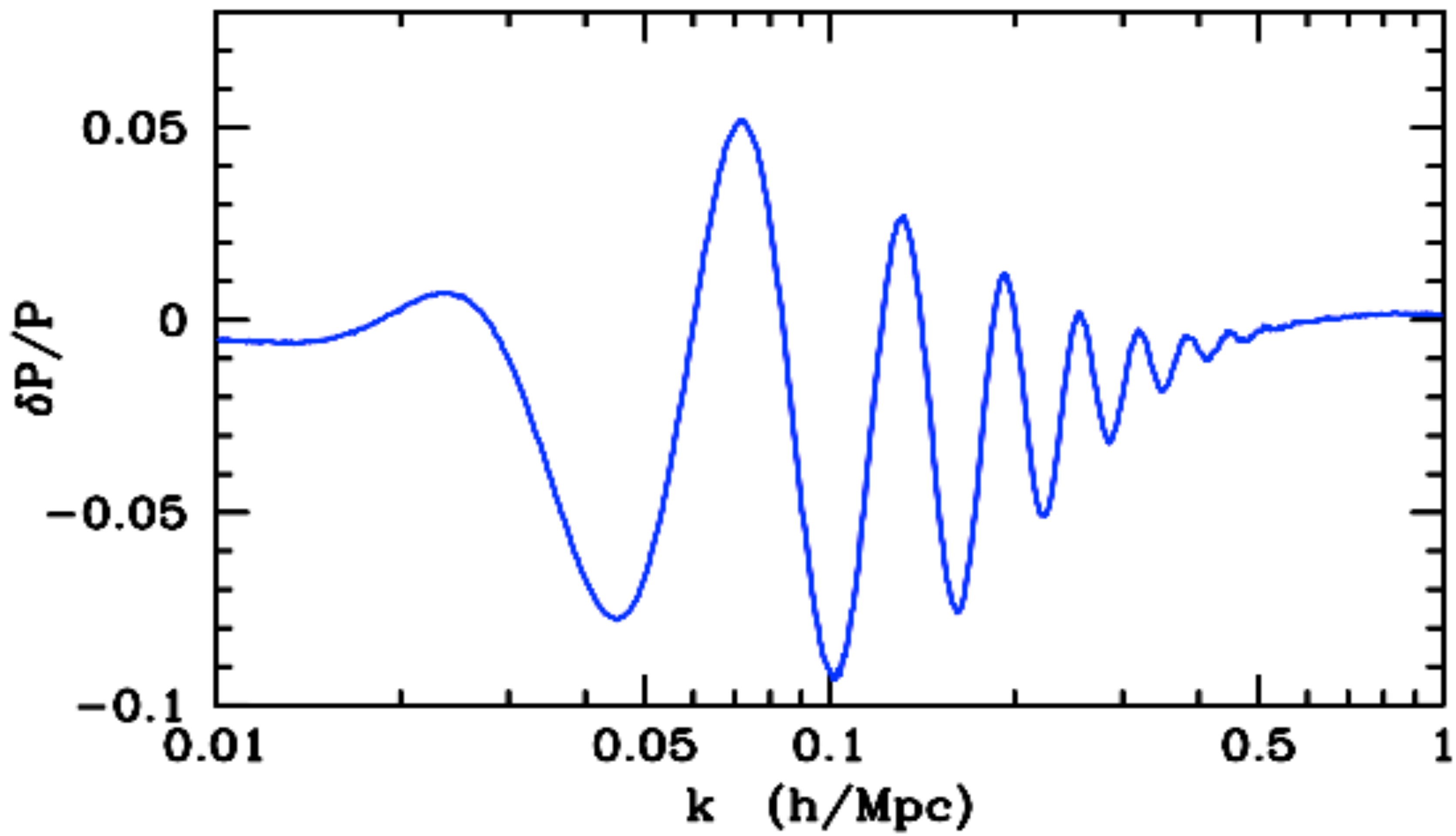
- **Note:** all of the matter sees the acoustic oscillations, not just the baryons.

Baryon (acoustic) oscillations



Divide out the gross trend …

A damped, almost harmonic sequence of "wiggles" in the power spectrum of the mass perturbations of amplitude *O*(10%).



Higher order effects

- The matter and radiation oscillations are not in phase, and the phase shift depends on *k*.
- There is a subtle shift in the oscillations with *k* due to the fact that the universe is expanding and becoming more matter dominated.
- The finite duration of decoupling and rapid change in mfp means the damping of the oscillations on small scales is not a simple Gaussian shape.
- But regardless, the spectrum is calculable and *s* can be inferred!

These features are frozen into the mass power spectrum, providing a known length scale that can be measured as a function of z.

Beyond the cartoon

- In Newtonian gauge the evolution of the baryon and photon perturbations is governed by:
  - Continuity equation(s):

$$\begin{aligned} \dot\delta_\gamma &= -\tfrac{4}{3}kV_\gamma & -4\dot\Phi \\ \dot\delta_b &= -kV_b & -3\dot\Phi \end{aligned} \qquad \left(a \to a[1+\Phi]\right)$$

  - Euler equation(s):

$$\begin{aligned} \dot V_\gamma &= k\left[\tfrac{1}{4}\delta_\gamma + \Psi - \tfrac{1}{6}\Pi_\gamma\right] - \dot\tau\left(V_\gamma - V_b\right) \\ \dot V_b &= -(\dot a/a)V_b + k\Psi + \dot\tau\left(V_\gamma - V_b\right)/R \end{aligned}$$

Fluid equations

- These equations can be easily derived by stress-energy conservation, but physically:
  - Densities are enhanced/reduced by converging/ diverging flows and by the stretching of space.
  - Accelerations are sources by gradients of the potential, and comoving velocities decay due to the expansion.
- Scattering of photons off free electrons couples drags V_γ-V_b to zero, leading to a baryon-photon fluid.
  - The protons follow the electrons via electromagnetic interactions.

Acoustic oscillations: photons

- Ignore for now the τ and Π terms.

$$\dot{\delta} = -\tfrac{4}{3}kV - 4\dot{\Phi}$$

$$\dot{V} = k\left(\tfrac{1}{4}\delta + \Psi\right)$$

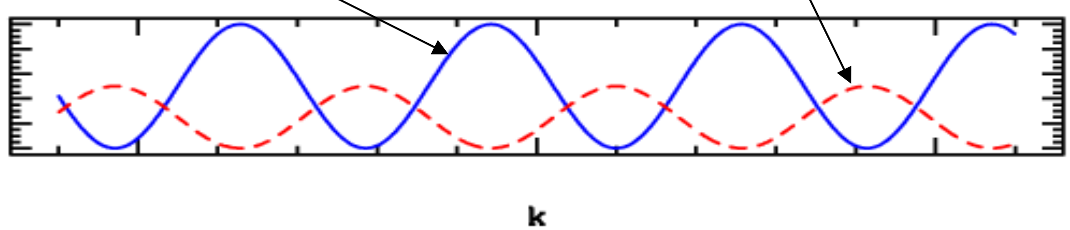
$$\Longrightarrow \quad \frac{\ddot{\delta}}{4} + \frac{k^2}{3}\frac{\delta}{4} = -\frac{1}{3}k^2\Psi - \ddot{\Phi}$$

- If Φ~const this becomes:

$$\frac{d^2}{d\eta^2}\left(\frac{\delta}{4} + \Psi\right) + k^2 c_s^2\left(\frac{\delta}{4} + \Psi\right) = 0 \quad \Rightarrow \quad \left(\frac{\delta}{4} + \Psi\right) = A\cos(ks) + \cdots$$

Effective temperature

$$(\Delta T)^2_{\mathrm{ls}} \sim \cos^2(kc_s t_{\mathrm{ls}}) + \text{velocity terms}$$



Matter curves space

- The fluctuations in the matter/radiation generate spatial curvature:

$$k^2\Phi = 4\pi G a^2 \sum \rho_i \delta_i + 3\frac{\dot{a}}{a}\left(\rho_i + p_i\right) V_i / k$$

$$k^2\left(\Phi + \Psi\right) = -8\pi G a^2 \sum p_i \Pi_i$$

Tight coupling I

- At early times the density is high and the scattering is rapid compared with the "travel time" across a wavelength.
- To lowest order $V_\gamma = V_b = V$ and the continuity equation(s) give:

$$
\begin{aligned}
\dot{\delta}_\gamma &= -\tfrac{4}{3} k V_\gamma & -4\dot{\Phi} \\
\dot{\delta}_b &= -k V_b & -3\dot{\Phi}
\end{aligned}
$$

$$
\begin{aligned}
\frac{d}{d\eta}\left[(1+R)\dot{\delta}_b\right] &= \frac{d}{d\eta}\left[(1+R)\left\{-kV - 3\dot{\Phi}\right\}\right] \\
&= -3\frac{d}{d\eta}\left[(1+R)\dot{\Phi}\right] - k\frac{d}{d\eta}\left[(1+R)V\right] \\
&= -3\frac{d}{d\eta}\left[(1+R)\dot{\Phi}\right] - k\dot{R}V - k(1+R)\dot{V}
\end{aligned}
$$

Tight coupling II

- Expand the Euler equation in powers of Compton mean-free-path over wavelength [or k/(dτ/dη)] to lowest order $V_\gamma = V_b = V$ and

$$\dot{V}_\gamma = k\left[\tfrac{1}{4}\delta_\gamma + \Psi - \tfrac{1}{6}\Pi_\gamma\right] - \dot{\tau}\left(V_\gamma - V_b\right)$$

$$\dot{V}_b = -(\dot{a}/a)V_b + k\Psi + \dot{\tau}\left(V_\gamma - V_b\right)/R$$

$$\begin{aligned}
\dot{V} + \tfrac{\dot{a}}{a}V &= k\Psi + R^{-1}\left[k(\delta_\gamma/4 + \Psi) - \dot{V}\right] \\
\left(\tfrac{1+R}{R}\right)\dot{V} + \tfrac{\dot{a}}{a}V &= \left(\tfrac{1+R}{R}\right)k\Psi + \tfrac{k}{R}\tfrac{\delta_\gamma}{4} \\
(1+R)\dot{V} &= (1+R)k\Psi + k(\delta_\gamma/4) - \tfrac{\dot{a}}{a}RV
\end{aligned}$$

Tight coupling III

- Combing these, and using $\delta_b=(3/4)\delta_\gamma$ for adiabatic fluctuations:

$$\frac{d}{d\eta}\left[(1+R)\dot{\delta}_b\right]+\frac{k^2}{3}\delta_b=-k^2(1+R)\Psi-\frac{d}{d\eta}\left[3(1+R)\dot{\Phi}\right]$$

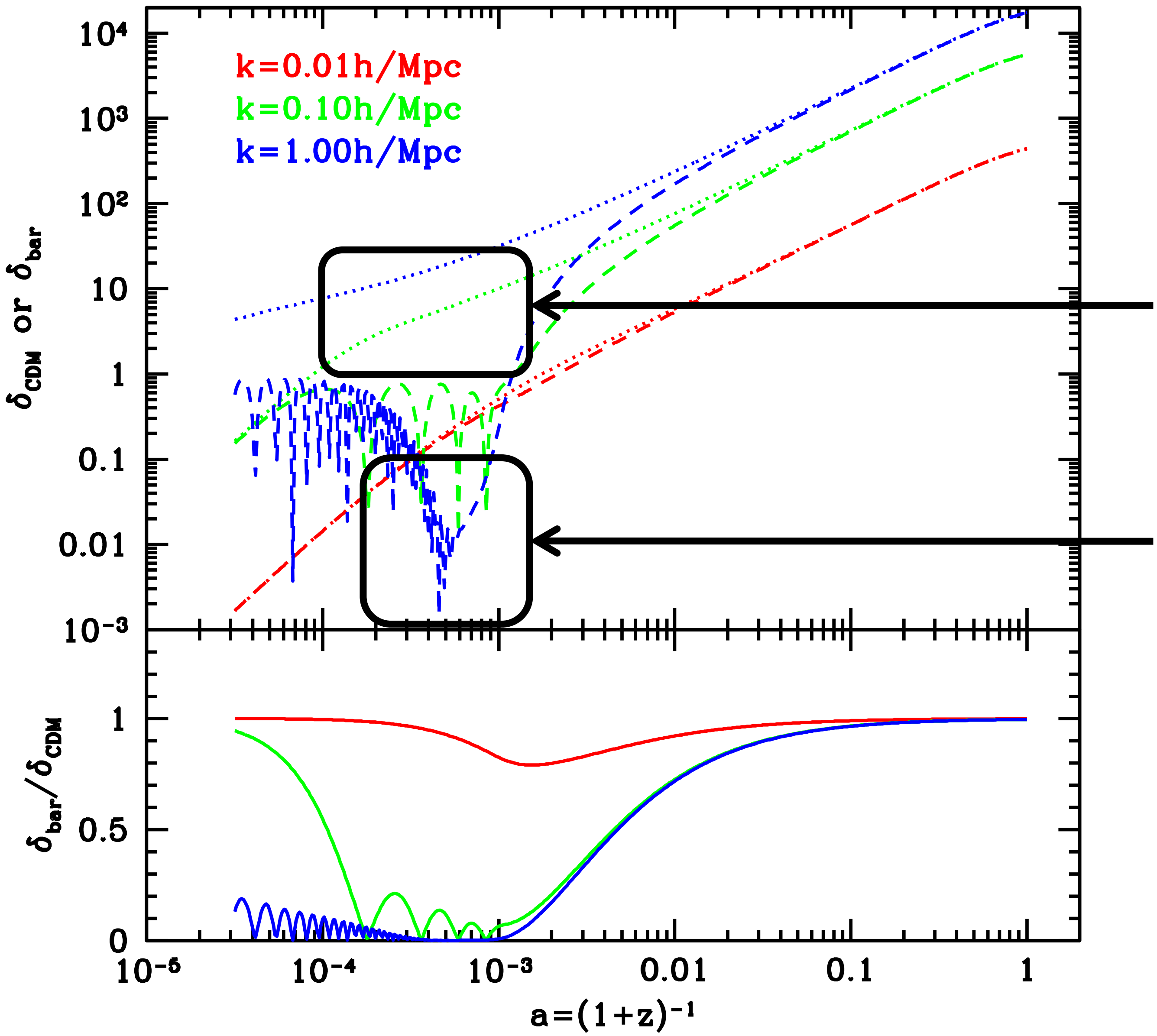
- A driven harmonic oscillator with natural frequency $c_s^{-2}=3(1+R)$.
- During tight-coupling the amplitude of the baryonic perturbation cannot grow
  - Harmonic motion with decaying amplitude [$(1+R)^{-1/4}$ in adiabatic limit.
- Baryons decouple when $\tau_b\sim 1$ ($\tau_b=\int\frac{\dot{\tau}\,d\eta}{1+R}$)

$$s=\int c_s(1+z)\,dt=\int\frac{c_s\,dz}{H(z)}=\frac{1}{\sqrt{\Omega_m H_0^2}}\frac{2c}{\sqrt{3z_{\rm eq}R_{\rm eq}}}\ \ln\frac{\sqrt{1+R_{\rm dec}}+\sqrt{R_{\rm dec}+R_{\rm eq}}}{1+\sqrt{R_{\rm eq}}}$$

Post-decoupling

- Once the photons have released the baryons, both the CDM and baryon perturbations grow with $\delta \sim a$ ($z \gg 1$).
- Density and velocity perturbations from tight-coupling must be matched onto growing mode solution.
  - Velocity overshoot.
- Note: for the period between horizon entry and decoupling all perturbation growth is suppressed. Changes shape of *P(k)* near “peak”.
- Oscillations have larger amplitude for higher ω_B and lower ω_m

Perturbation evolution



Baryons slow the growth of the DM. "Stagflation".

Oscillations at high *k* are damped.

Diffusion/Silk damping

- If we expand to next order in k/[dτ/dη] and assume R, Φ and Ψ are slowly varying we get a dispersion relation

$$\omega = \pm k c_s + \frac{ik^2}{6\dot{\tau}}\left[\frac{R^2}{(1+R)^2} + \frac{16}{15}\frac{1}{1+R}\right]$$

- which indicates (diffusion) damping of the oscillations with scale:

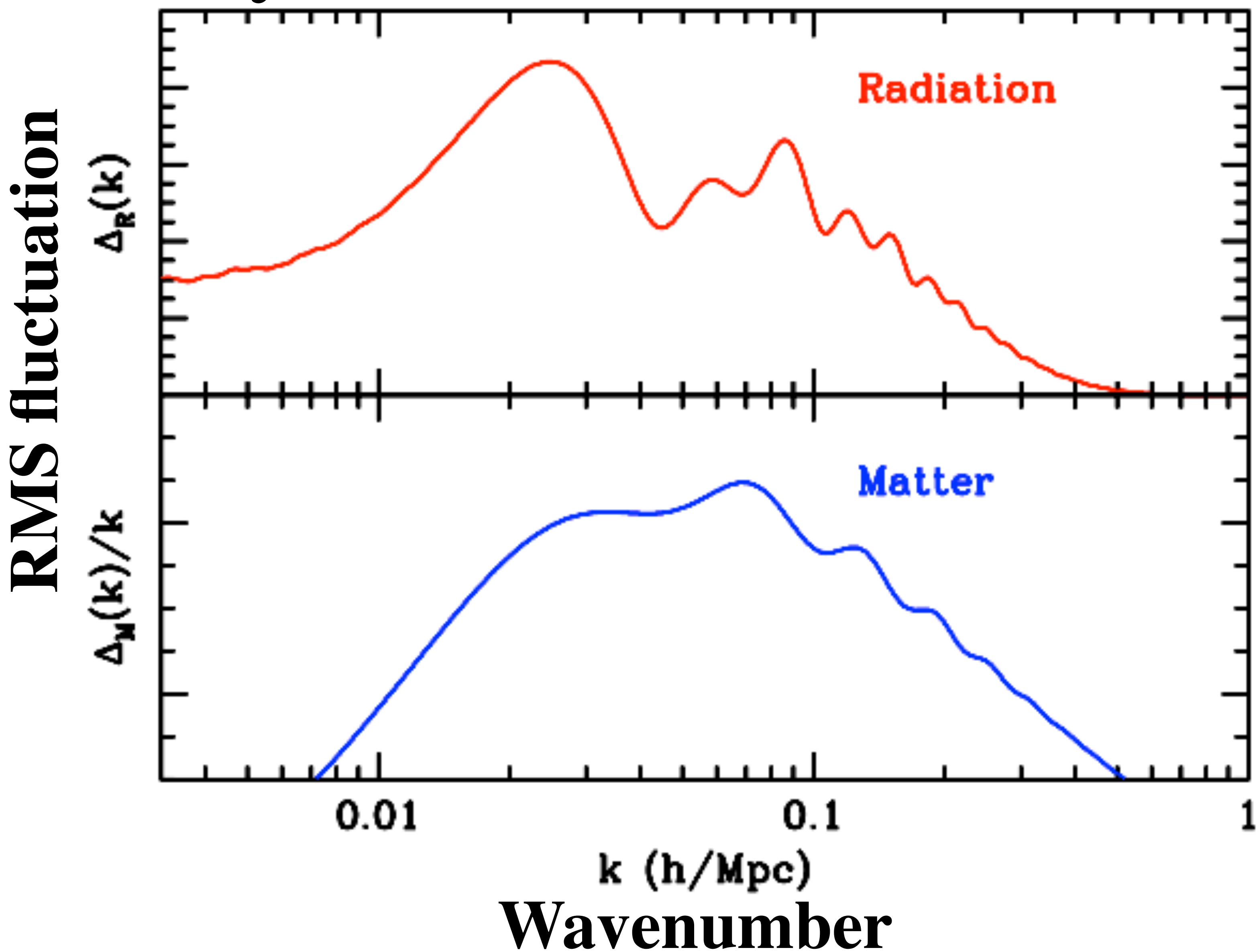
$$k_D^{-2} = \frac{1}{6}\int d\eta\ \frac{1}{\dot{\tau}}\ \frac{R^2 + 16(1+R)/15}{(1+R)^2}$$

Silk (1967),
Kaiser (1983),
Hu & White (1997)

  – Note $k_D \sim ([d\tau/d\eta]/\eta)^{1/2}$: geometric mean of mfp & horizon

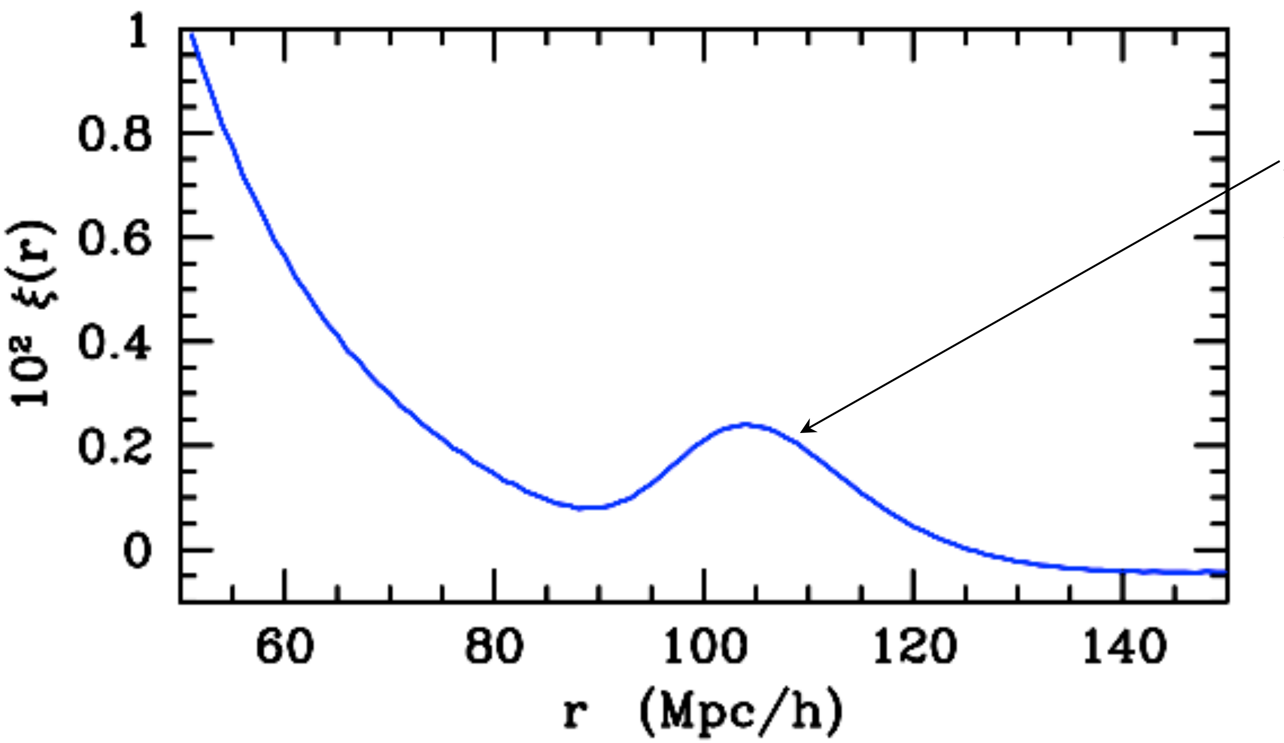
- The acoustic signal is thus an (almost) harmonic series of peaks with a quasi-exponential damping at $k_D \sim 0.1$ *h*/Mpc.
  – True effect is more complicated due to rapid changes during recombination.

Baryon (acoustic) oscillations



In configuration space

- The configuration space picture offers some important insights,and will be useful when we consider non-linearities and bias.
- In configuration space we measure not power spectra but correlation functions: $\xi(r)=\int P(k)e^{ikr}d^3k=\int \Delta^2(k)j_0(kr)\,d\ln k$..
- A harmonic sequence would be a δ-function in r, the shift in frequency and diffusion damping broaden the feature.



Acoustic feature at ~100 Mpc/h with width ~10Mpc/h (Silk scale)

Configuration space

In configuration space one uses a Green's function method to solve the equations, rather than expanding *k*-mode by *k*-mode. (Bashinsky & Bertschinger 2000)

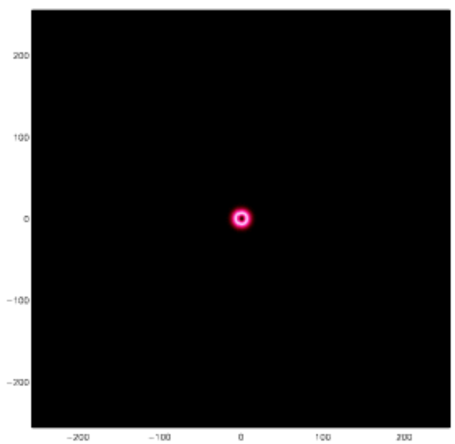
To linear order Einstein's equations look similar to Poisson's equation relating ϕ and δ, but upon closer inspection one finds that the equations are hyperbolic: they describe traveling waves.

[effects of local stress-energy conservation, causality, …]

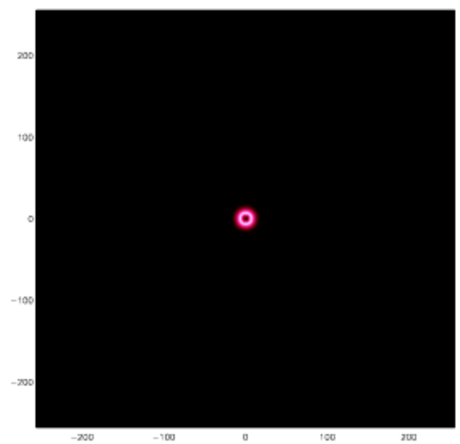
The acoustic wave

Start with a single perturbation. The plasma is totally uniform except for an excess of matter at the origin.

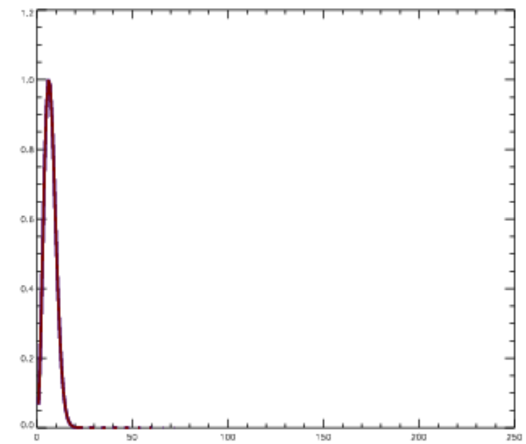
High pressure drives the gas+photon fluid outward at speeds approaching the speed of light.



Baryons



Photons

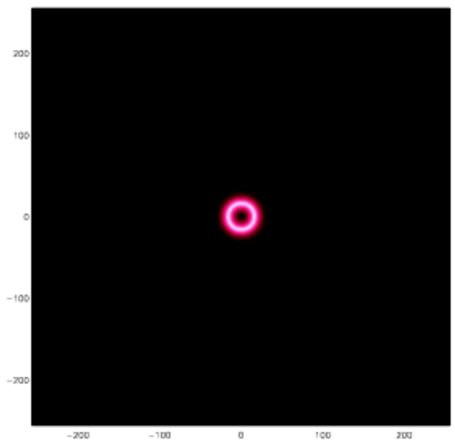


Mass profile

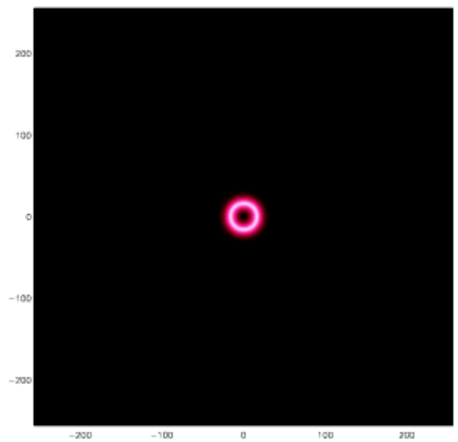
Eisenstein, Seo & White (2006)

The acoustic wave

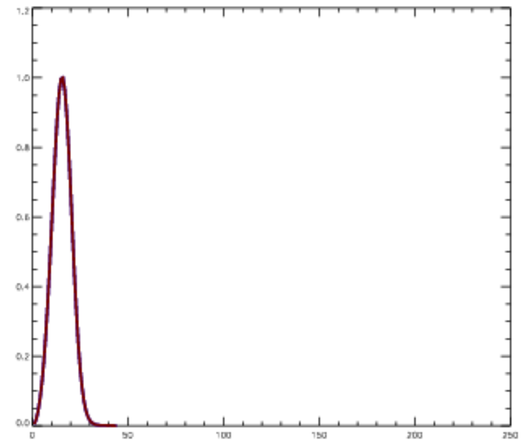
Initially both the photons and the baryons move outward together, the radius of the shell moving at over half the speed of light.



Baryons

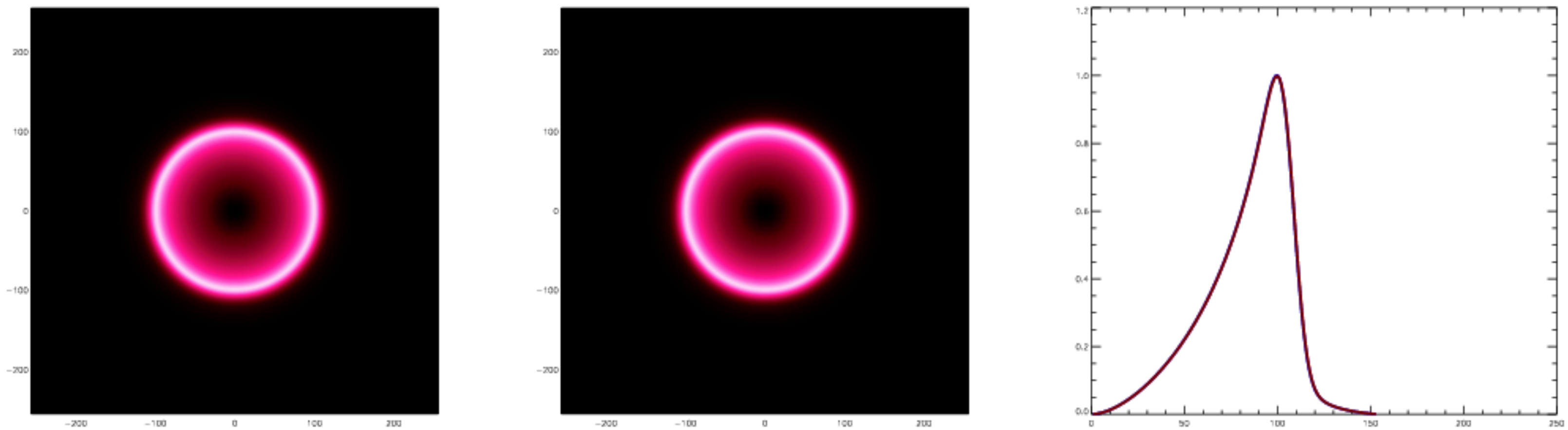


Photons



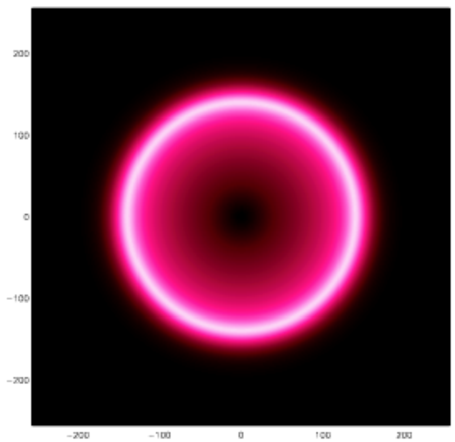
The acoustic wave

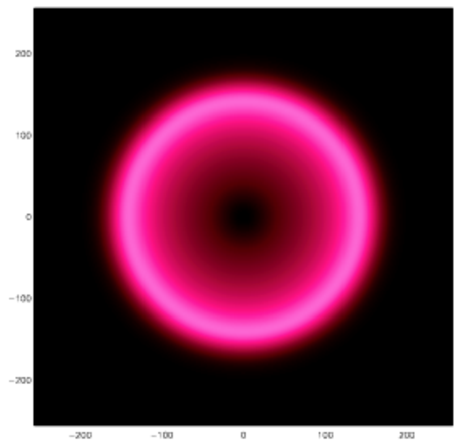
This expansion continues for 10^5 years

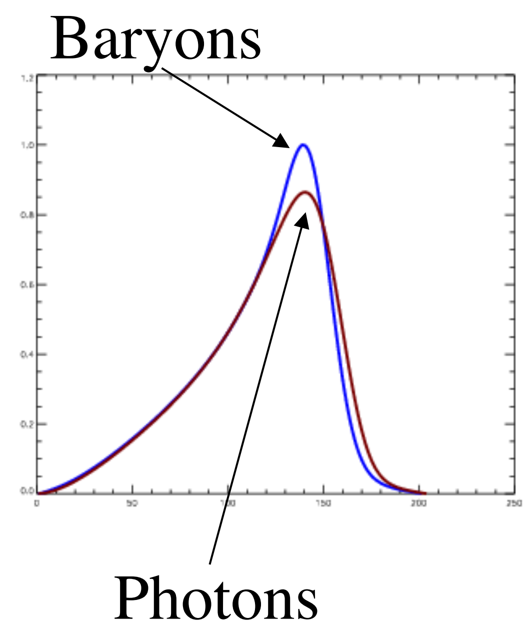


The acoustic wave

After 10^5 years the universe has cooled enough the protons capture the electrons to form neutral Hydrogen. This decouples the photons from the baryons. The former quickly stream away, leaving the baryon peak stalled.

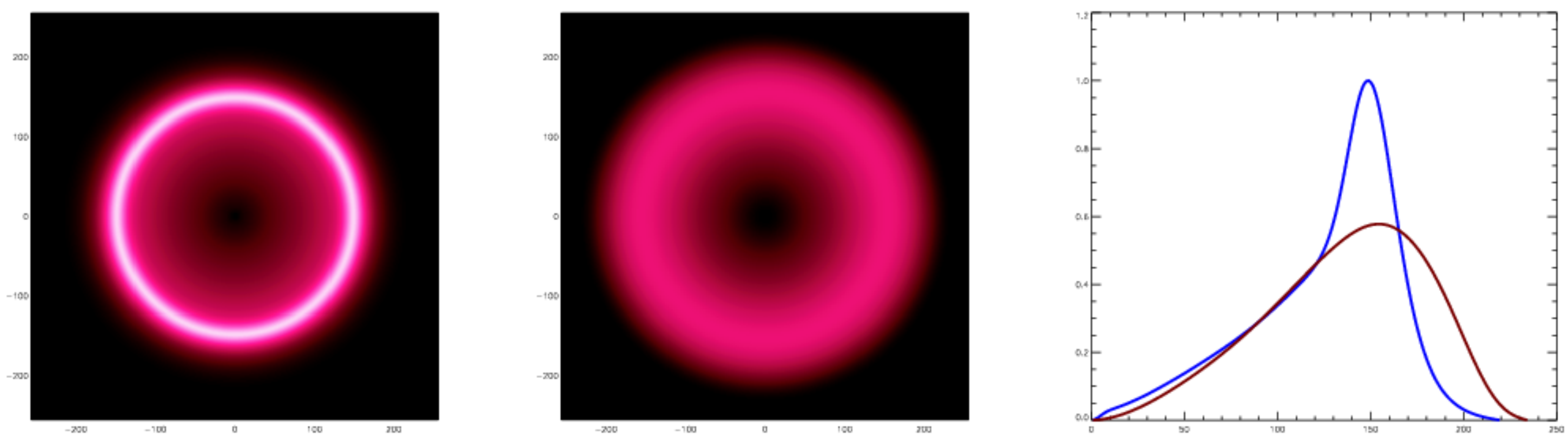




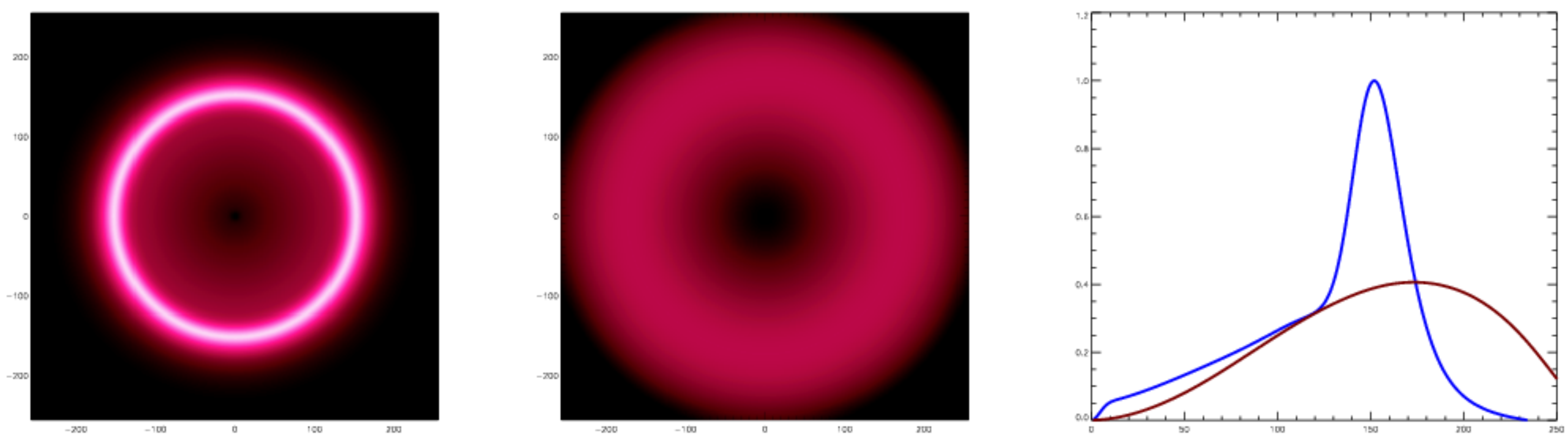


The acoustic wave

The photons continue to stream away while the baryons, having lost their motive pressure, remain in place.



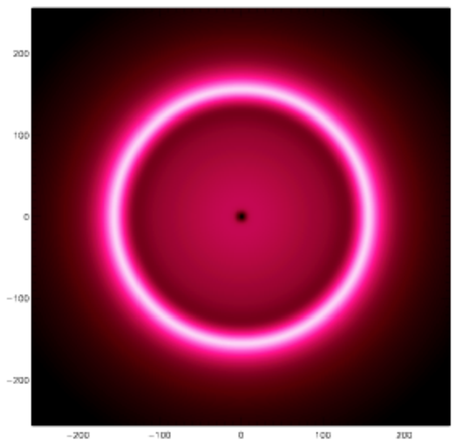
The acoustic wave

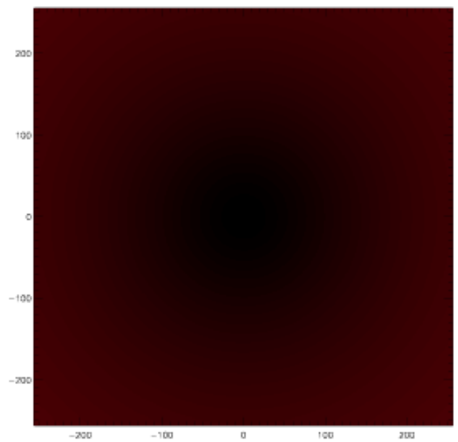


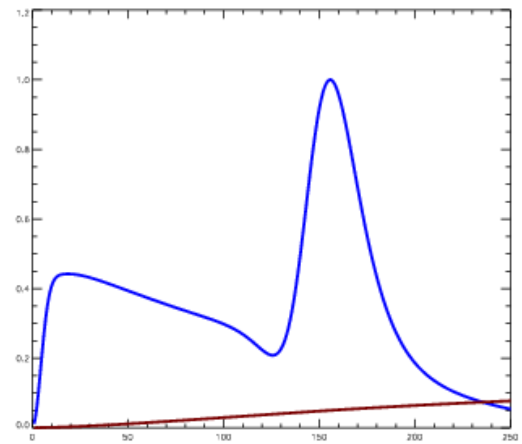
The acoustic wave

The photons have become almost completely uniform, but the baryons remain overdense in a shell 100Mpc in radius.

In addition, the large gravitational potential well which we started with starts to draw material back into it.



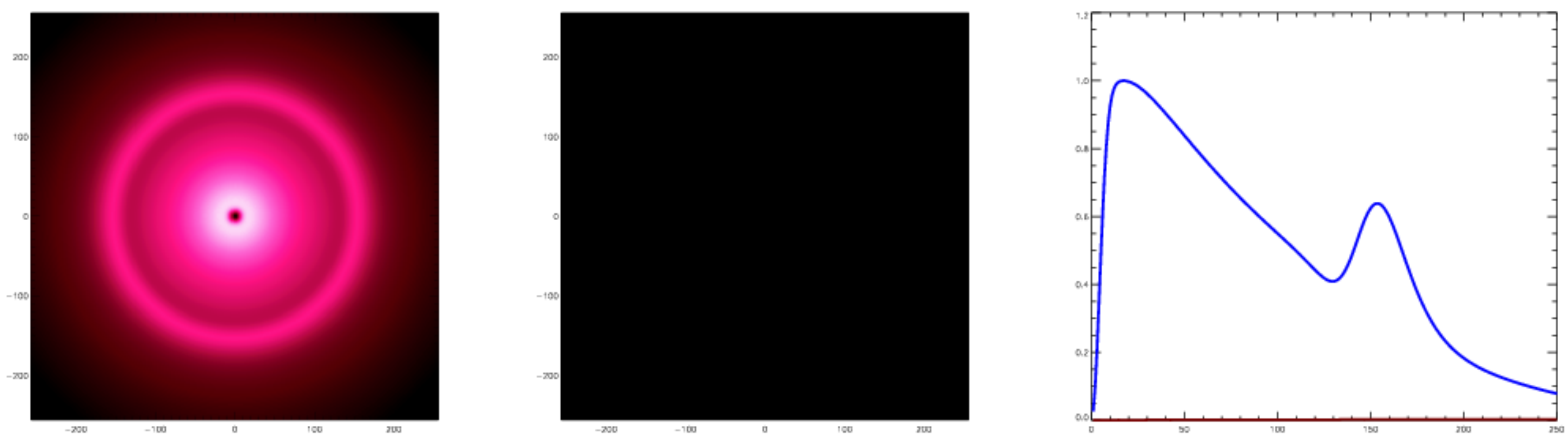




The acoustic wave

As the perturbation grows by $\sim 10^3$ the baryons and DM reach equilibrium densities in the ratio Ω_b/Ω_m.

The final configuration is our original peak at the center (which we put in by hand) and an “echo” in a shell roughly 100Mpc in radius.

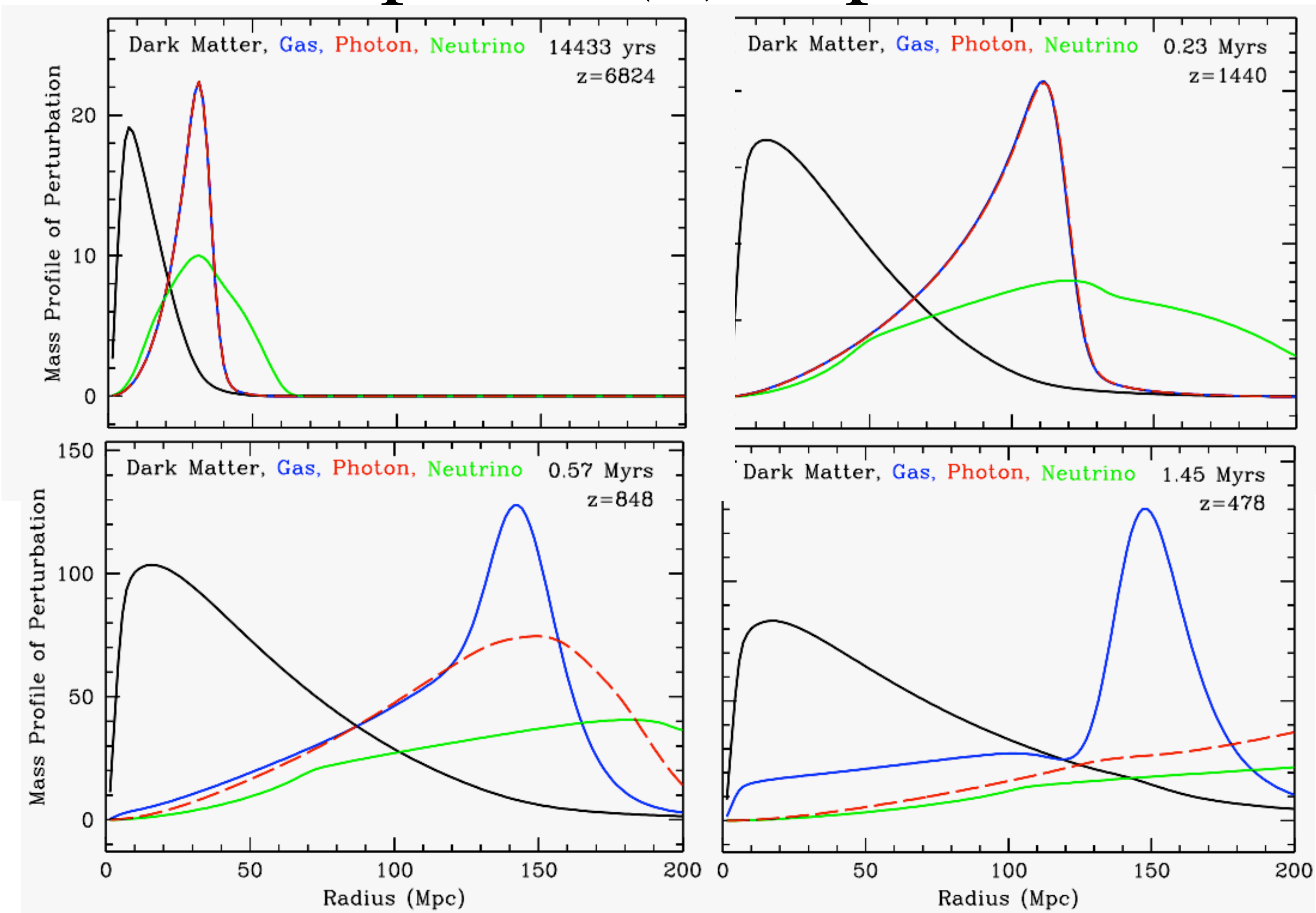


Further (non-linear) processing of the density field acts to broaden and very slightly shift the peak -- but galaxy formation is a local phenomenon with a length scale ~10Mpc, so the action at r=0 and $r\sim$100Mpc are essentially decoupled. We will return to this …

Aside:broad-band shape of *P(k)*

- This picture also allows us a new way of seeing why the DM power spectrum has a "peak" at the scale of M-R equality.
- Initially our DM distribution is a δ-function.
- As the baryon-photon shell moves outwards during radiation domination, its gravity "drags" the DM, causing it to spread.
- The spreading stops once the energy in the photon-baryon shell no longer dominates: after M-R equality.
- The spreading of the δ-function $\rho(r)$ is a smoothing, or suppression of high-*k* power.

Shape of *P(k)* in pictures



Eisenstein, Seo & White (2007)

Features of baryon oscillations

- Firm prediction of models with $\Omega_b>0$
- Positions well predicted once (physical) matter and baryon density known - calibrated by the CMB.
- Oscillations are "sharp", unlike other features of the power spectrum.
- Internal cross-check:
  - d_A should be the integral of $H^{-1}(z)$.
- Since have $d(z)$ for several z's can check spatial flatness (addition law for distances).
- Ties low-z distance measures (e.g. SNe) to absolute scale defined by the CMB (in Mpc, not h^{-1}Mpc).
  - Allows ~1% measurement of h using trigonometry!

Those pesky details …

- I have argued (convincingly?) that we understand and can calculate the real space, linear theory, matter power spectrum with exquisite accuracy and that it contains highly useful features for cosmology.
- Unfortunately we don't measure the linear theory matter power spectrum in real space.
- We measure:
  - the non-linear
  - galaxy power spectrum
  - in redshift space
- How do we handle this?

Recent BAO "theory"

With the basic measurement demonstrated/validated, theoretical attention has been divided into five areas

1. Constraints at $z \sim 10^3$.
2. Understanding the effects of non-linearity, bias & redshift space distortions.
3. Understanding how to perform "reconstruction".
4. Studying BAO in the IGM.
5. Looking at statistical estimators, covariance matrices, etc.

DE or early universe weirdness?

- Key to computing ***s*** is our ability to model CMB anisotropies.
- Want to be sure that we don't mistake an error in our understanding of $z \sim 10^3$ for a property of the DE!
- What could go wrong in the early universe?
  - Recombination.
  - Misestimating c_s or ρ_B/ρ_γ.
  - Misestimating H(z>>1) (e.g. missing radiation).
  - Strange thermal history (e.g. decaying ν).
  - Isocurvature perturbations.
  - …
- It seems that future measurements of CMB anisotropies (e.g. with Planck) constrain ***s*** well enough for this measurement even in the presence of odd high-*z* physics.

Eisenstein & White (2004); White (2006)

Effects of non-linearity: mass

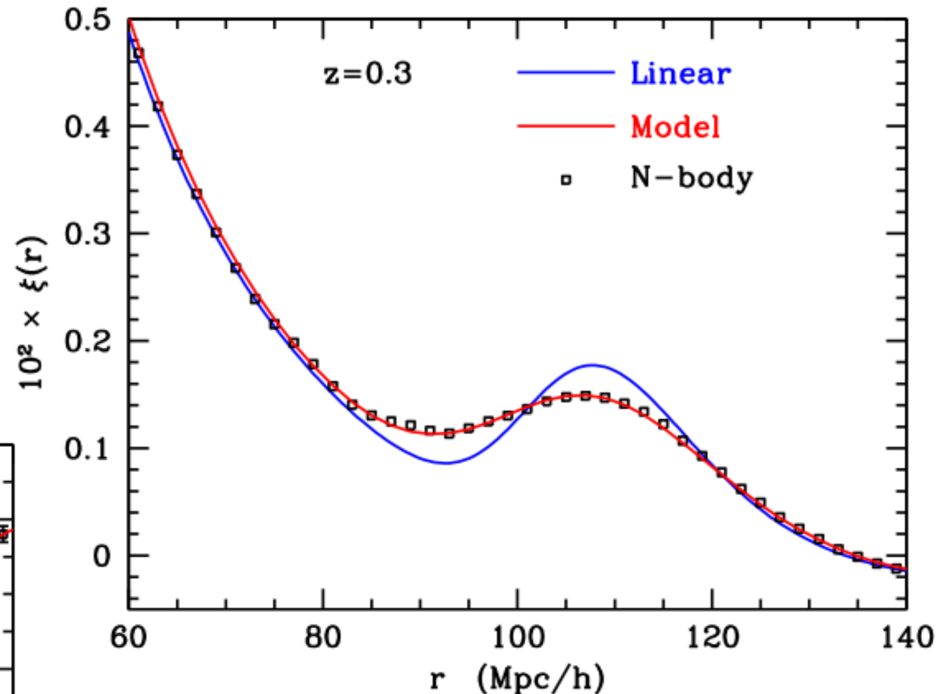
As large-scale structure grows, neighboring objects "pull" on the baryon shell around any point. This causes a broadening of the peak and additional non-linear power on small scales. From simulations or PT (of various flavors) one finds:

$$\Delta^2(k) = \left\{\Delta^2_{\rm lin}(k) + \cdots\right\} \exp\left[-k^2\Sigma^2/2\right] + \Delta^2_{22} + \cdots$$

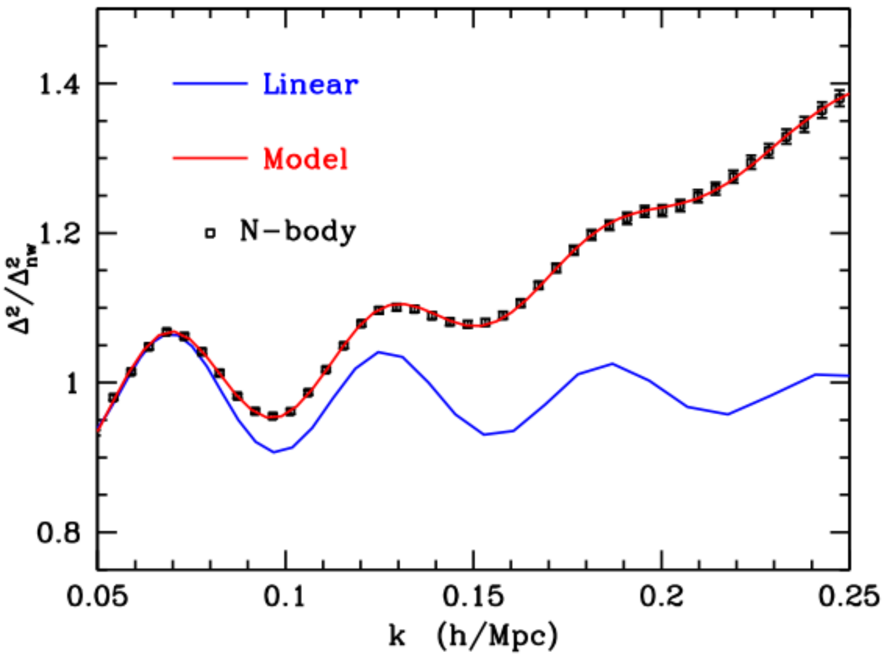
This does a reasonable job of providing a "template" low-z spectrum, and it allows us to understand where the information lives in Fourier space [forecasting].

Bharadwaj (1996); Eisenstein, Seo & White (2007); Smith, Scoccimarro & Sheth (2007); Eisenstein et al. (2007); Matsubara (2007); Padmanabhan, White & Cohn (2009); Padmanabhan & White (2009); Seo et al. (2009); Noh et al. (2009); Mehta et al. (2010); …

Non-linearities smear the peak



Loss of contrast and excess power from non-linear collapse.



Broadening of feature due to Gaussian smoothing and ~0.5% shift due to mode coupling.

Information on the acoustic scale

- For a Gaussian random field Var[x^2]=2Var[x]^2, so our power spectrum errors are go as the square of the (total) power measured.
  - Measured power is *P+1/n*
- For a simple 1D model

(Seo & Eisenstein 2006)

$$\sigma_{\ln s}^{-2} = \frac{V}{2} \int \frac{d^3k}{(2\pi)^3} \left(\frac{\partial P / \partial \ln s}{P + \bar{n}^{-1}} \right)^2$$

- Note that *δP/δlns* depends only on the wiggles while *P+1/n* depends on the whole spectrum.
- The wiggles are (exponentially) damped at high *k*.
- A more complete treatment keeps the angle-dependence due to redshift space distortions.
  - Such Fisher forecasts agree well with the results of numerical simulations.

Lagrangian perturbation theory

- A different approach to PT, which has been radically developed recently by Matsubara and is *very* useful for BAO.
  - Buchert89, Moutarde++91, Bouchet++92, Catelan95, Hivon++95.
  - Matsubara (2008a; PRD, 77, 063530)
  - Matsubara (2008b; PRD, 78, 083519)
- Relates the current (Eulerian) position of a mass element, **x**, to its initial (Lagrangian) position, **q**, through a displacement vector field, Ψ.
  - Note **q** is a position, not a wave-vector!

Lagrangian perturbation theory

$$
\begin{aligned}
\delta(\mathbf{x}) &= \int d^3q \; \delta_D(\mathbf{x}-\mathbf{q}-\mathbf{\Psi}) - 1 \\
\delta(\mathbf{k}) &= \int d^3q \; e^{-i\mathbf{k}\cdot\mathbf{q}} \left(e^{-i\mathbf{k}\cdot\mathbf{\Psi}(\mathbf{q})} - 1 \right) \; .
\end{aligned}
$$

$$
\frac{d^2\mathbf{\Psi}}{dt^2} + 2H\frac{d\mathbf{\Psi}}{dt} = -\nabla_x \phi \left[\mathbf{q} + \mathbf{\Psi}(\mathbf{q}) \right]
$$

$$
\begin{aligned}
\mathbf{\Psi}^{(n)}(\mathbf{k}) &= \frac{i}{n!} \int \prod_{i=1}^{n} \left[\frac{d^3k_i}{(2\pi)^3} \right] (2\pi)^3 \delta_D \left(\sum_i \mathbf{k}_i - \mathbf{k} \right) \\
&\times \; \mathbf{L}^{(n)}(\mathbf{k}_1, \cdots, \mathbf{k}_n, \mathbf{k}) \delta_0(\mathbf{k}_1) \cdots \delta_0(\mathbf{k}_n)
\end{aligned}
$$

Kernels

$$\mathbf{L}^{(1)}(\mathbf{p}_1) = \frac{\mathbf{k}}{k^2} \tag{1}$$

$$\mathbf{L}^{(2)}(\mathbf{p}_1, \mathbf{p}_2) = \frac{3}{7}\frac{\mathbf{k}}{k^2}\left[1 - \left(\frac{\mathbf{p}_1 \cdot \mathbf{p}_2}{p_1 p_2}\right)^2\right] \tag{2}$$

$$\mathbf{L}^{(3)}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = \cdots \tag{3}$$

$$\mathbf{k} \equiv \mathbf{p}_1 + \cdots + \mathbf{p}_n$$

Standard LPT

- If we expand the exponential and keep terms consistently in δ_0 we regain a series $\delta=\delta^{(1)}+\delta^{(2)}+\ldots$ where $\delta^{(1)}$ is linear theory and e.g.

$$\begin{aligned}\delta^{(2)}(\mathbf{k}) &= \frac{1}{2}\int \frac{d^3k_1 d^3k_2}{(2\pi)^3}\delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\delta_0(\mathbf{k}_1)\delta_0(\mathbf{k}_2) \\ &\times \left[\mathbf{k}\cdot\mathbf{L}^{(2)}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k})+\mathbf{k}\cdot\mathbf{L}^{(1)}(\mathbf{k}_1)\mathbf{k}\cdot\mathbf{L}^{(1)}(\mathbf{k}_2)\right]\end{aligned}$$

- which regains “SPT”.
  - The quantity in square brackets is F_2.

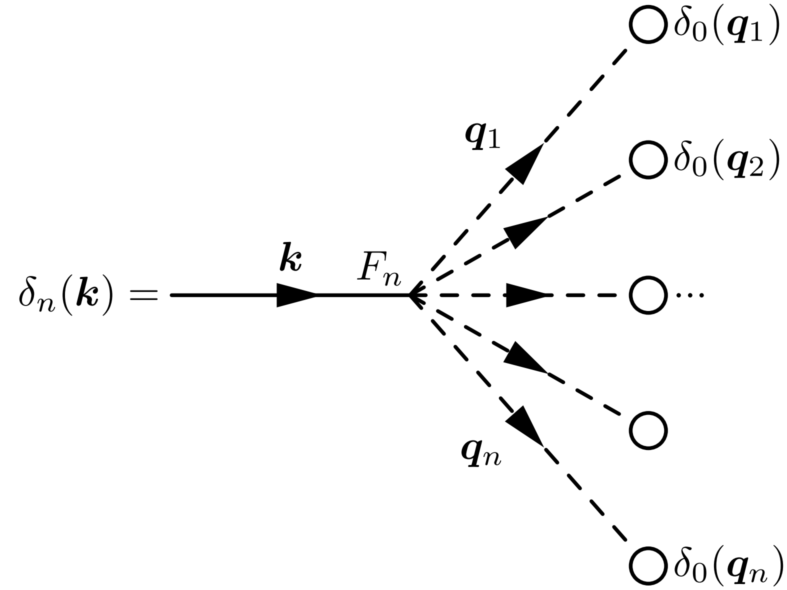
$$F_2(\mathbf{k}_1,\mathbf{k}_2)=\frac{5}{7}+\frac{2}{7}\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)^2}{k_1^2k_2^2}+\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)}{2}\left(k_1^{-2}+k_2^{-2}\right)$$

Power spectrum

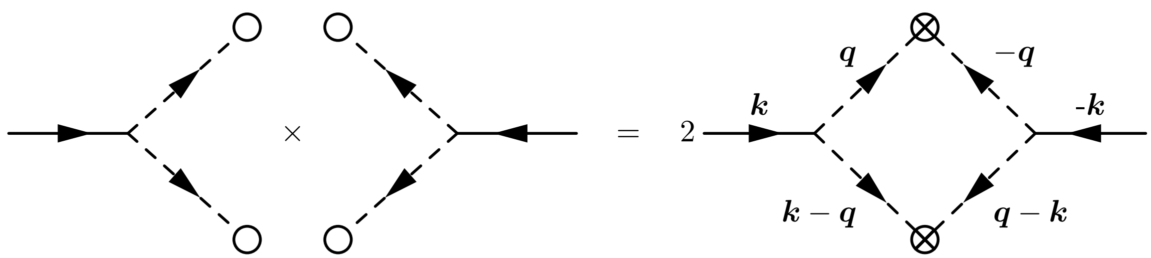
- If the initial fluctuations are Gaussian only expectation values even in δ_0 survive:
  - $P(k) \sim \langle[\delta^{(1)}+\delta^{(2)}+\delta^{(3)}+\ldots][\delta^{(1)}+\delta^{(2)}+\delta^{(3)}+\ldots]\rangle$
  - $\quad = P^{(1,1)} + 2P^{(1,3)} + P^{(2,2)} + \cdots$
- with terms like $\langle\delta^{(1)}\delta^{(2)}\rangle$ vanishing because they reduce to $\langle\delta_0\delta_0\delta_0\rangle$.

Perturbation theory: diagrams

Just as there is a diagrammatic short-hand for perturbation theory in quantum field theory, so there is in cosmology:



$$\equiv \quad (2\pi)^3 \delta_D(\boldsymbol{q}+\boldsymbol{q}')P_0(q),$$



$$= 2\int \frac{d^3q}{(2\pi)^3} F_2(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q}) F_2(-\boldsymbol{q}, \boldsymbol{q}-\boldsymbol{k}) P_0(q) P_0(|\boldsymbol{k}-\boldsymbol{q}|)$$

Example: 2nd order

$$
\begin{aligned}
P^{(1,3)}(k) &= \frac{1}{252}\frac{k^3}{4\pi^2}P_L(k)\int_0^\infty dr\ P_L(kr)\left[\frac{12}{r^2}-158+100r^2-42r^4\right.\\
&\left.+\frac{3}{r^2}(r^2-1)^3(7r^2+2)\ln\left|\frac{1+r}{1-r}\right|\right],
\end{aligned}
$$

$$
\begin{aligned}
P^{(2,2)}(k) &= \frac{1}{98}\frac{k^3}{4\pi^2}\int_0^\infty dr\ P_L(kr)\int_{-1}^{1} dx\ P_L\left(k\sqrt{1+r^2-2rx}\right)\\
&\times \frac{(3r+7x-10rx^2)^2}{(1+r^2-2rx)^2}.
\end{aligned}
$$

Perturbation theory enables the generation of truly impressive looking equations which arise from simple angle integrals. Like Feynman integrals, they are simple but look erudite!

LPT power spectrum

- Alternatively we can use the expression for $\delta_{\mathbf{k}}$ to write

$$P(k) = \int d^3q\ e^{-i\vec{k}\cdot\vec{q}} \left(\left\langle e^{-i\vec{k}\cdot\Delta\vec{\Psi}} \right\rangle - 1 \right)$$

- where $\Delta\Psi=\Psi(\mathbf{q})-\Psi(0)$.
- Expanding the exponential and plugging in for $\Psi^{(n)}$ gives the usual results.
- **BUT** Matsubara suggested a different and very clever approach.

Cumulants

- The cumulant expansion theorem allows us to write the expectation value of the exponential in terms of the exponential of expectation values.
- Expand the terms $(\mathbf{k}\Delta\boldsymbol{\Psi})^N$ using the binomial theorem.
- There are two types of terms:
  - Those depending on Ψ at same point.
    - This is independent of position and can be factored out of the integral.
  - Those depending on Ψ at different points.
    - These can be expanded as in the usual treatment.

Example

- Imagine Ψ is Gaussian with mean zero.
- For such a Gaussian: $\langle e^X \rangle = \exp[\sigma^2/2]$.

$$P(k) = \int d^3q\, e^{-i\mathbf{k}\cdot\mathbf{q}} \left(\left\langle e^{-ik_i \Delta\Psi_i(\mathbf{q})} \right\rangle - 1 \right)$$

$$\left\langle e^{-i\mathbf{k}\cdot\Delta\boldsymbol{\Psi}(q)} \right\rangle = \exp\left[-\frac{1}{2} k_i k_j \left\langle \Delta\Psi_i(\mathbf{q}) \Delta\Psi_j(\mathbf{q}) \right\rangle \right]$$

$$k_i k_j \left\langle \Delta\Psi_i(\mathbf{q}) \Delta\Psi_j(\mathbf{q}) \right\rangle = 2k_i^2 \langle \Psi_i^2(\mathbf{0}) \rangle - 2k_i k_j \xi_{ij}(\mathbf{q})$$

Keep exponentiated.

Expand

Resummed LPT

- The first corrections to the power spectrum are then:

$$P(k) = e^{-(k\Sigma)^2/2} \left[P_L(k) + P^{(2,2)}(k) + \widetilde{P}^{(1,3)}(k) \right] ,$$

- where $P^{(2,2)}$ is as in SPT but part of $P^{(1,3)}$ has been "resummed" into the exponential prefactor.

- The exponential prefactor is identical to that obtained from
  - The peak-background split (Eisenstein++07)
  - Renormalized Perturbation Theory (Crocce++08).
- Non-linearities, or mode coupling, erase the acoustic signature (Meiksin, White & Peacock 1999).
  - Fewer k-modes to measure.
  - Peak is "broadened" making it harder to centroid.
  - Much of the contribution to Σ comes from low k!

Beyond real-space mass

- One of the more impressive features of Matsubara's approach is that it can gracefully handle both biased tracers and redshift space distortions.
- In redshift space $\mathbf{\Psi} \to \mathbf{\Psi} + \frac{\widehat{\mathbf{z}} \cdot \dot{\mathbf{\Psi}}}{H} \widehat{\mathbf{z}}$
- For bias local in Lagrangian space:

$$\delta_{\rm obj}(\mathbf{x}) = \int d^3q \ F\left[\delta_L(\mathbf{q})\right] \delta_D(\mathbf{x} - \mathbf{q} - \mathbf{\Psi})$$

- we obtain

$$P(k) \quad = \quad \int d^3q \ e^{-i\mathbf{k}\cdot\mathbf{q}} \left[\int \frac{d\lambda_1}{2\pi} \frac{d\lambda_2}{2\pi} F(\lambda_1) F(\lambda_2) \left\langle e^{i[\lambda_1 \delta_L(\mathbf{q}_1) + \lambda_2 \delta_L(\mathbf{q}_2)] + i\mathbf{k}\cdot\Delta\mathbf{\Psi}} \right\rangle - 1 \right]$$

- which can be massaged with the same tricks as we used for the mass.
- If we assume halos/galaxies form at peaks of the initial density field ("peaks bias") then explicit expressions for the integrals of F exist.

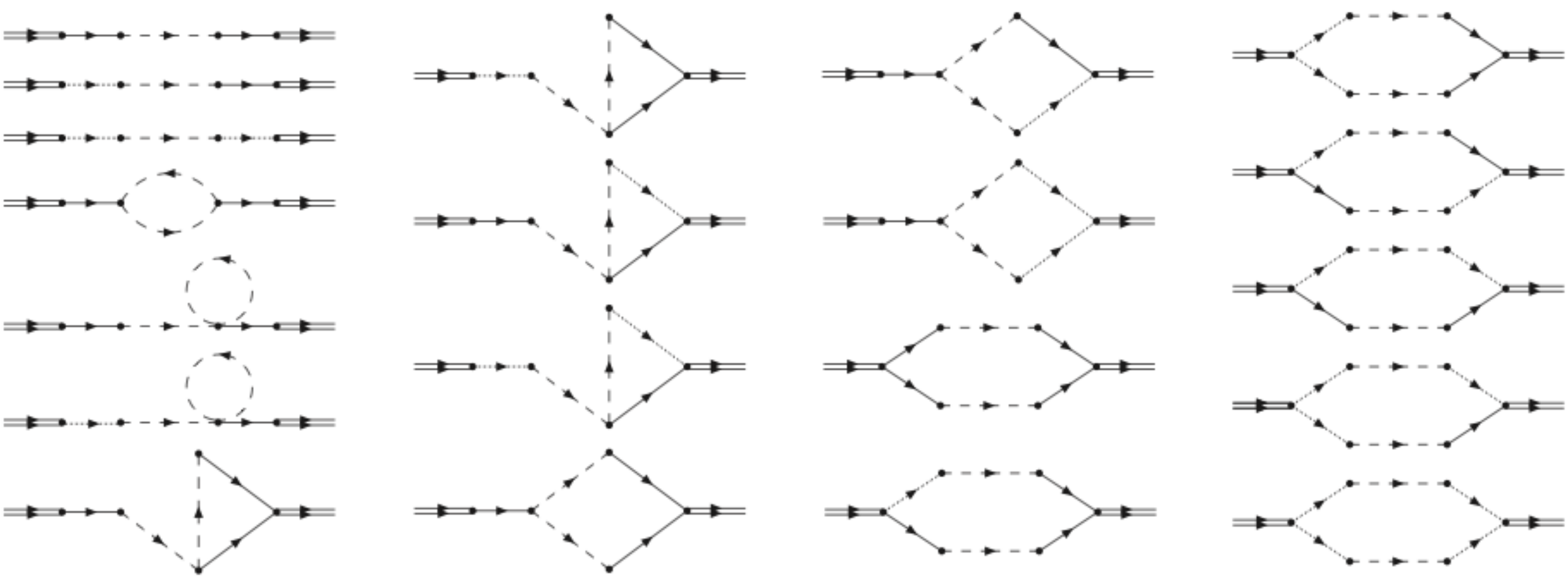
The answer

$$P^{(s)}_{\rm obj} = e^{-\left[1+f(f+2)\mu^2\right]k^2\Sigma^2/2}\left[\left(b+f\mu^2\right)^2 P_L + \sum_{n,m}\mu^{2n}f^m E_{nm}\right]$$

Note angle dependence of damping.

Zel'dovich damping

Mode coupling terms up to E_{44}. These terms involve b_1 and b_2.



Effects of non-linearity on BAO

- Non-linear evolution has 3 effects on the power spectrum:
  - It generates “excess” high *k* power, reducing the contrast of the wiggles.
  - It damps the oscillations.
  - It generates an out-of-phase component.
- In configuration space:
  - Generates “excess” small-scale power.
  - Broadens the peak.
  - Shifts the peak.

Understanding higher order

- We want to fit for the position of the acoustic feature while allowing for variations in the broadband shape (due e.g. to biasing).
  - $P_{fit}(k) = B(k)\, P_w(k,\alpha) + A(k)$
  - *B(k)* and *A(k)* are smooth functions.
    - Can take B(k)=const and A(k) as a spline, polynomial, Pade, ...
  - α measures shift relative to "fiducial" cosmology.
  - $P_w(k,\alpha)$ is a template.
    - Numerous arguments suggest $P_w(k,\alpha)=\exp[-k^2\Sigma^2/2]P_L(k/\alpha)$.
    - Take Σ *to be a free parameter, perhaps with a prior.*
- How does this do?

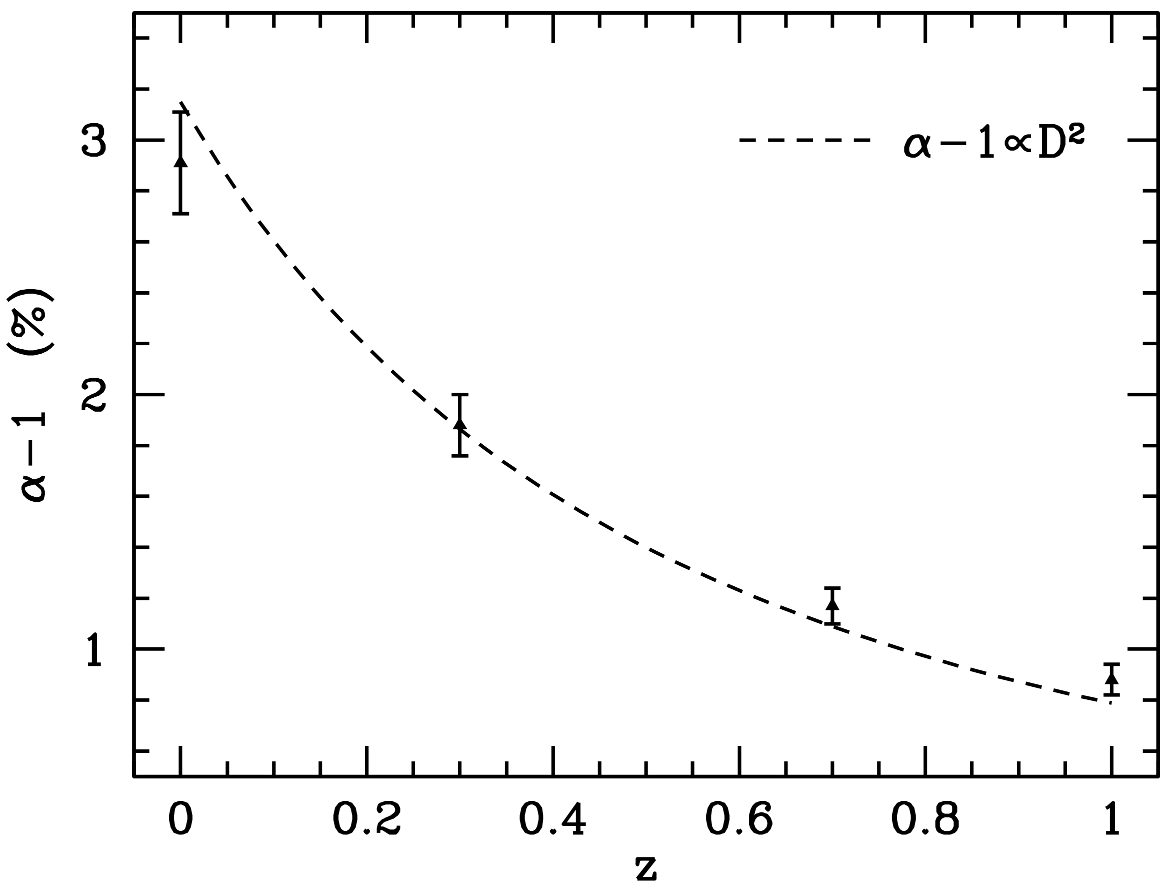
Argument from
Padmanabhan & White (2009)

Measuring shifts in cCDM

- Any "shift" in the acoustic scale is small in ΛCDM, and therefore hard to study.
- Work with a "crazy" cosmology
  - Ω_m=1, Ω_B=0.4, h=0.5, n=1, σ_8=1.
  - Sound horizon 50h^{-1}Mpc, not 100h^{-1}Mpc.
- The fitted shifts are (α-1 in percent):

z	DM	xδ_L	w/P_{22}
0.0	2.91 ± 0.20	-0.2 ±0.1	-0.03 ± 0.16
0.3	1.88 ± 0.12	-0.2 ±0.1	-0.38 ± 0.09
0.7	1.17 ± 0.07	-0.1 ±0.1	-0.12 ± 0.05
1.0	0.88 ± 0.06	-0.1 ±0.1	-0.04 ± 0.04

Shifts vs time



Amplitude of the shift vs. time (redshift) for the mass.

Shifts are consistent with D^2 scaling (dotted) suggesting an origin from 2^{nd} order terms …

Where do the shifts come from?

Recall in PT we can write $\delta=\delta^{(1)}+\delta^{(2)}+\ldots$ or
$P = \{P_{11} + P_{13} + P_{15} + \ldots\} + \{P_{22} + \ldots\} = P_{1n} + P_{mn}$.
We can isolate these two types of terms by considering the cross-spectrum of the final with the initial field, which doesn't contain P_{mn}.

z	DM	$x\delta_L$	w/P_{22}
0.0	2.91 ± 0.20	-0.2 ±0.1	-0.03 ± 0.16
0.3	1.88 ± 0.12	-0.2 ±0.1	-0.38 ± 0.09
0.7	1.17 ± 0.07	-0.1 ±0.1	-0.12 ± 0.05
1.0	0.88 ± 0.06	-0.1 ±0.1	-0.04 ± 0.04

Shifts in the cross-spectrum are an order of magnitude smaller than shifts in the auto-spectrum!

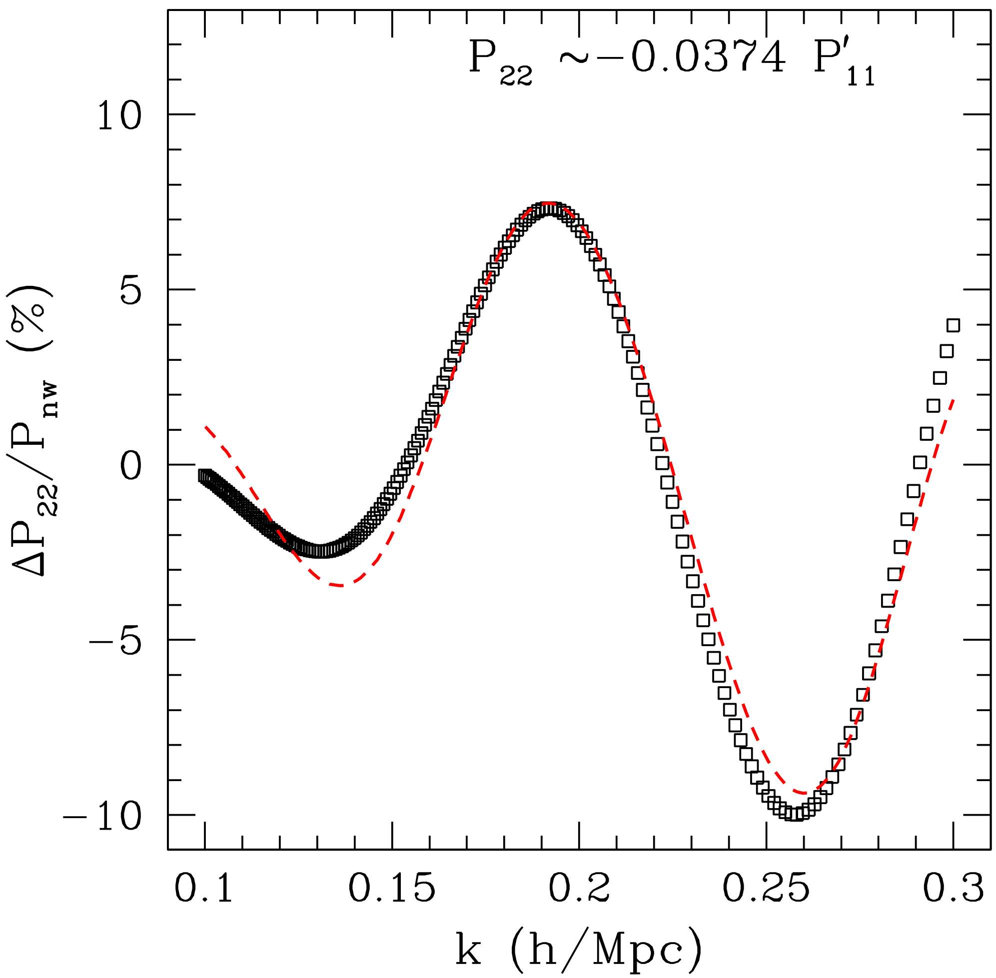
$$P_{1n}(k) \sim P_L(k) \int \prod_k \left[d^3 q_k P_L(q_k)\right] F_n(\cdots)$$

Broad kernel suppresses oscillations.

Mode-coupling

- By contrast the P_{mn} terms involve integrals of products of P_Ls times peaked kernels.
- Example: $P_{22} \sim \int P_L P_L\ F_2$ and F_2 is sharply peaked around $k_1 \approx k_2 \approx k/2$.
- Thus the $\int P_L P_L$ term contains an out-of-phase oscillation
  – $P_L \sim \ldots + \sin(kr)$: $P_L P_L F_2 \sim \sin^2(kr/2) \sim 1+\cos(kr)$
- Since cos(x)~d/dx sin(x) this gives a "shift" in the peak
  – $P(k/\alpha) \sim P(k) - (\alpha-1)\ dP/d\ln k + \ldots$

Mode-coupling approximates derivative



Up to an overall factor the mode-coupling term, P_{22}, is well approximated by $dP_L/d\ln k$.

Modified template

- This discussion suggests a modified template, which has just as many free parameters as our old template:

$$
\begin{aligned}
P_{\rm w}(k,\alpha) &= \exp\left(-\frac{k^2\Sigma^2}{2}\right) P_L(k/\alpha) \\
&+ \exp\left(-\frac{k^2\Sigma_1^2}{2}\right) P_{22}(k/\alpha)\,.
\end{aligned}
$$

- This removes most of the shift.

z	DM	$x\delta_L$	w/P_{22}
0.0	2.91 ± 0.20	-0.2 ±0.1	-0.03 ± 0.16
0.3	1.88 ± 0.12	-0.2 ±0.1	-0.38 ± 0.09
0.7	1.17 ± 0.07	-0.1 ±0.1	-0.12 ± 0.05
1.0	0.88 ± 0.06	-0.1 ±0.1	-0.04 ± 0.04

Biased tracers?

- In order to remove the shift we needed to know the relative amplitude of P_{11} and P_{22}.
- What do we do for biased tracers?
  - Eulerian bias

$$P_h = \left(b_1^E\right)^2 (P_{11} + P_{22}) + b_1^E b_2^E \left(\frac{3}{7} Q_8 + Q_9\right) + \frac{(b_2^E)^2}{2} Q_{13} + \cdots$$

  - Lagrangian bias

$$\begin{aligned} P_h &= \exp\left[-\frac{k^2\Sigma^2}{2}\right] \left\{ \left(1 + b_1^L\right)^2 P_{11} + P_{22} + b_1^L \left[\frac{6}{7} Q_5 + 2Q_7\right] + b_2^L \left[\frac{3}{7} Q_8 + Q_9\right] \right. \\ &+ \left. \left(b_1^L\right)^2 \left[Q_9 + Q_{11}\right] + 2 b_1^L b_2^L Q_{12} + \frac{1}{2} \left(b_2^L\right)^2 Q_{13} \right\} + \cdots \end{aligned}$$

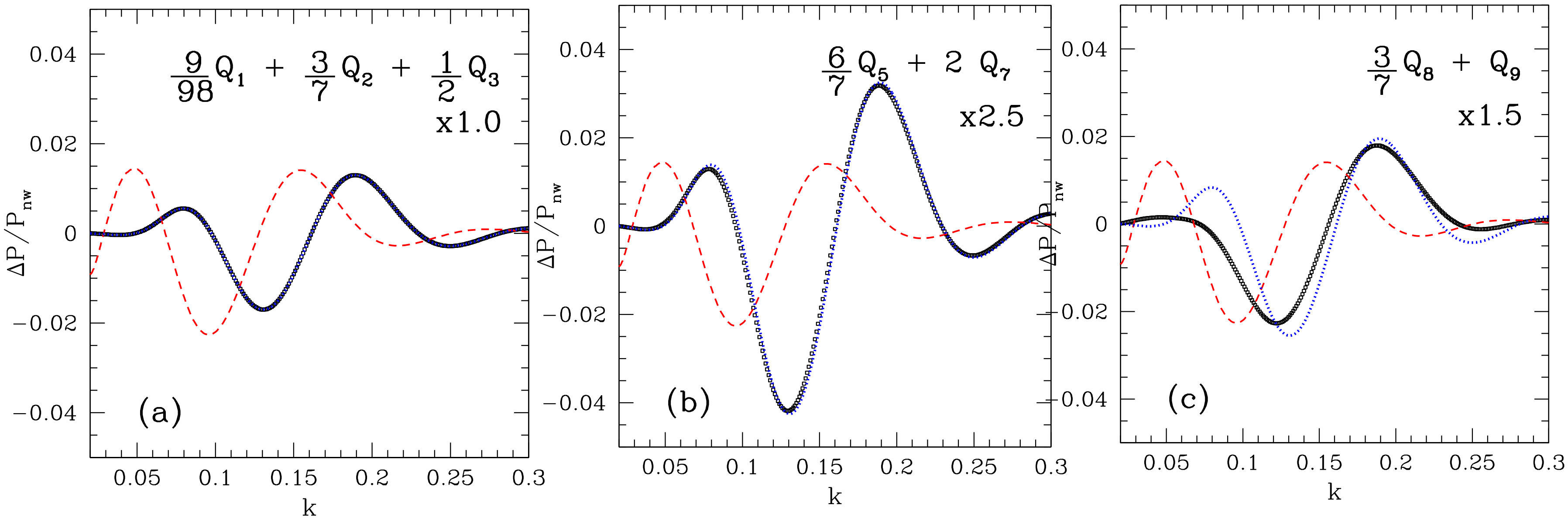
Mode-coupling integrals

$$Q_n(k) = \frac{k^3}{4\pi^2} \int_0^\infty dr\, P_L(kr) \int_{-1}^1 dx\, P_L(k\sqrt{1+r^2-2rx})\widetilde{Q}_n(r,x)$$

$$\begin{aligned}
\widetilde{Q}_1 &= \frac{r^2(1-x^2)^2}{y^2}, \quad \widetilde{Q}_2 = \frac{(1-x^2)rx(1-rx)}{y^2}, \\
\widetilde{Q}_3 &= \frac{x^2(1-rx)^2}{y^2}, \quad \widetilde{Q}_4 = \frac{1-x^2}{y^2}, \\
\widetilde{Q}_5 &= \frac{rx(1-x^2)}{y}, \quad \widetilde{Q}_6 = \frac{(1-3rx)(1-x^2)}{y}, \\
\widetilde{Q}_7 &= \frac{x^2(1-rx)}{y}, \quad \widetilde{Q}_8 = \frac{r^2(1-x^2)}{y}, \\
\widetilde{Q}_9 &= \frac{rx(1-rx)}{y}, \quad \widetilde{Q}_{10} = 1-x^2, \\
\widetilde{Q}_{11} &= x^2,\ \widetilde{Q}_{12} = rx,\ \widetilde{Q}_{13} = r^2
\end{aligned}$$

(Matsubara 2008)

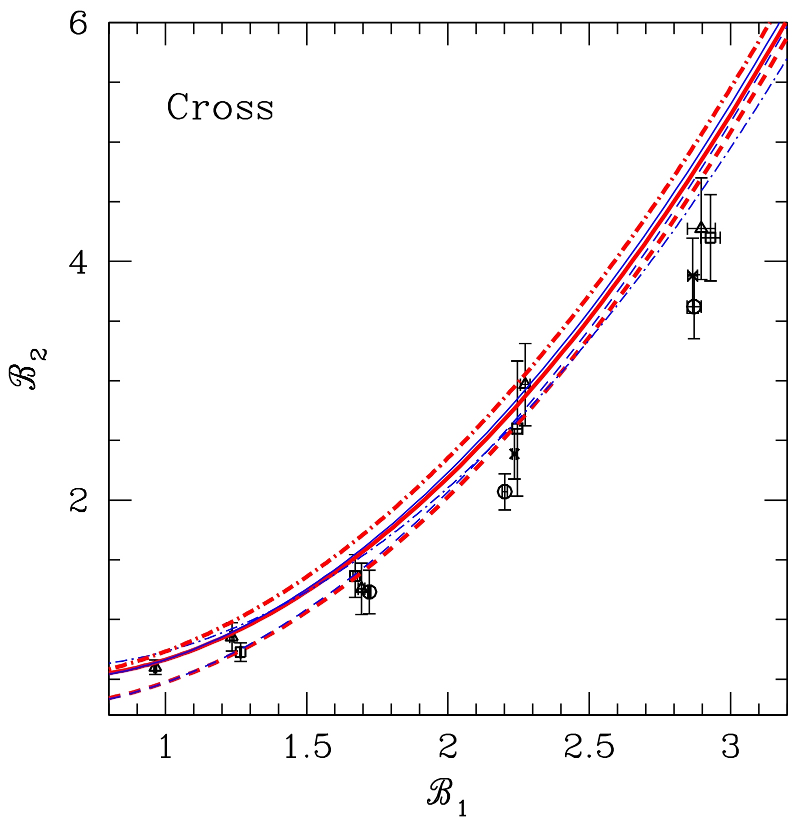
Out-of-phase?

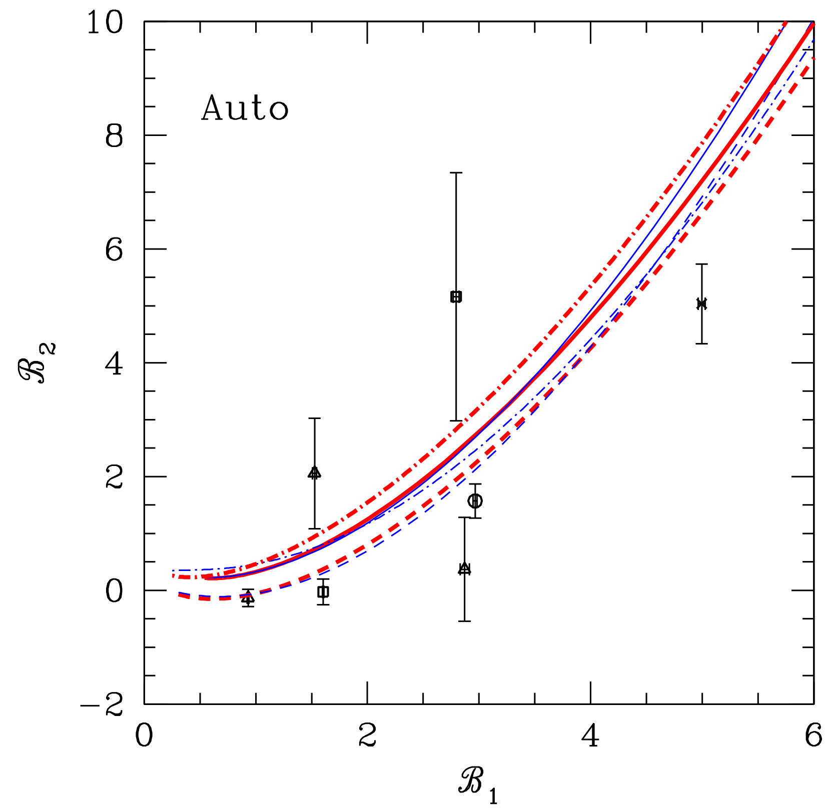


The numerous combinations that come in are also well approximated by the (log-)derivative of P_{11}! All of these terms can be effectively written as:

$$P_h = \exp\left(-\frac{k^2\Sigma^2}{2}\right)\left[\mathcal{B}_1 P_L + \mathcal{B}_2 P_{22}\right].$$

Models do lie on a narrow line



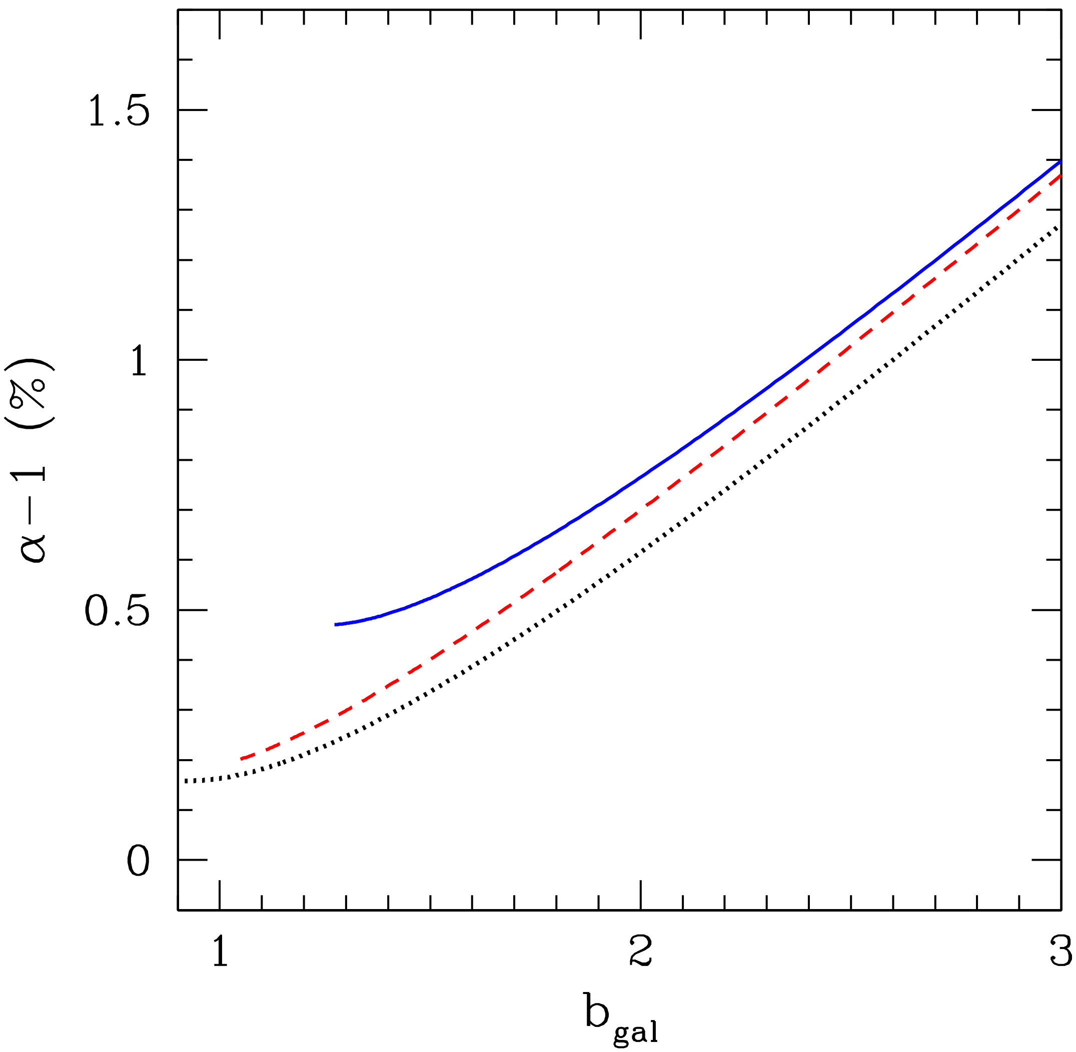


$$P_{\rm w}(k,\alpha) = b_1 \left[\exp\left(-\frac{k^2\Sigma^2}{2}\right) P_L(k/\alpha) + \exp\left(-\frac{k^2\Sigma_1^2}{2}\right) \frac{\mathcal{B}_2}{\mathcal{B}_1} P_{22}(k/\alpha) \right]$$

Implications for ΛCDM?

- Shifts caused by P_{22}, well approximated by $dP_L/d\ln k$.
  - True also for ΛCDM, same scaling coeff.
- Additional shifts for biased tracers approximate $dP_L/d\ln k$.
  - True also for ΛCDM, same scaling coeff.
- Simple model explains B_1-B_2 relation.
  - True also for ΛCDM.
  - Can also be measured from simulations.
- For ΛCDM the shifts are an order of magnitude smaller than for cCDM.
  - α-1~0.5% x D^2 x B_2/B_1

Shifts for galaxies



Shifts at z=0 for

Halos of mass M
Halos above M
N~[1+M/M_1]

At higher z the shift decreases as D^2.

Recall, the final error in BAO scale is the *uncertainty* in this correction, not the size of the correction itself!

Redshift space

- In resummed LPT we can also consider the redshift space power spectrum for biased tracers.
- For the isotropic P(k) find a similar story though now the scaling coefficients depend on $f \sim d\ln D/d\ln a$.
  - Expressions become more complex, but the structure is unchanged.
- The amplitude of the shift increases slightly.

Perturbation theory & BAO

- Meiksin, White & Peacock, 1999
  - Baryonic signatures in large-scale structure
- Crocce & Scoccimarro, 2007
  - Nonlinear Evolution of Baryon Acoustic Oscillations
- Nishimichi et al., 2007
  - Characteristic scales of BAO from perturbation theory
- Matsubara, 2008ab
- Jeong & Komatsu, 2007, 2008
  - Perturbation theory reloaded I & II
- Pietroni, 2008
  - Flowing with time
- Padmanabhan et al., 2009; Noh et al. 2009
  - Reconstructing baryon oscillations: A Lagrangian theory perspective
  - Reconstructing baryon oscillations.
- Taruya et al., 2009
  - Non-linear Evolution of Baryon Acoustic Oscillations from Improved Perturbation Theory in Real and Redshift Spaces

Galaxy bias

- The hardest issue is galaxy bias.
  - Galaxies don't faithfully trace the mass
- ... but galaxy formation "scale" is << 100Mpc so effects are "smooth".
  - In *P*(*k*) effect of bias can be approximated as a smooth multiplicative function and a smooth additive function.
- Work is on-going to investigate these effects:
  - Seo & Eisenstein (2005)
  - White (2005)
  - Schulz & White (2006)
  - Eisenstein, Seo & White (2007)
  - Percival et al. (2007)
  - Huff et al. (2007)
  - Angulo et al. (2007)
  - Smith et al. (2007)
  - Padmanabhan et al. (2008, 2009)
  - Seo et al. (2008)
  - Matsubara (2008)
  - Noh et al. (2009)

$$\Delta^2_g(k) = B^2(k)\, \Delta^2(k) + C(k)$$

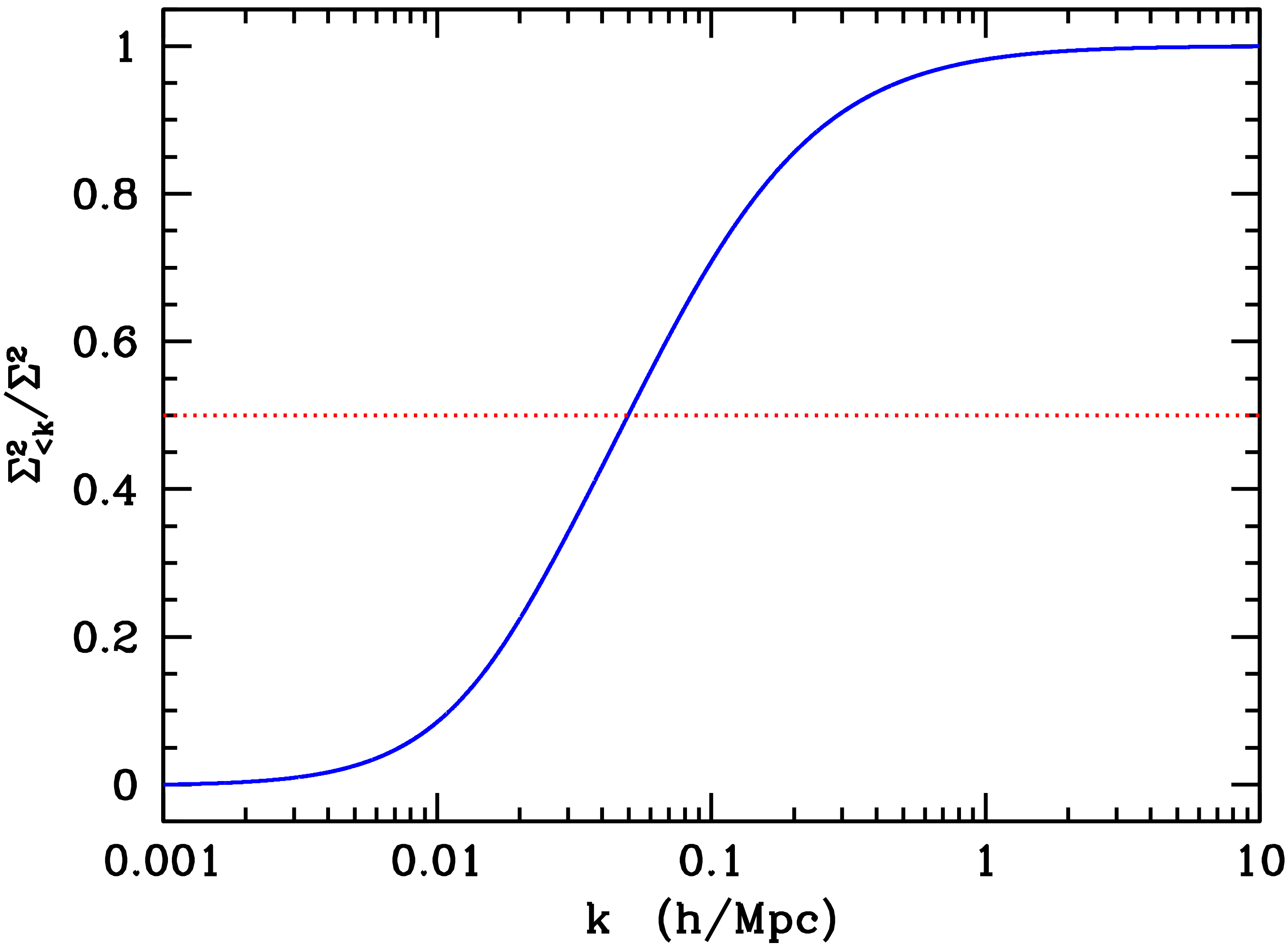
Rational functions or polynomials or splines.

Reconstruction

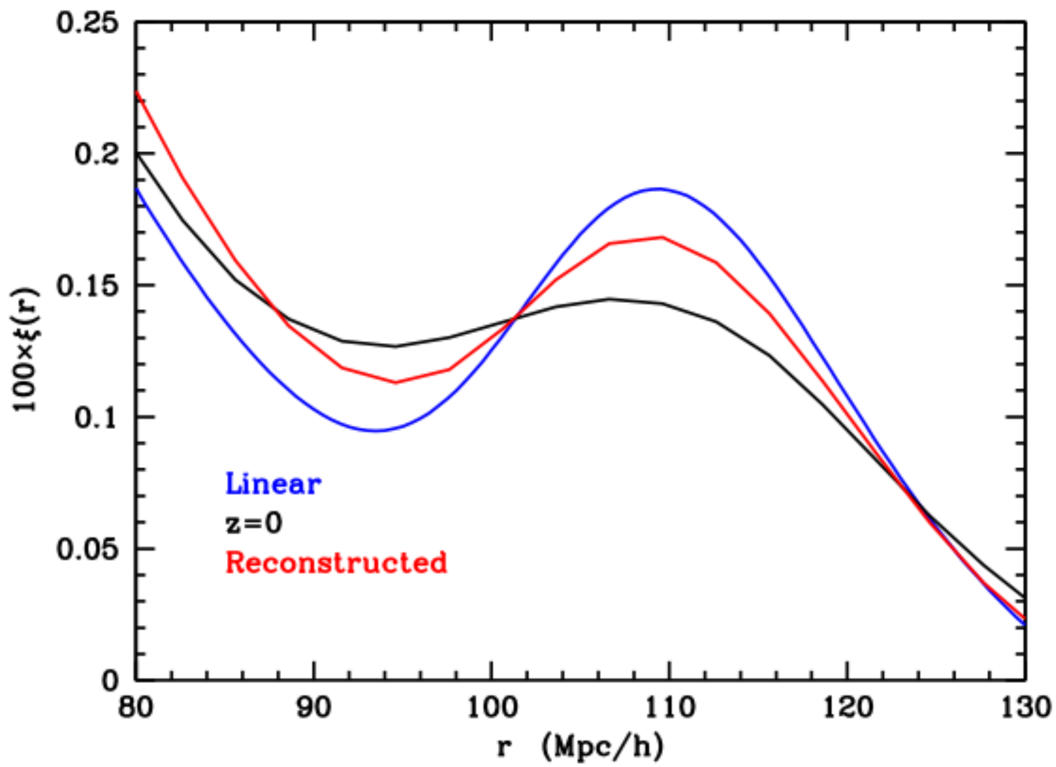
- The broadening of the peak comes from the “tugging” of large-scale structure on the baryon “shell”.
- We measure the large-scale structure and hence the gravity that “tugged”.
- Half of the displacement in the shell comes from “tugs” on scales ~100 Mpc/h
- Use the observations to “undo” non-linearity (Eisenstein++07)
  - Measure $\delta(x)$, infer $\phi(x)$, hence displacement.
  - Move the galaxies back to their original positions.
- Putting information from the phases back into $P(k)$.
- There were many ideas about this for measuring velocities in the 80’s and 90’s; but not much of it has been revisited for reconstruction (yet).

Eisenstein++07; Huff++07; Seo et al.++08,09; Wagner++08; Padmanabhan++09; Mehta++09; Noh++09; …

Contributions to the displacement



Reconstruction



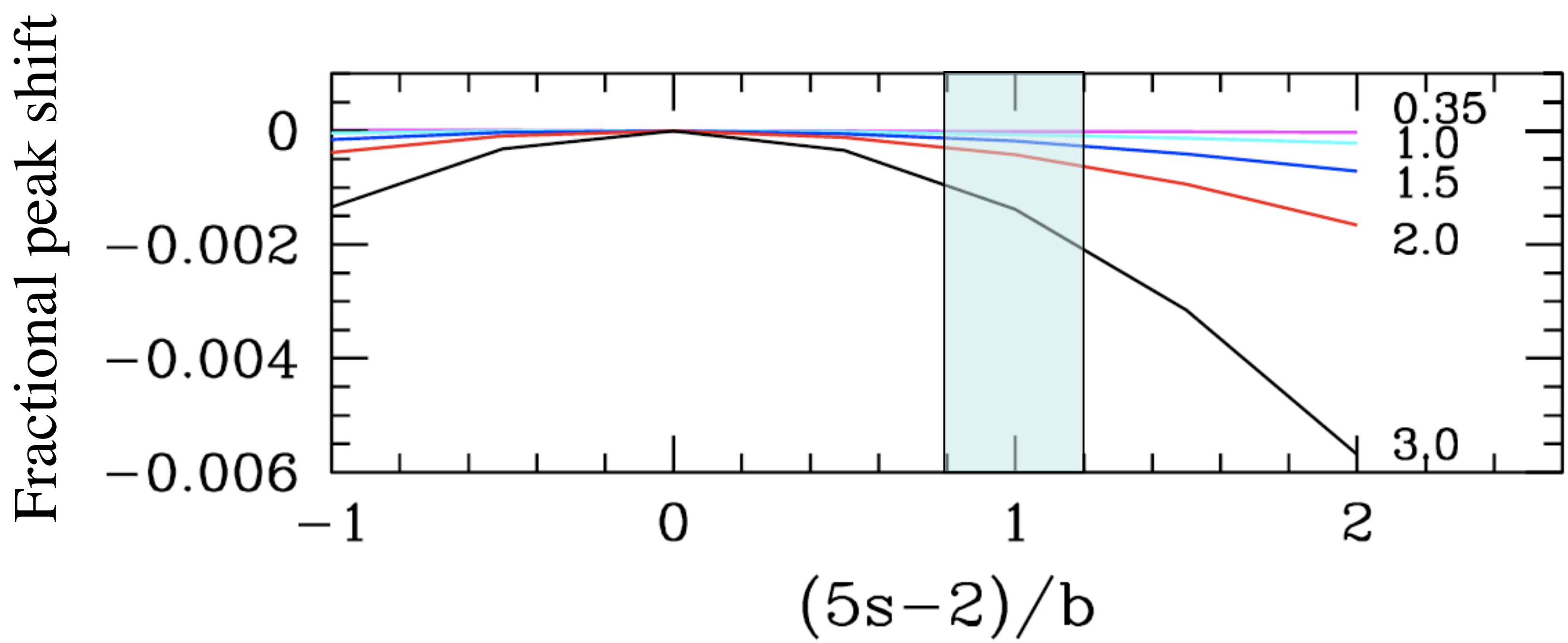
Reconstruction helps to sharpen the peak in the correlation function which is smeared by non-linear evolution.

This seems relatively "easy", **BUT**, to date reconstruction hasn't been demonstrated on non-simulated data.

Lensing

Hui, Gaztanaga & LoVerde: effects of lensing on the correlation function.
For next-generation experiments effect is small.
Eventually may be measurable: template known.

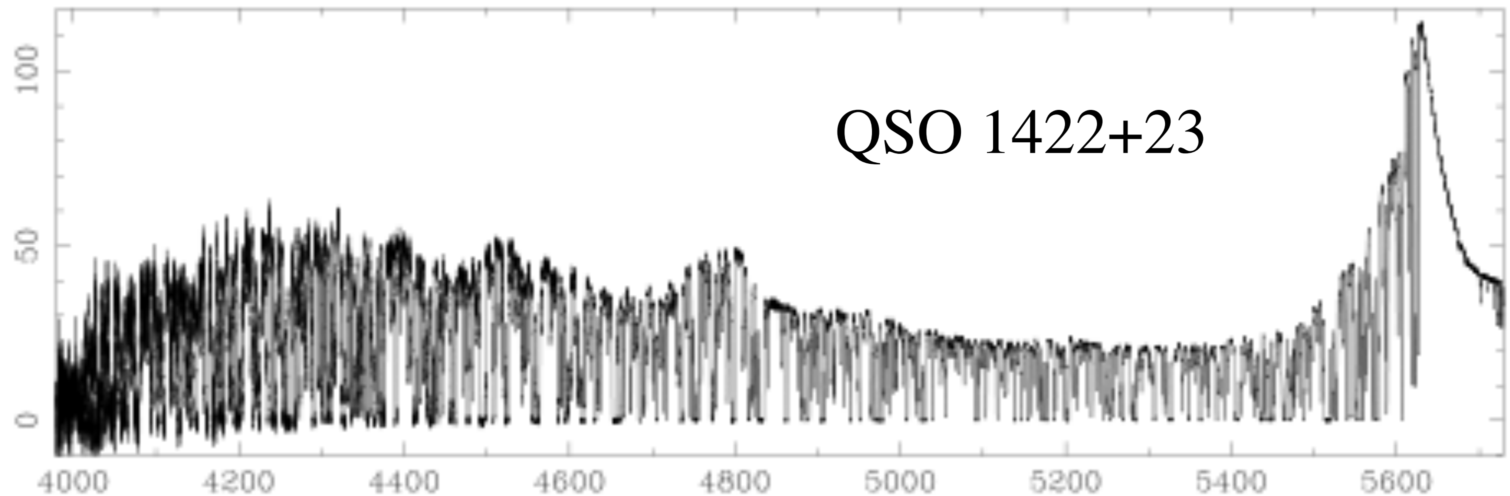
$$\xi_{obs}(R,z) = \xi\left(\sqrt{R^2+z^2}\right) + f(R)z + g(R)$$

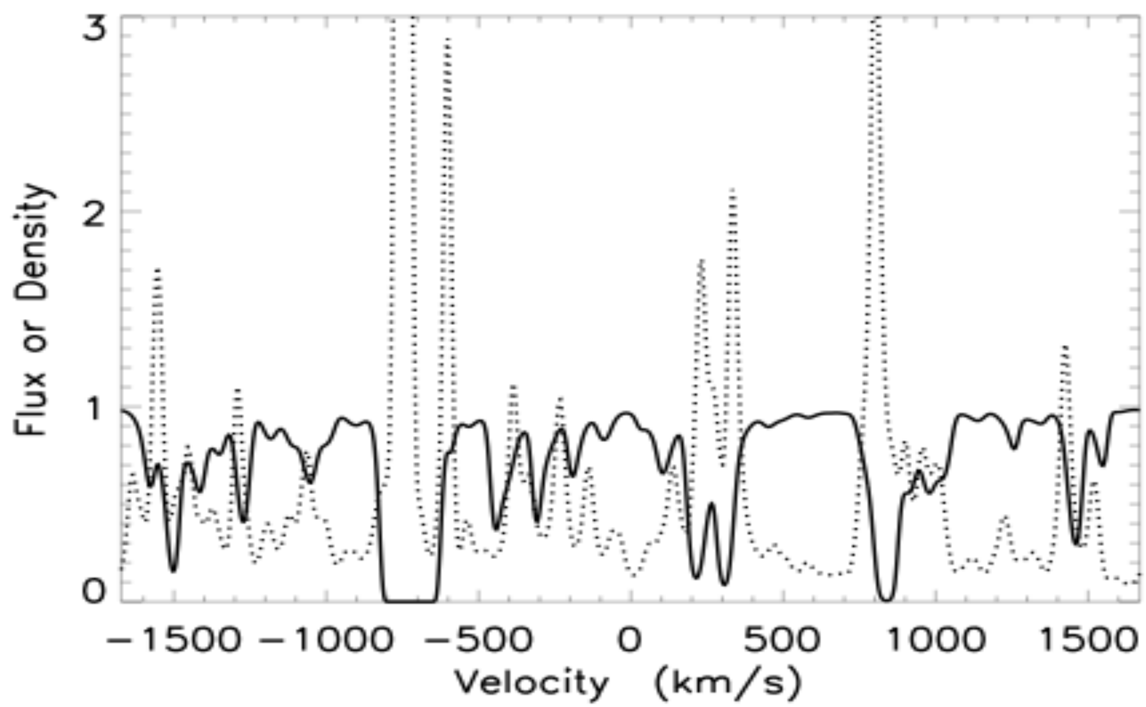


BAO and the IGM

- Distance constraints become tighter as one moves to higher z
  - More volume per sky area.
  - Less non-linearity.
- Expensive if use galaxies as tracers.
- Any tracer will do: HI
  - 21cm from HI in galaxies: SKA or custom expt.
  - Lyα from IGM as probed by QSOs.
    - If IGM is in photo-ionization equilibrium
      - Absorption traces mass in a calculable way.
      - Flux(λ) ~ $\exp[-A(1+\delta)^{\beta}]$ (Cen++94, Hui & Gnedin 97, Croft++98)
    - A dense grid of QSO sightlines could probe BAO
      - (White 2003, McDonald & Eisenstein 2007, Slosar++09, White++10)

Spectrum ‘=’ density





Orientation: distances & redshifts

z	**λ_α**	**$\Delta\chi$**	**$d\lambda/d\chi$**	**$dv/d\chi$**
2.0	3657	575	1.11	91
2.5	4255	546	1.37	97
3.0	4863	518	1.66	102

FGPA

- Physics of the forest is straightforward.
  - Gas making up the IGM is in photo-ionization (but not thermal) equilibrium with a (uniform?) ionization field which results in a tight ρ-T relation for the absorbing material
    - $T = T_0 (\rho/\rho_0)^{\gamma-1}$
    - Expect $\gamma \sim 1$ at reionization to ~1.5 at late time and $T_0 \sim 2.10^4$K
  - The HI density is proportional to a power of the baryon density.
    - For $z<5$, $x_e \sim 1$ so $n_e \sim n_p \sim n_b$ thus $n_{HI} \sim \alpha(T)\, n_b^2/\Gamma \sim n_b^p$

FGPA

- Physics of the forest is straightforward.
  - Since pressure forces are sub-dominant on “large” scales, the gas traces the dark matter (0.1-10 Mpc/h).
  - The structure in the QSO spectrum thus traces, in a calculable way, the fluctuations in the matter density along the line-of-sight to the QSO. The Ly-α forest arises from overdensities ~ 1.

$$\tau(u) \propto \int dx \left[\frac{\rho(x)}{\bar{\rho}}\right]^2 T(x)^{-0.7} \frac{e^{-(u-u_0)^2/b^2}}{b} \quad \text{with} \quad b = \sqrt{2k_B T/m_{\mathrm{H}}}$$

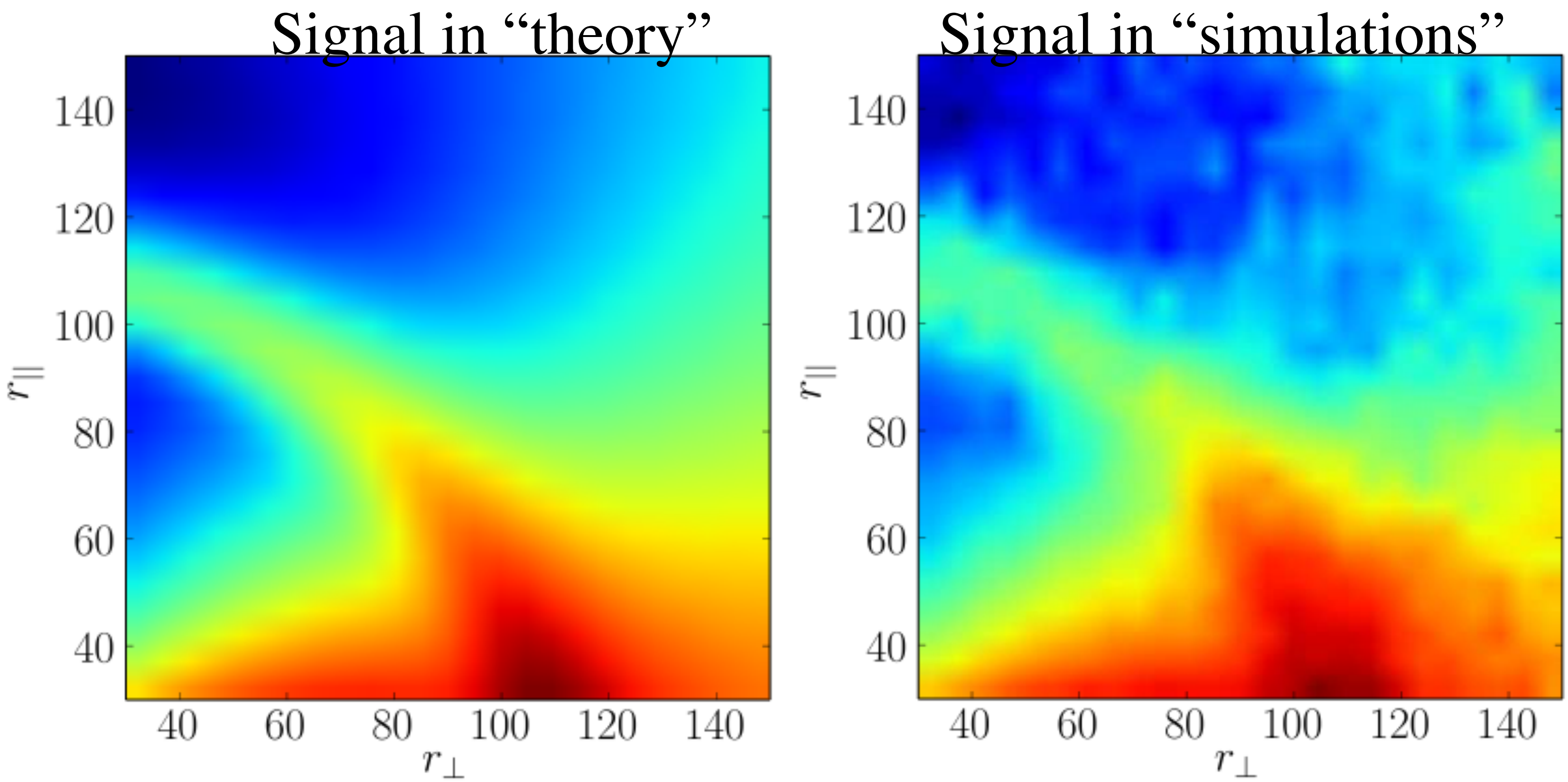
  - Observed flux is $e^{-\tau}$ (times quasar continuum, plus noise, etc.)
  - The pre-factor is in principle calculable (depends e.g. on Γ) but is usually fixed by an external data point, typically <F>, or fit to the data.

On large scales

- Now on large-scales we have that the flux is some (complicated) function of the density.
  - Flux traces mass, with a bias.
- Expect to see a BAO signal in the flux.
- Differences with the galaxies
  - Projection/finite sampling.
  - Signal is $e^{-\tau}$, so downweights high-δ.
  - Need to be slightly careful about redshift space distortions (τ conserved, not *n*).

BAO at high z

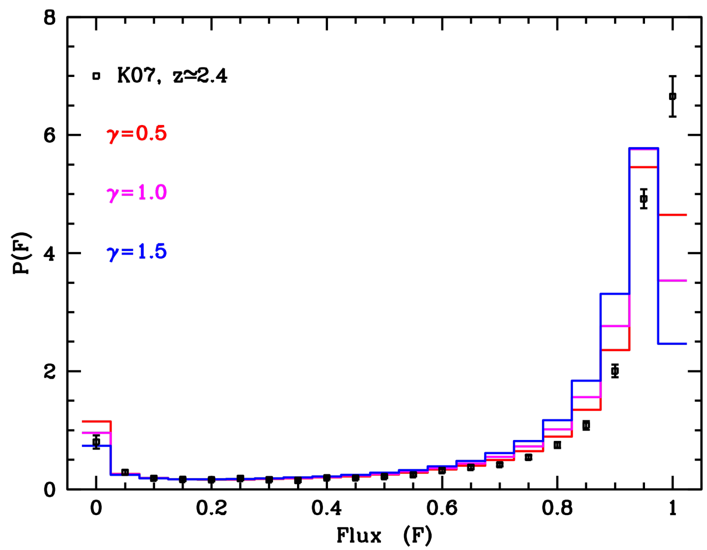
Slosar, Ho, White & Louis (2009)

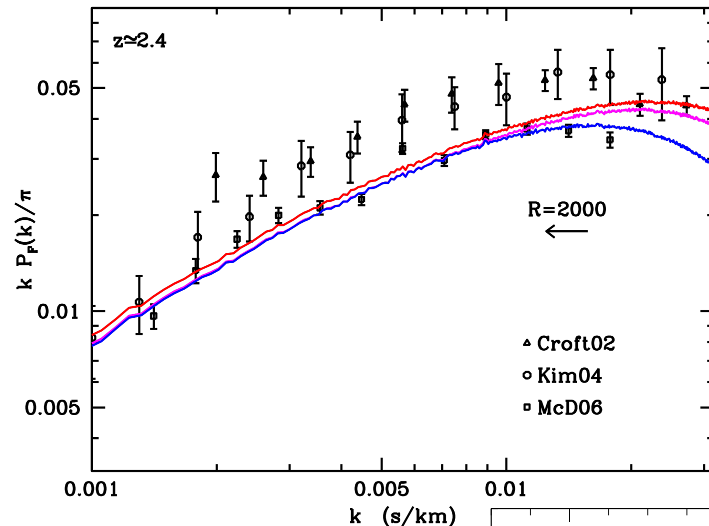


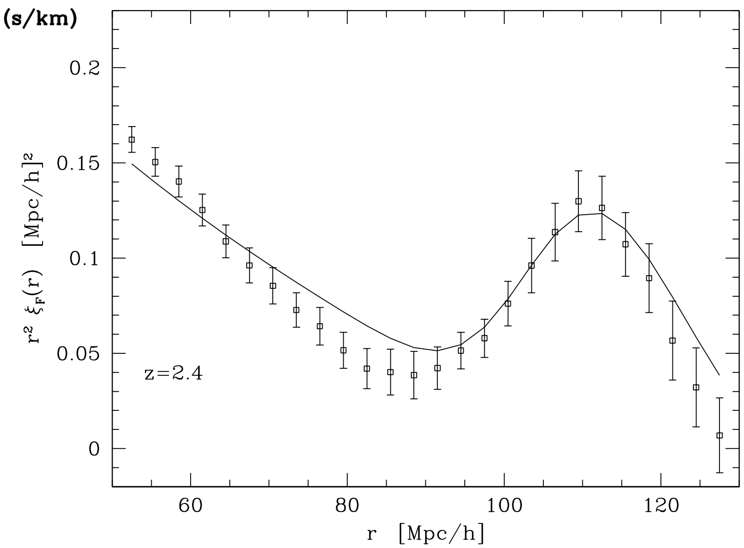
BAO feature survives in the LyA flux correlation function, because on large scales flux traces density. Relatively insensitive to astrophysical effects.

Small-scales: Roadrunner

Previous simulations were fine for BAO-scale, but lacked resolution to give a reasonable small-scale model: pipeline tests, error bars, …







With Roadrunner we simultaneously resolve large (100Mpc) and small (100kpc) scales in a single simulation!

White, Pope, Carlson, Heitmann, Habib, Fasel, Daniel & Lukic (2010)

Lower dimensional fields

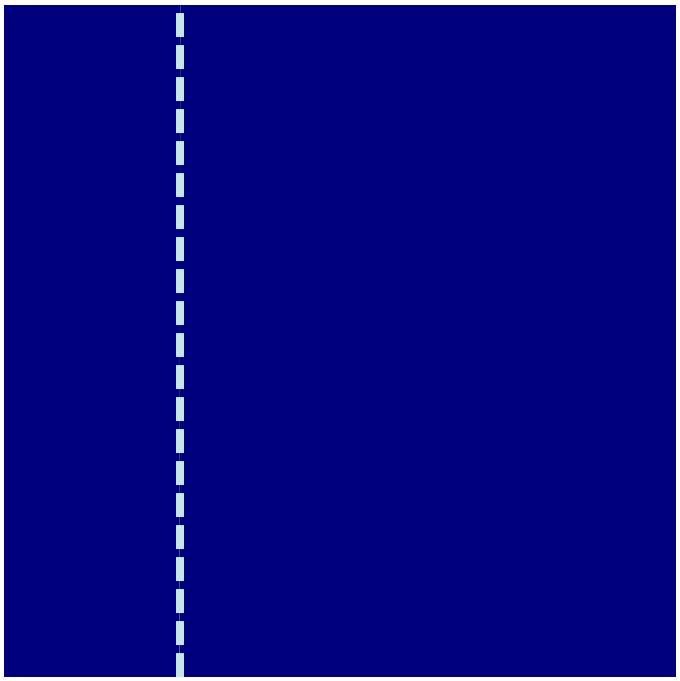
- Imagine $\delta(\boldsymbol{x})$ is a 3D stochastic field.
- Let $W(\boldsymbol{x})$ be a window function we multiply the field by in configuration space
  - $\delta_W(\boldsymbol{x}) = \delta(\boldsymbol{x})W(\boldsymbol{x})$.
- In Fourier space
  - $\delta_W(\boldsymbol{k}) = [\delta * W](\boldsymbol{k})$.
  - $P_W(k)=[P*W^2](k)$.
- For a 1D field along z: $W(\boldsymbol{x})=\delta_D(x)\delta_D(y)\boldsymbol{1}(z)$
  - $W(\boldsymbol{k})=\boldsymbol{1}(k_x)\boldsymbol{1}(k_y)\delta_D(k_z)$

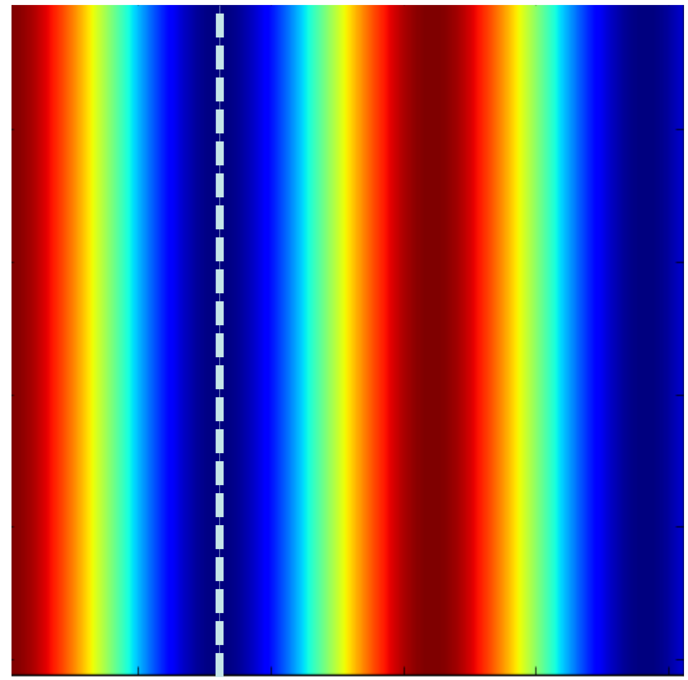
$$\Delta^2_{1D}(k) = \frac{kP(k)}{\pi} = k\int_k^\infty \frac{d^3\mathbf{k}'}{(2\pi)^3}\,\frac{P(k')}{k'}$$

Power at k_{1D} comes from $k_{3D} \geq k_{1D}$.

Aliasing

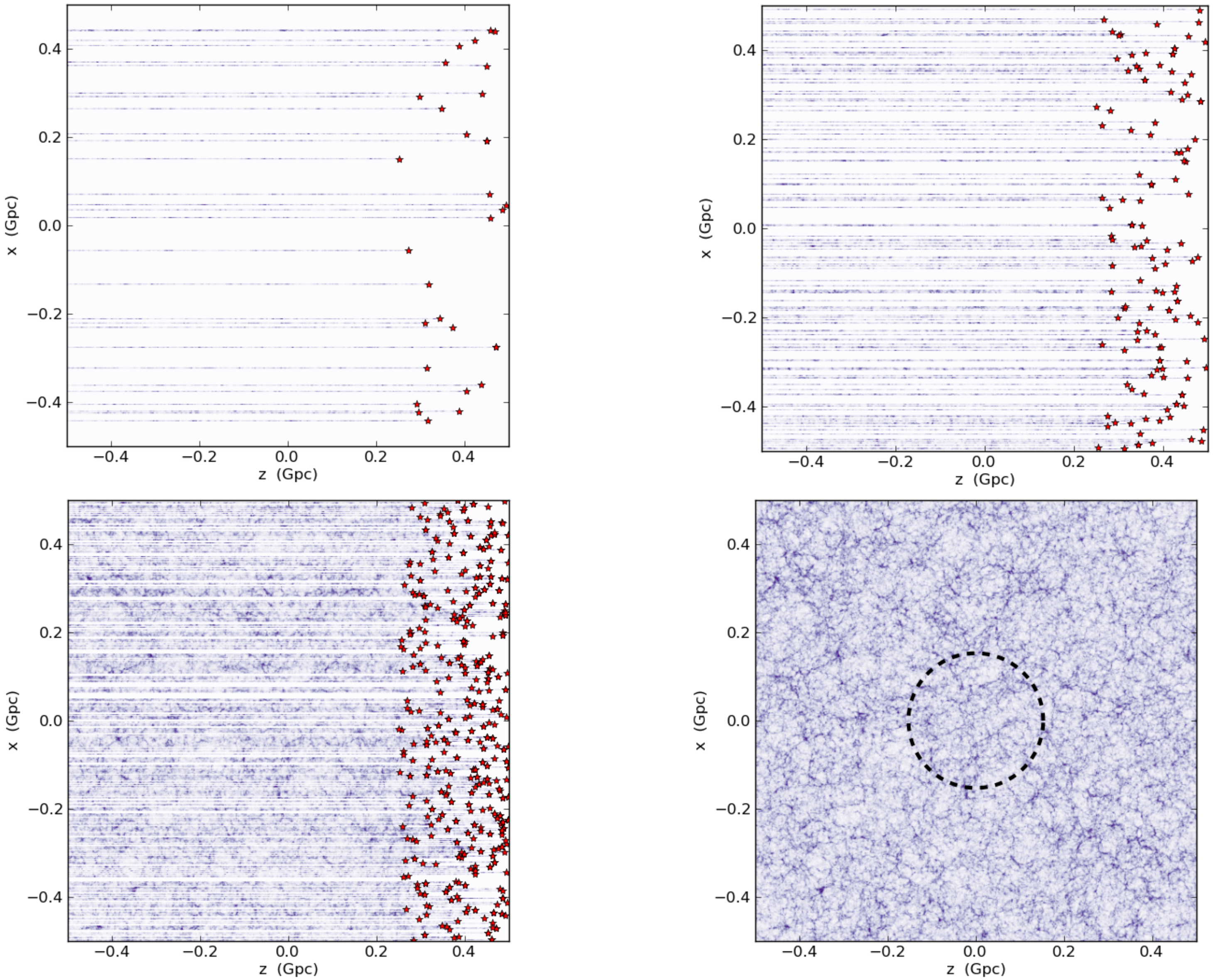
$$\Delta^2_{1D}(k) = \frac{kP(k)}{\pi} = k\int_k^\infty \frac{d^3\mathbf{k}'}{(2\pi)^3}\,\frac{P(k')}{k'}$$





Can't tell the difference between a constant field (k_x=k_y=k_z=0) and one varying transverse to the line-of-sight (k_x>0 or k_y>0)

Skewer density



Skewer density

- Looking along a finite number of sightlines leads to power aliasing.
- As the number of sightlines increases this aliasing is tamed – eventually reach sample variance.
- Variance arising from aliasing equals sample variance at a critical 2D number density of sightlines:

$$\bar{n}_{\rm crit} = \frac{\Delta^2_{1D}(k)}{k\,P(k)/\pi} \approx 0.01 h^2\,{\rm Mpc}^{-2} \qquad , \qquad \Delta^2_{1D} = k\int_k^\infty \frac{d^3k}{(2\pi)^3}\,\frac{P(k)}{k}$$

- corresponding to about 50 quasars/sq. deg.
  – Number for QSOs at a fixed *z*.

White et al. (2010: The "Roadrunner" simulations)

New surveys, new statistics

- Estimating the 2-point function from survey data is an old problem.
  - Most techniques we use today were developed decades ago when surveys were in a very different regime.
    - Landy-Szalay: optimal for small N_{gal} in the no-clustering regime.
- New modes of operation.
  - Surveys are much larger, boundaries often less important, but
  - Signals are smaller and
  - Careful attention to errors is critical for proper statistical inference.
  - Frequently (always?) compare observations to simulations.
    - Does the statistic "play well" with periodic simulation boxes?
- Higher order statistics?
  - N-point functions.
    - On large scales structure is pretty Gaussian. Necessary?
    - Reconstruction??
  - Marked correlation functions with e.g. mark ρ.

Ongoing work

- Templates for fitting data, able to account for non-linearity, redshift space distortions and galaxy bias.
- New estimators optimized for large-scale signals calibrated by numerical simulations.
- Models for the covariance matrices, calibrated by simulations.
- More sophisticated reconstruction algorithms.
- Some “new” ideas, and experimental approaches …

Conclusions

- Baryon oscillations are a firm prediction of CDM models.
- Method is “simple” geometry, with few systematics.
- The acoustic signature has been detected in the SDSS!
- With enough samples of the density field, we can measure $d_A(z)$ and $H^{-1}(z)$ to the percent level and thus constrain DE.
  - Was Einstein right?
- Require “only” a large redshift survey - we have >20 years of experience with redshift surveys.
- Exciting possibility of doing high z portion with QSO absorption lines, rather than galaxies.
- It may be possible to “undo” non-linearity.

The End