Baryon acoustic oscillations and Non-linearity

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Summer School on Redshift Surveys
and
Large-Scale Structure
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Places to look

- These lectures
  - http://mwhite.berkeley.edu/BAO/beijing10.pdf
- Reviews
  - Dark energy and cosmic sound
  - (New Astronomy Reviews, 49, 360, 2005)
- Web sites
  - http://cdm.berkeley.edu/doku.php?id=baopages
  - http://cmb.as.arizona.edu/~eisenste/acousticpeak/
  - http://cosmo.nyu.edu/~eak306/BAF

Notation

$$
\begin{aligned}
\delta(\mathbf{x}) &= \frac{\rho(\mathbf{x}) - \bar{\rho}}{\bar{\rho}} = \frac{\delta\rho}{\rho}(\mathbf{x}) \\
\delta(\mathbf{k}) &= \int d^3x \ \delta(\mathbf{x})\, e^{i\mathbf{k}\cdot\mathbf{x}} \\
\langle \delta(\mathbf{k})\delta^\star(\mathbf{k}') \rangle &= (2\pi)^3 \delta_D(\mathbf{k} - \mathbf{k}') P(k) \\
\Delta^2(k) &= \frac{k^3 P(k)}{2\pi^2} \\
\xi(x) &= \int \frac{d^3k}{(2\pi)^3} P(k) e^{i\mathbf{k}\cdot\mathbf{x}} \\
&= \int \frac{dk}{k} \ \Delta^2(k) j_0(kr)
\end{aligned}
$$

Outline

- Overview of BAO/issues
  - Dark energy and standard rulers.
  - Cosmic sound: baryon acoustic oscillations.
  - Theoretical issues.
  - Modeling issues.
  - Prospects and conclusions.
- Beyond the cartoon: BAO, non-linearity
  - BAO beyond the cartoon
    - Tight coupling and acoustic oscillations
    - Diffusion damping and the Silk scale
    - Details, details, details, …
  - Non-linearity
    - Zel’dovich approximation
    - Spherical, top-hat collapse
    - Perturbation theory
    - Direct simulation
- Putting it “all” together
  - Broadening and shifting the peak.
  - Bias and redshift space distortions.
  - Reconstruction.
  - BAO at high z: the LyαF & 21cm
  - Putting it all together.

Dark energy

- There are now several independent ways to show that the expansion of the Universe is accelerating.
- This indicates that:

a) Our theory of gravity (General Relativity) is wrong.
b) The universe is dominated by a material which violates the strong energy condition: $\rho+3p>0$.

- If (b) then it cannot be any “classical” fluid, but some weird “quantum stuff” which dominates the energy density of the Universe (today). We refer to it as “dark energy”.
- The most <u>prosaic</u> explanation is Einstein’s cosmological constant, which can be interpreted as the energy of empty space.

Probing DE via cosmology

- We "see" dark energy through its effects on the expansion of the universe:

$$H^2(z) = \frac{8\pi G}{3} \sum_i \rho_i(z)$$

- Three (3) main approaches
  - Standard candles
    - measure d_L (integral of H^{-1})
  - Standard rulers
    - measure d_A (integral of H^{-1}) and $H(z)$
  - Growth of fluctuations.
    - Crucial for testing extra ρ components vs modified gravity.

Standard rulers

- Suppose we had an object whose length (in *meters*) we knew as a function of cosmic epoch.
- By measuring the angle ($\Delta\theta$) subtended by this ruler ($\Delta\chi$) as a function of redshift we map out the angular diameter distance d_A

$$\Delta\theta = \frac{\Delta\chi}{d_A(z)} \qquad\qquad d_A(z) = \frac{d_L(z)}{(1+z)^2} \propto \int_0^z \frac{dz'}{H(z')}$$

- By measuring the redshift interval (Δz) associated with this distance we map out the Hubble parameter $H(z)$

$$c\Delta z = H(z)\ \Delta\chi$$

Ideal properties of the ruler?

To get competitive constraints on dark energy we need to be able to see changes in *H(z)* at the 1% level -- this would give us "statistical" errors in DE equation of state (w=p/ρ) of ~10%.

- We need to be able to calibrate the ruler accurately over most of the age of the universe.
- We need to be able to measure the ruler over much of the volume of the universe.
- We need to be able to make ultra-precise measurements of the ruler.

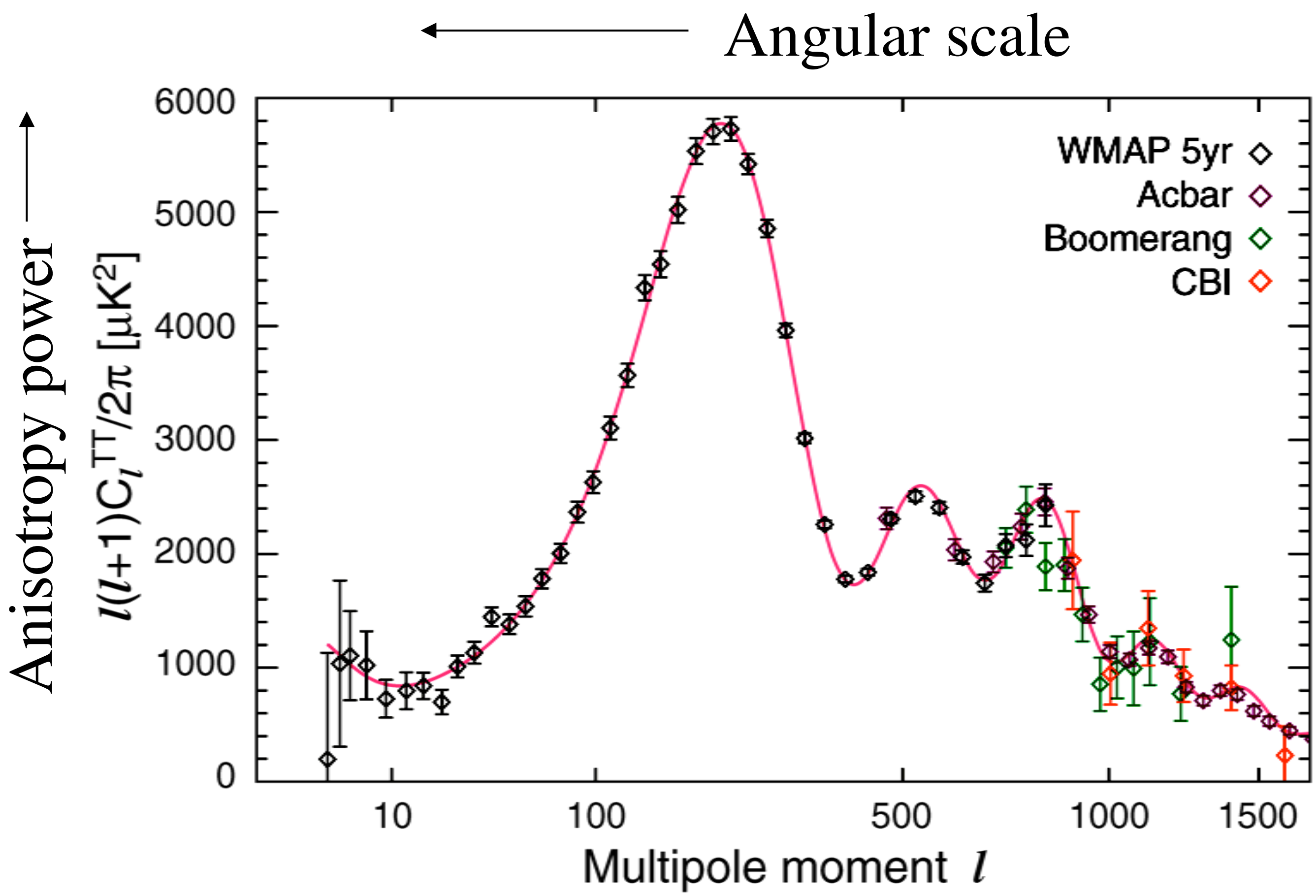
Where do we find such a ruler?

- Cosmological objects can probably never be uniform enough.
- We believe that the laws of physics haven't changed over the relevant time scales.
  - Use features arising from physical processes in the early Universe.
- Use statistics of the large-scale distribution of matter and radiation.
  - If we work on large scales or early times perturbative treatment is valid and calculations under control.

Sunyaev & Zel'dovich (1970); Peebles & Yu (1970); Doroshkevitch, Sunyaev & Zel'dovich (1978); …; Hu & White (1996); Cooray, Hu, Huterer & Joffre (2001); **Eisenstein** (2003); Seo & Eisenstein (2003); Blake & Glazebrook (2003); Hu & Haiman (2003); …

Back to the beginning …

The CMB power spectrum



The current CMB data are in excellent agreement with the theoretical predictions of a ΛCDM model.

Hinshaw et al. (2008)

The cartoon

- At early times the universe was hot, dense and ionized. Photons and matter were tightly coupled by Thomson scattering.
  - Short m.f.p. allows fluid approximation.
- Initial fluctuations in density and gravitational potential drive acoustic waves in the fluid: compressions and rarefactions with $\delta_\gamma \propto \delta_b$.
- Consider a (standing) plane wave perturbation of comoving wavenumber *k.*
- If we expand the Euler equation to first order in the Compton mean free path over the wavelength we obtain a driven harmonic oscillator:

$$\frac{d}{d\tau}\left[m_{\rm eff}\frac{d\delta_b}{d\tau}\right]+\frac{k^2}{3}\delta_b=F[\Psi] \qquad m_{\rm eff}=1+3\rho_b/4\rho_\gamma$$

The cartoon

- These perturbations show up as temperature fluctuations in the CMB.
- Since $\rho \sim T^4$ for a relativistic fluid the temperature perturbations look like:

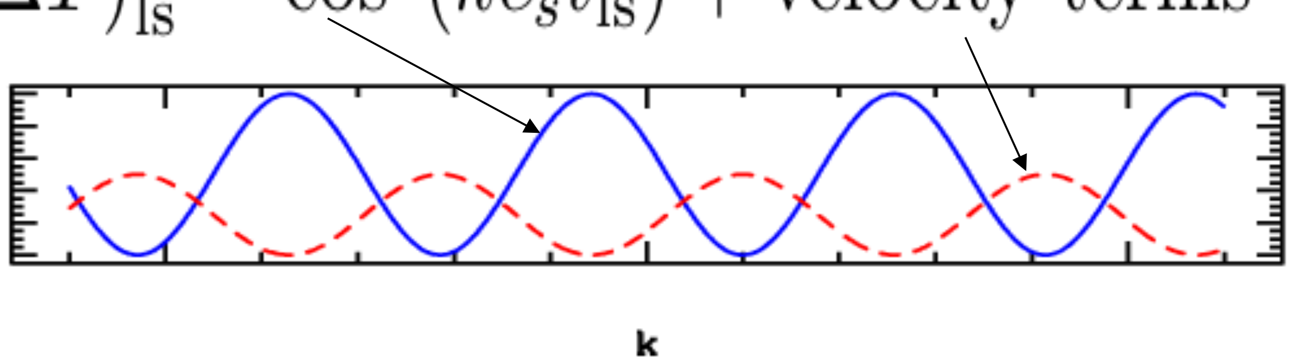
$$\Delta T \sim \delta\rho_\gamma^{1/4} \sim A(k)\cos(kc_s t) \qquad \text{[harmonic wave]}$$

- … plus a component due to the velocity of the fluid (the Doppler effect).

The cartoon

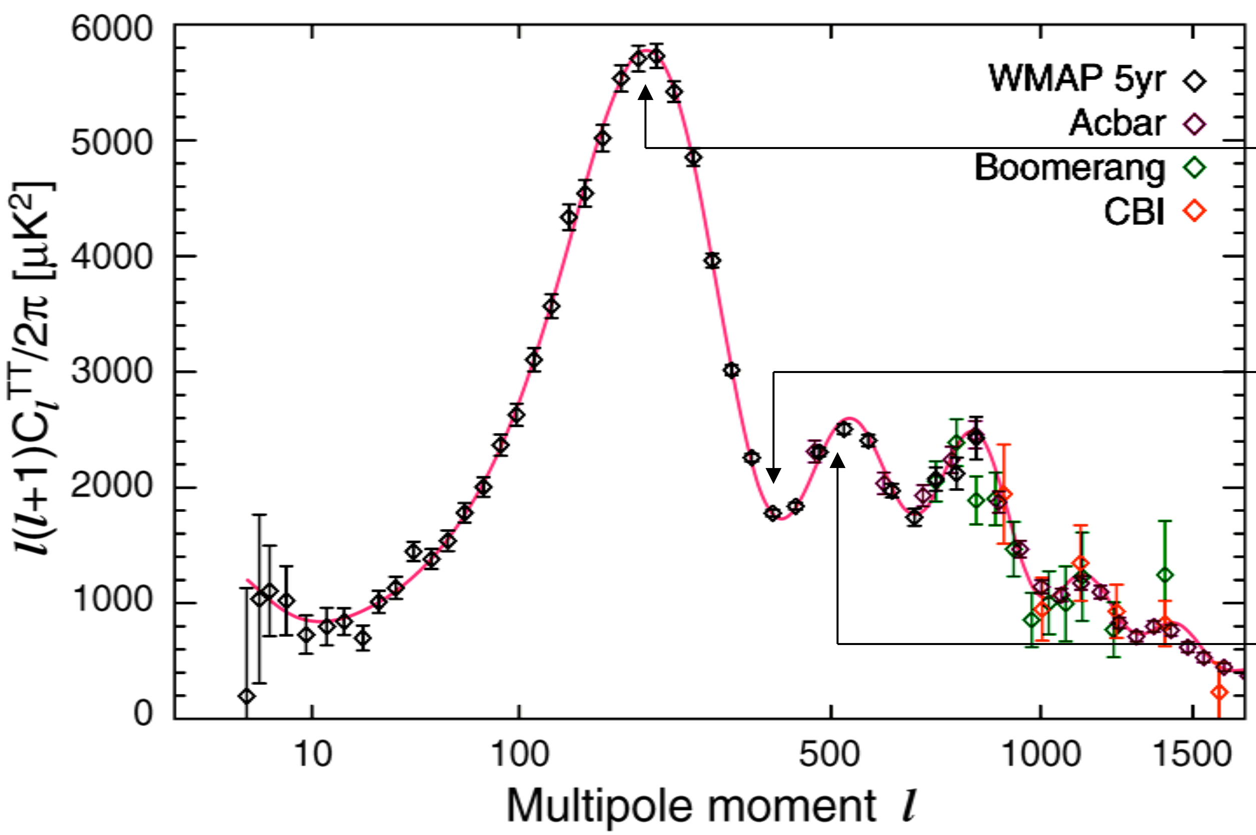
- A sudden "recombination" decouples the radiation and matter, giving us a snapshot of the fluid at "last scattering".

$$(\Delta T)^2_{\rm ls} \sim \cos^2(k c_s t_{\rm ls}) + \text{velocity terms}$$



- These fluctuations are then projected on the sky with $\lambda \sim r_{ls}\theta$ or $l \sim k\, r_{ls}$

Acoustic oscillations seen!



First "compression", at $kc_st_{ls}=\pi$. Density maxm, velocity null.

Velocity maximum

First "rarefaction" peak at $kc_st_{ls}=2\pi$

Acoustic scale is set by the *sound horizon* at last scattering: $s = c_st_{ls}$

CMB calibration

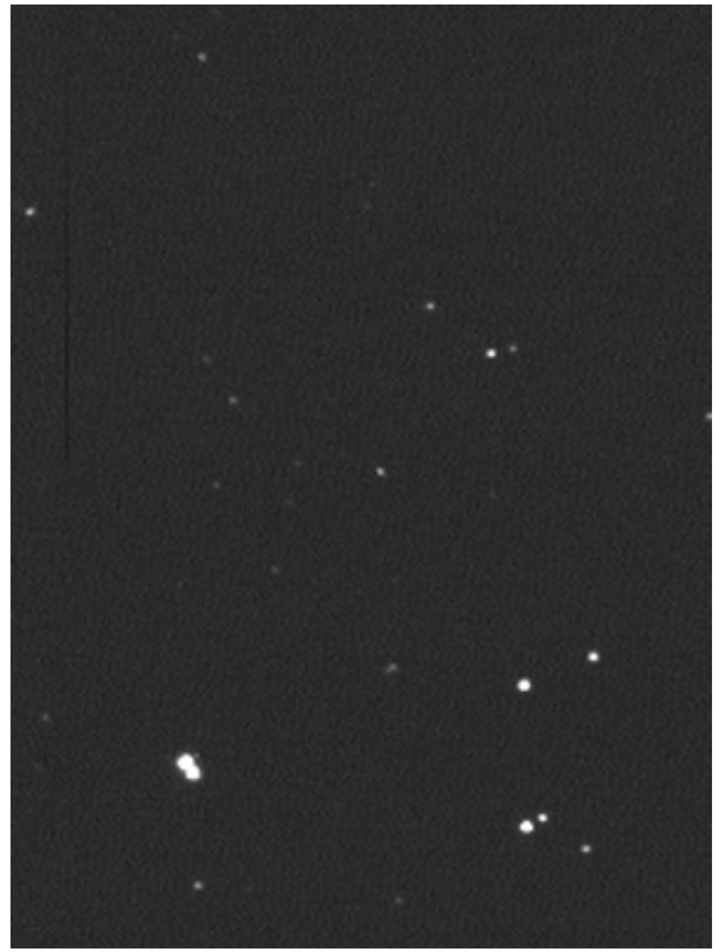
- Not coincidentally the sound horizon is extremely well determined by the structure of the acoustic peaks in the CMB.

$$\begin{aligned} s &= 146.8 \pm 1.8 \text{ Mpc} \\ &= (4.53 \pm 0.06) \times 10^{24} \text{m} \end{aligned}$$

WMAP 5th yr data

Dominated by uncertainty in ρ_m from poor constraints near 3rd peak in CMB spectrum. (Planck will nail this!)

Ariane 5 lifts off with Herschel and Planck on board on 14 May 2009 at 15:12:02 CEST.

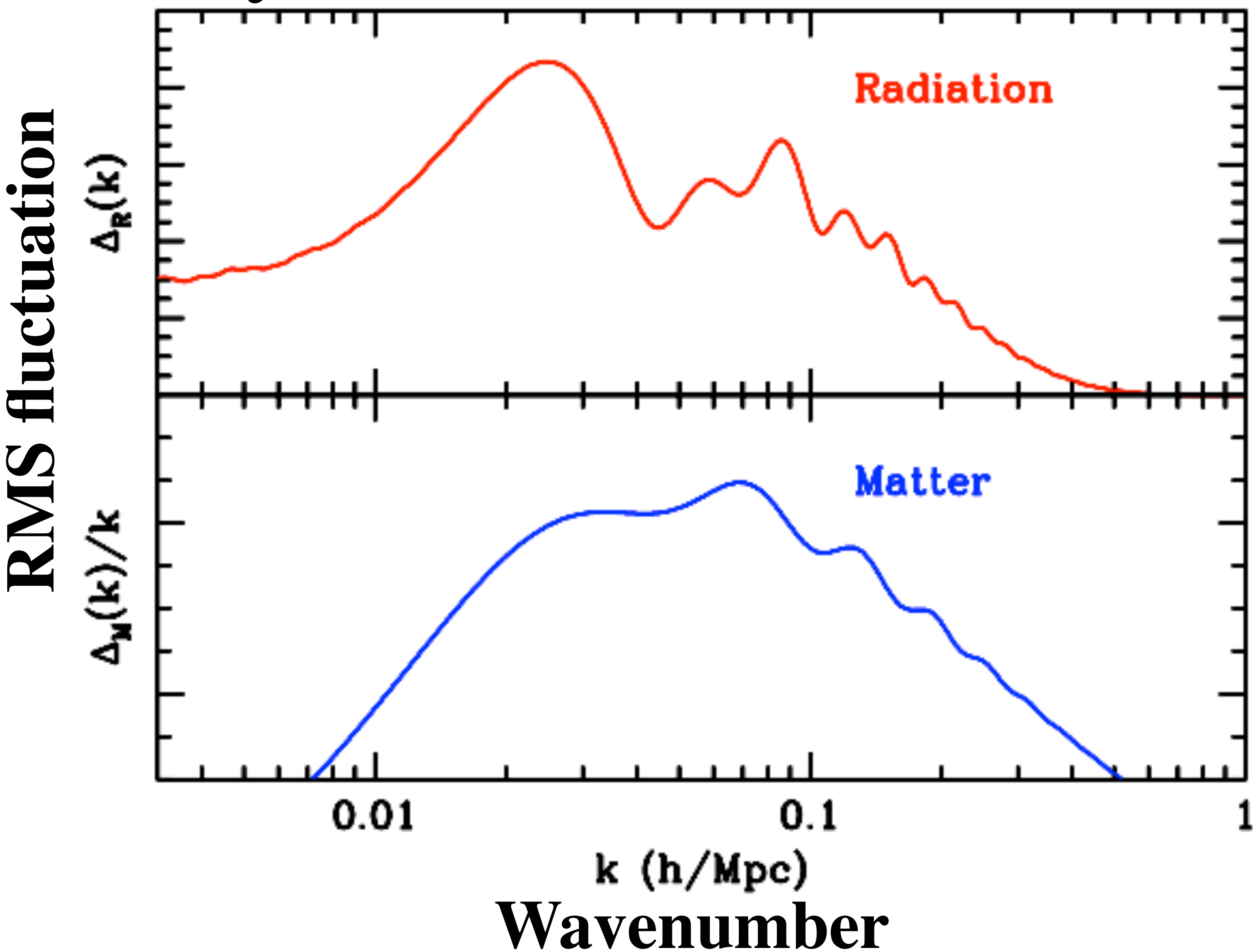


Baryon oscillations in *P(k)*

- Since the baryons contribute ~15% of the total matter density, the total gravitational potential is affected by the acoustic oscillations with scale set by *s*.
- This leads to small oscillations in the matter power spectrum *P*(*k*).
  - No longer order unity, like in the CMB
  - Now suppressed by $\Omega_b/\Omega_m \sim 0.1$

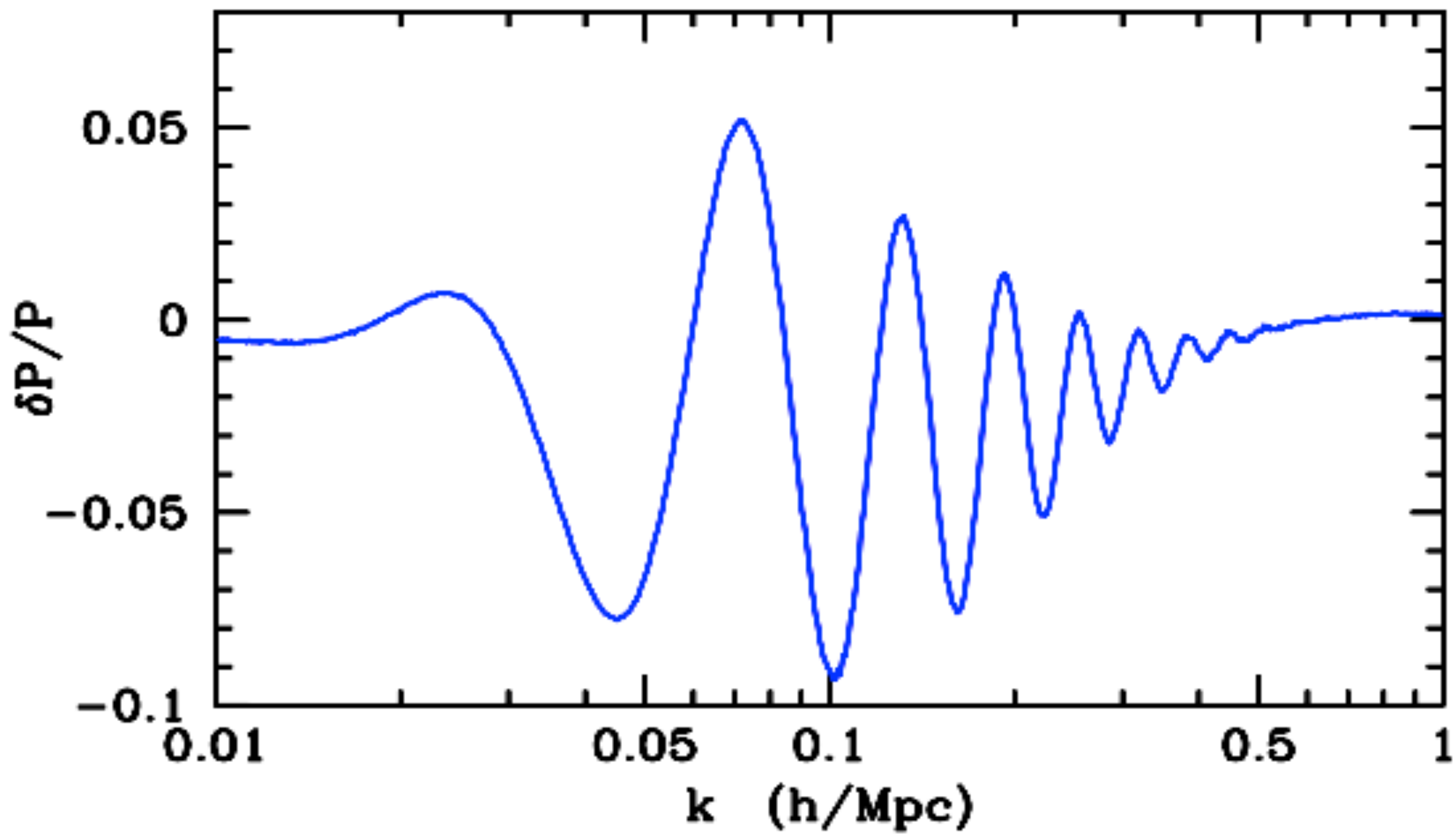
- **Note:** all of the matter sees the acoustic oscillations, not just the baryons.

Baryon (acoustic) oscillations



Divide out the gross trend …

A damped, almost harmonic sequence of “wiggles” in the power spectrum of the mass perturbations of amplitude *O*(10%).

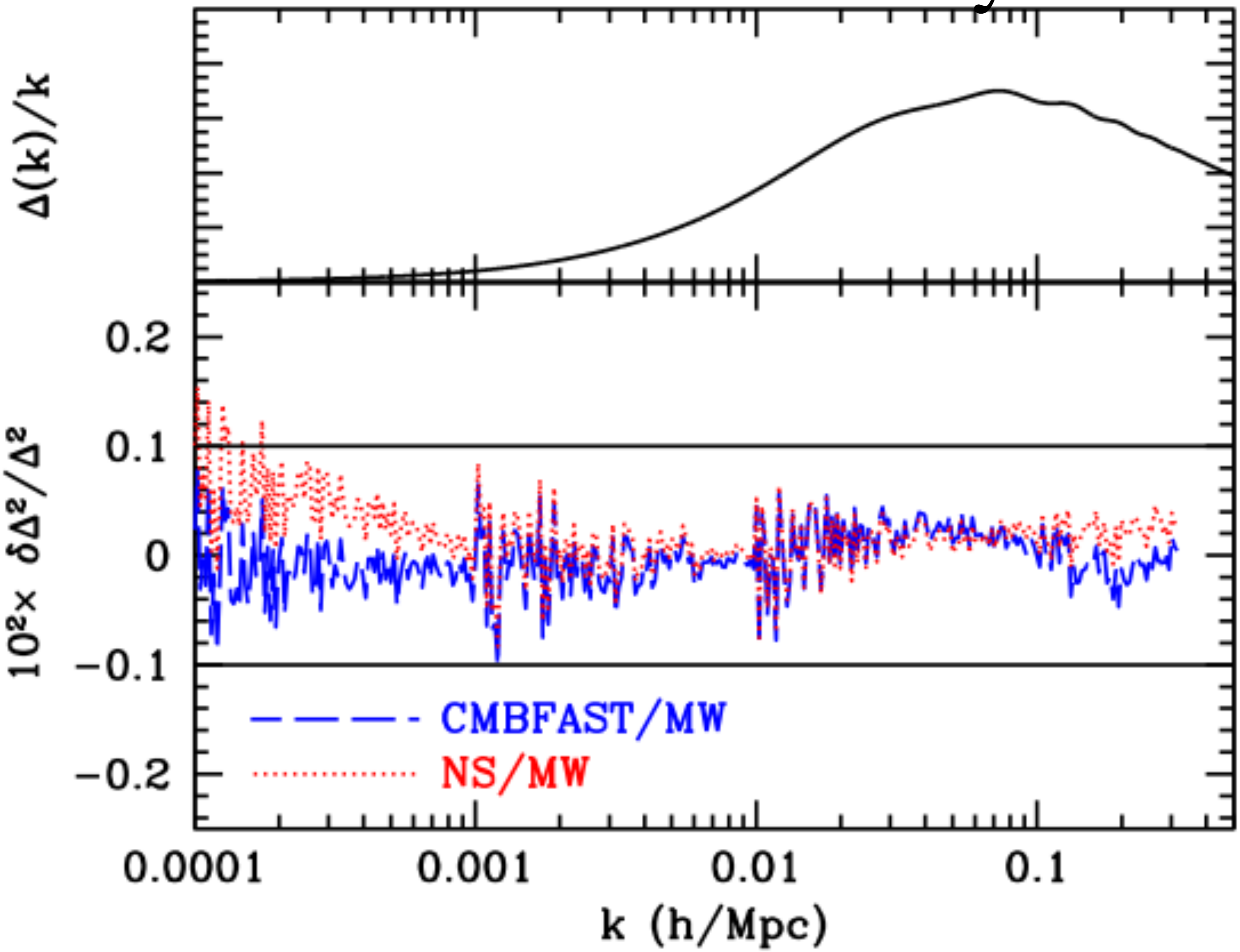


Higher order effects

- The matter and radiation oscillations are not in phase, and the phase shift depends on *k*.
- There is a subtle shift in the oscillations with *k* due to the fact that the universe is expanding and becoming more matter dominated.
- The finite duration of decoupling and rapid change in mfp means the damping of the oscillations on small scales is not a simple Gaussian shape.
- But regardless, the spectrum is calculable and *s* can be inferred!

These features are frozen into the mass power spectrum, providing a known length scale that can be measured as a function of *z*.

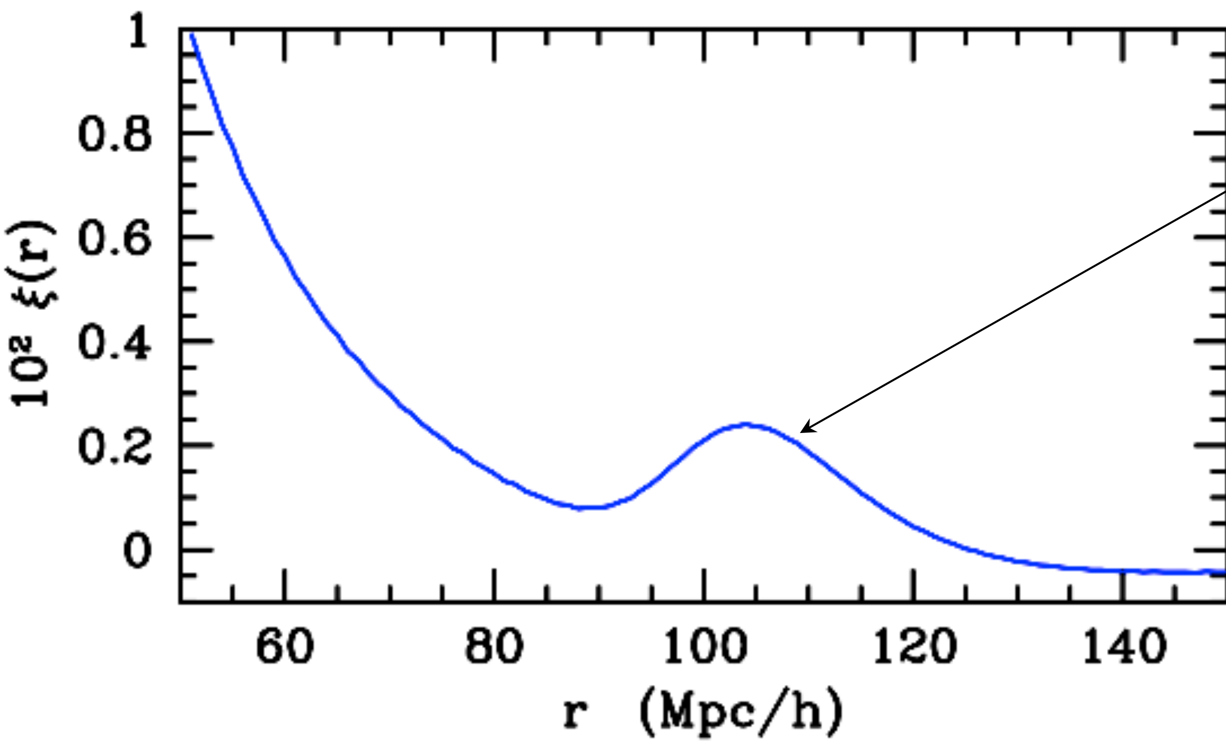
Numerical stability



Seljak, Sugiyama, White & Zaldarriaga (2003)

In configuration space

- The configuration space picture offers some important insights,and will be useful when we consider non-linearities and bias.
- In configuration space we measure not power spectra but correlation functions: $\xi(r)=\int P(k)e^{ikr}d^3k=\int \Delta^2(k)j_0(kr)\, d\ln k$..
- A harmonic sequence would be a δ-function in r, the shift in frequency and diffusion damping broaden the feature.



Acoustic feature at ~100 Mpc/h with width ~10Mpc/h (Silk scale)

Configuration space

In configuration space on uses a Green's function method to solve the equations, rather than expanding *k*-mode by *k*-mode. (Bashinsky & Bertschinger 2000)

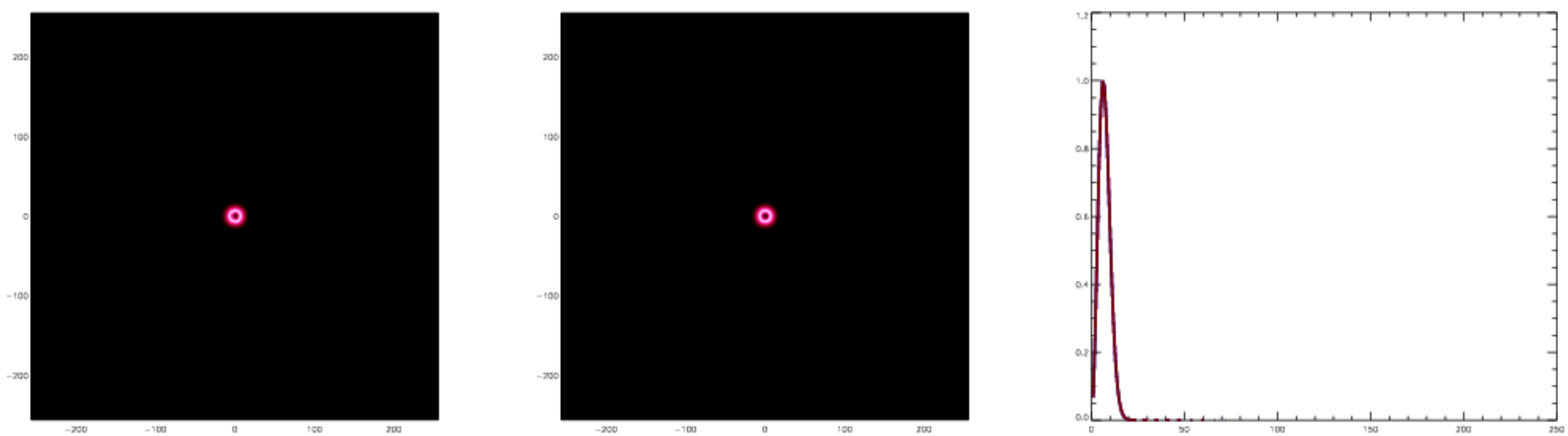
To linear order Einstein's equations look similar to Poisson's equation relating ϕ and δ, but upon closer inspection one finds that the equations are hyperbolic: they describe traveling waves.

[effects of local stress-energy conservation, causality, …]

The acoustic wave

Start with a single perturbation. The plasma is totally uniform except for an excess of matter at the origin.

High pressure drives the gas+photon fluid outward at speeds approaching the speed of light.



Baryons

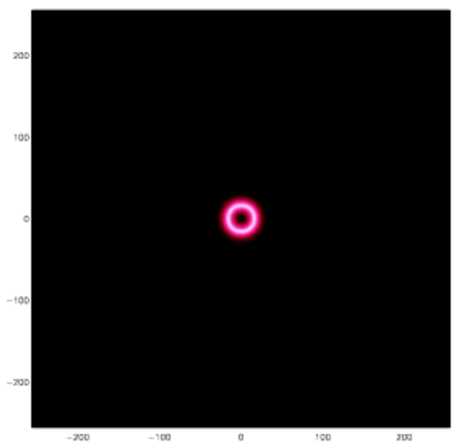
Photons

Mass profile

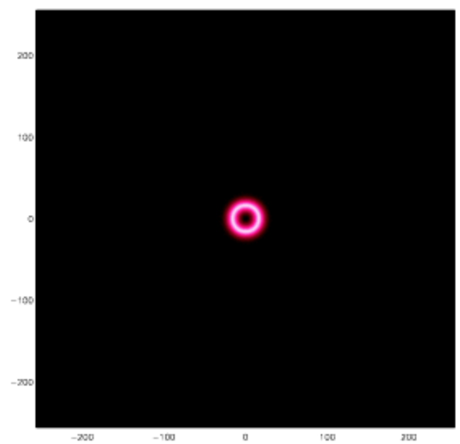
Eisenstein, Seo & White (2006)

The acoustic wave

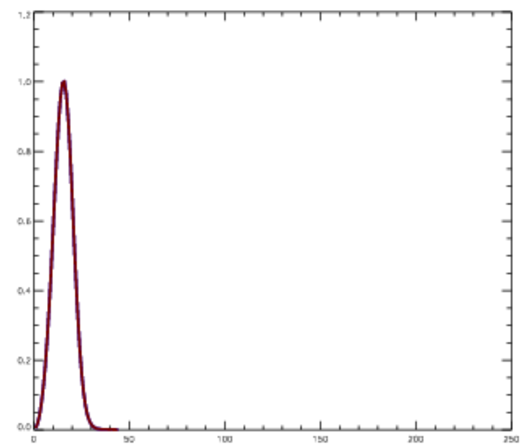
Initially both the photons and the baryons move outward together, the radius of the shell moving at over half the speed of light.



Baryons

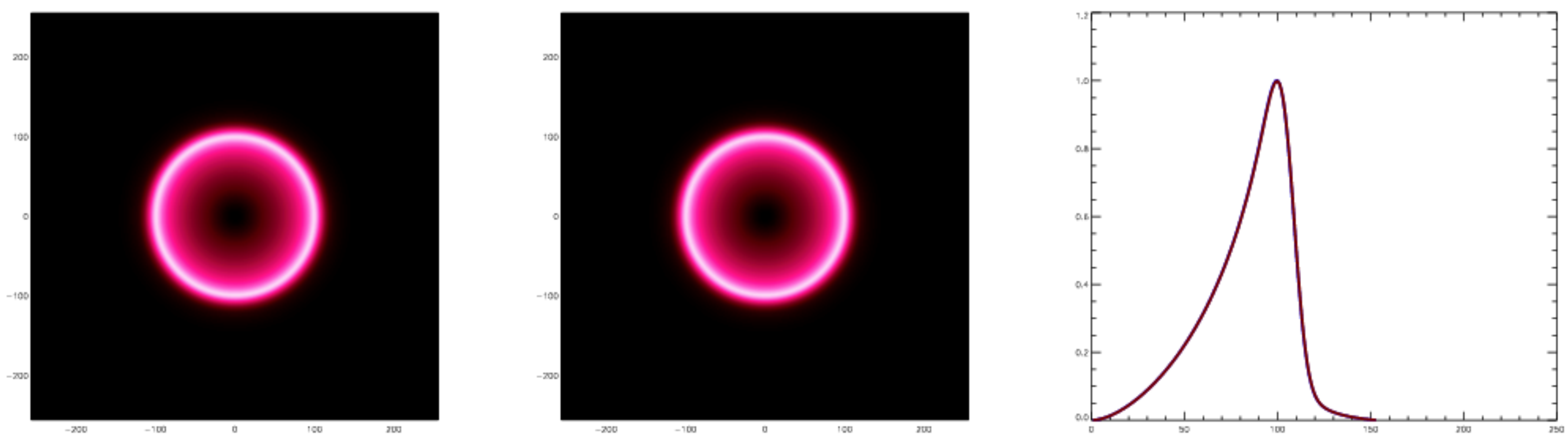


Photons



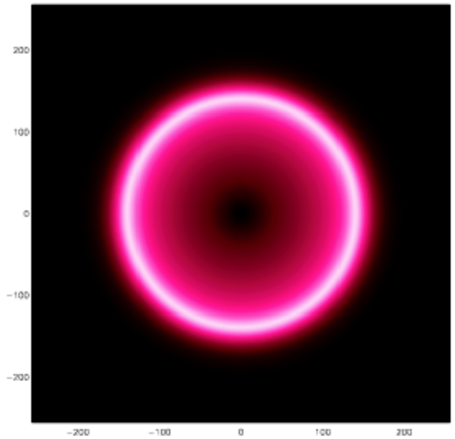
The acoustic wave

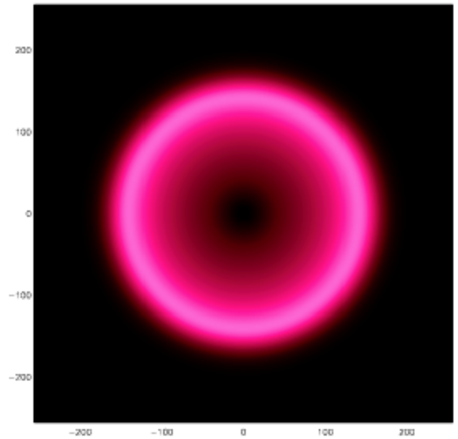
This expansion continues for 10^5 years

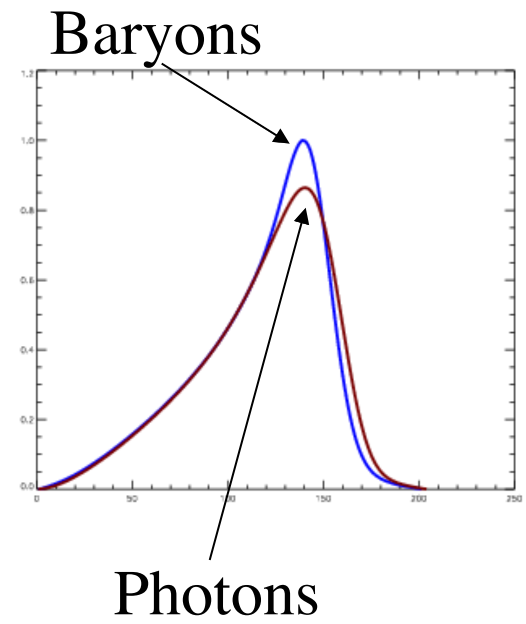


The acoustic wave

After 10^5 years the universe has cooled enough the protons capture the electrons to form neutral Hydrogen. This decouples the photons from the baryons. The former quickly stream away, leaving the baryon peak stalled.

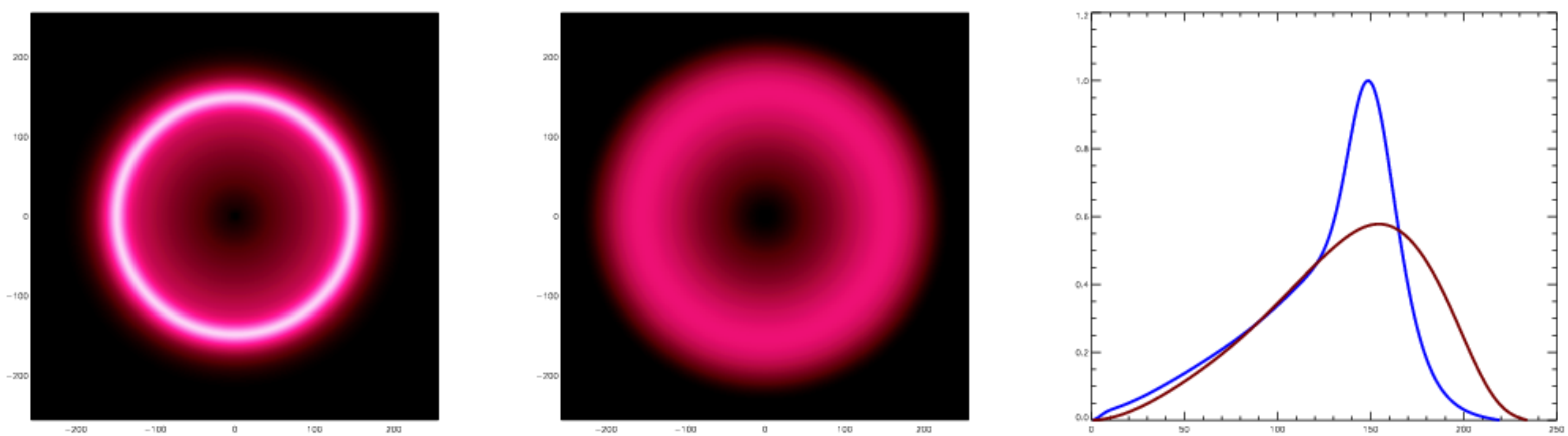




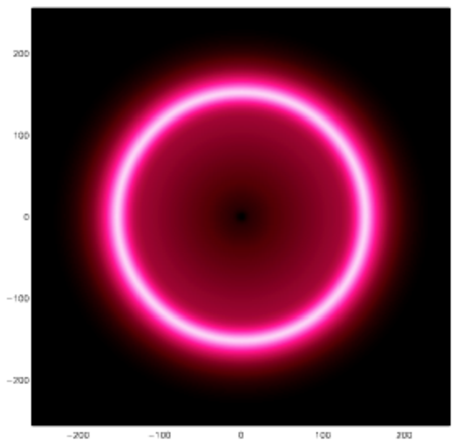


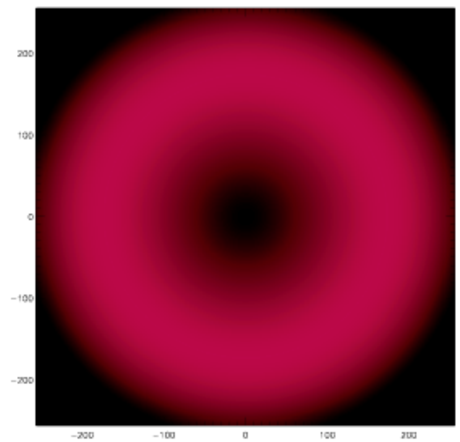
The acoustic wave

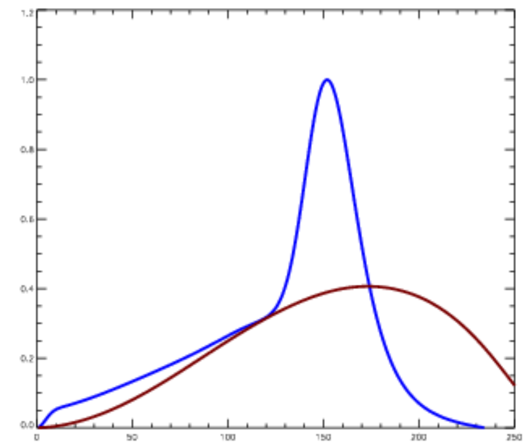
The photons continue to stream away while the baryons, having lost their motive pressure, remain in place.



The acoustic wave

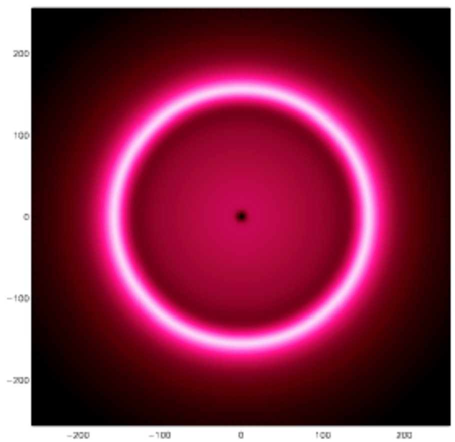


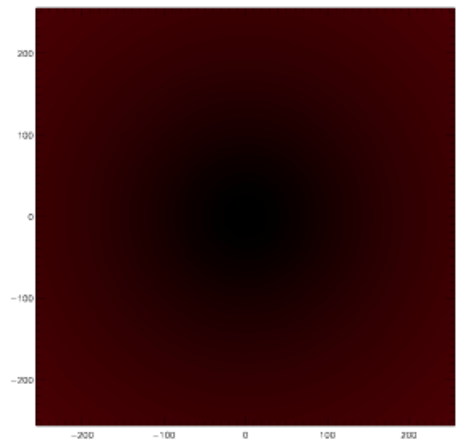


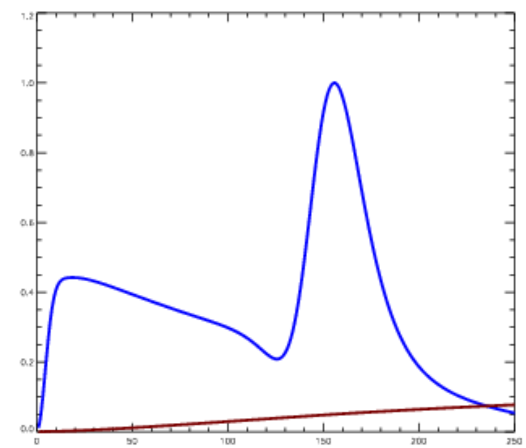


The acoustic wave

The photons have become almost completely uniform, but the baryons remain overdense in a shell 100Mpc in radius.
In addition, the large gravitational potential well which we started with starts to draw material back into it.



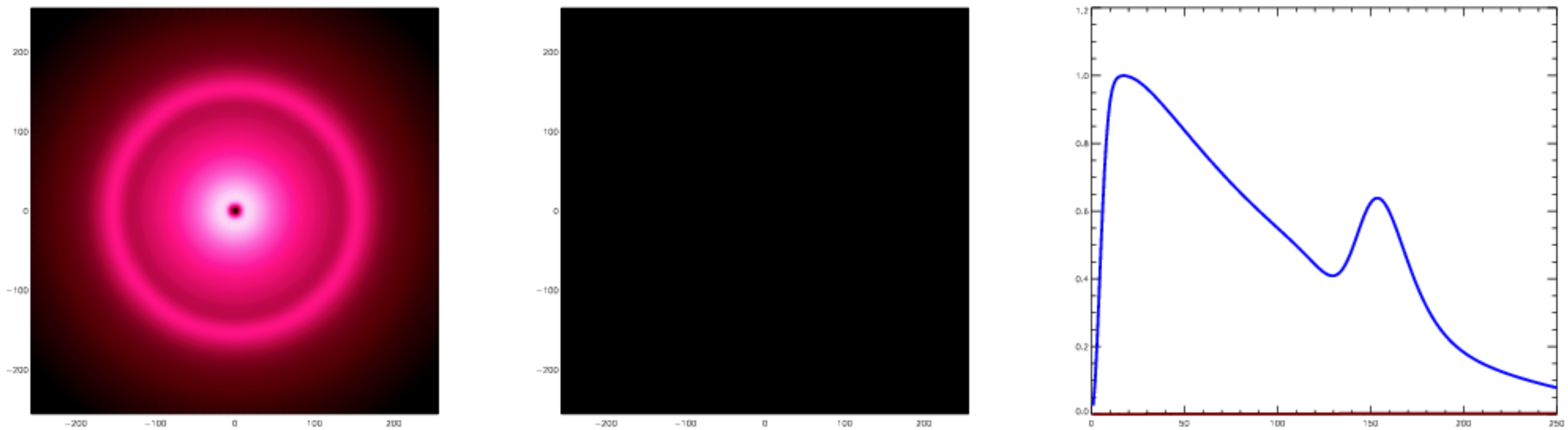




The acoustic wave

As the perturbation grows by ~10^3 the baryons and DM reach equilibrium densities in the ratio Ω_b/Ω_m.

The final configuration is our original peak at the center (which we put in by hand) and an "echo" in a shell roughly 100Mpc in radius.

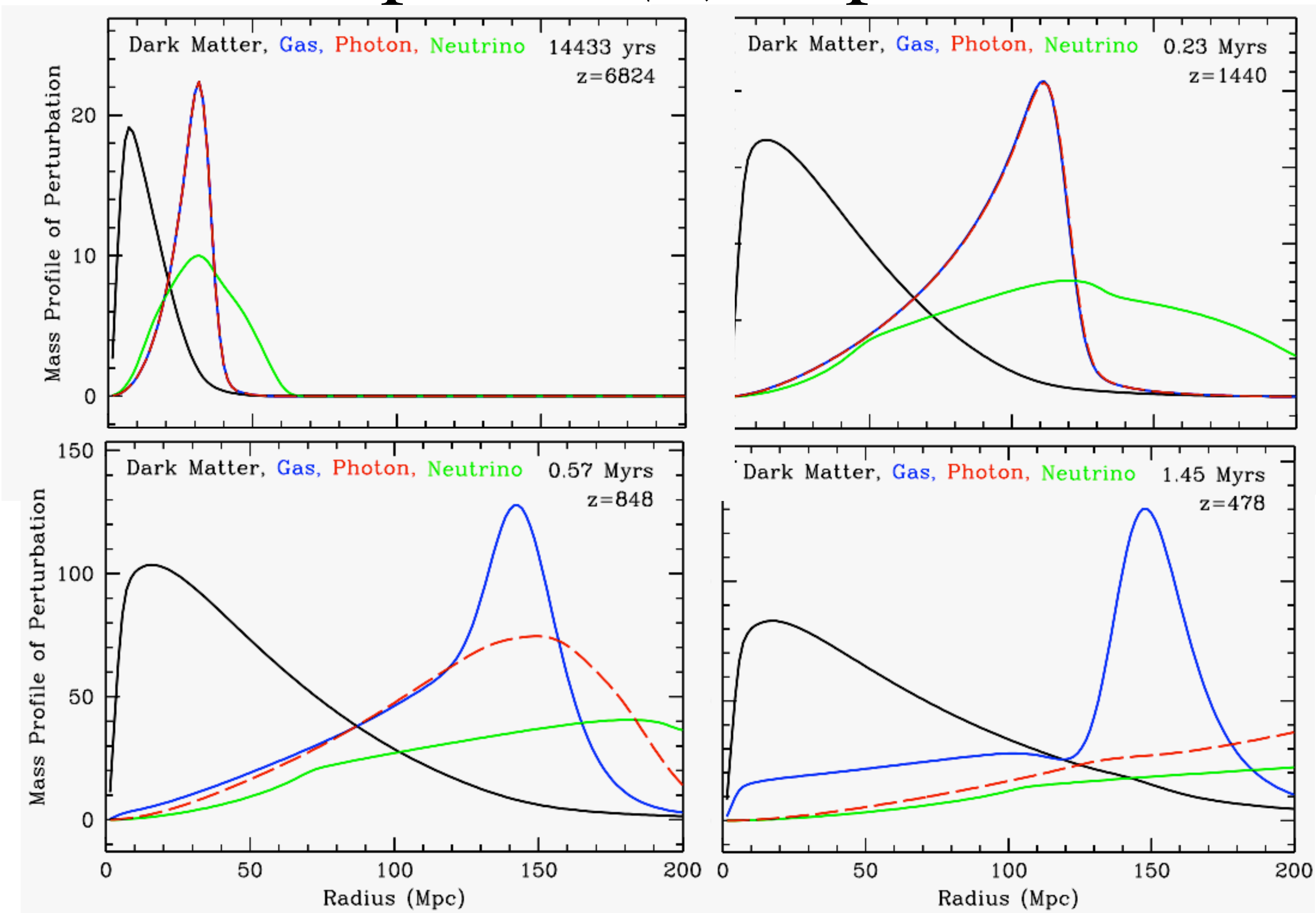


Further (non-linear) processing of the density field acts to broaden and very slightly shift the peak -- but galaxy formation is a local phenomenon with a length scale ~10Mpc, so the action at $r=0$ and $r\sim100$Mpc are essentially decoupled. We will return to this …

Aside:broad-band shape of *P(k)*

- This picture also allows us a new way of seeing why the DM power spectrum has a "peak" at the scale of M-R equality.
- Initially our DM distribution is a δ-function.
- As the baryon-photon shell moves outwards during radiation domination, its gravity "drags" the DM, causing it to spread.
- The spreading stops once the energy in the photon-baryon shell no longer dominates: after M-R equality.
- The spreading of the δ-function ρ(r) is a smoothing, or suppression of high-*k* power.

Shape of *P(k)* in pictures



Eisenstein, Seo & White (2007)

Features of baryon oscillations

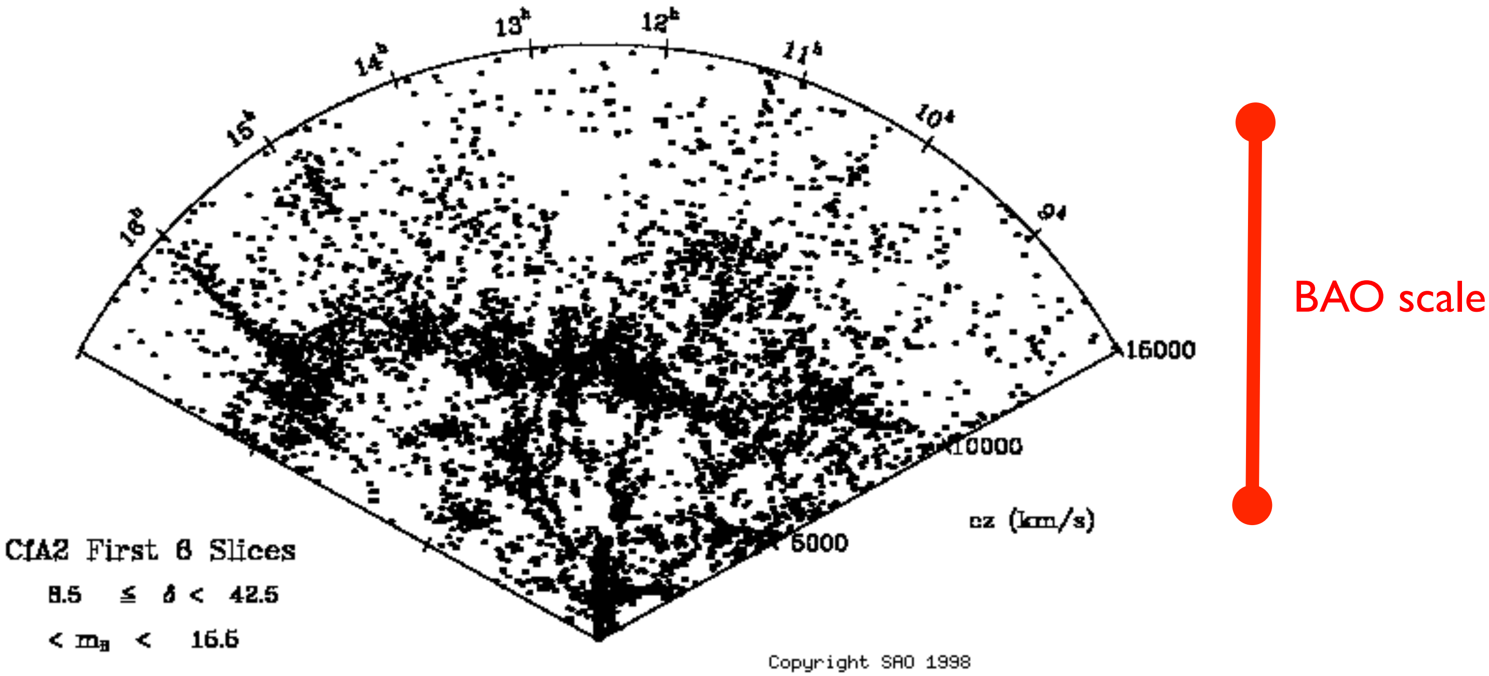
- Firm prediction of models with Ω_b>0
- Positions well predicted once (physical) matter and baryon density known - calibrated by the CMB.
- Oscillations are "sharp", unlike other features of the power spectrum.
- Internal cross-check:
  - d_A should be the integral of $H^{-1}(z)$.
- Since have $d(z)$ for several z's can check spatial flatness (addition law for distances).
- Ties low-z distance measures (e.g. SNe) to absolute scale defined by the CMB (in Mpc, not h^{-1}Mpc).
  - Allows ~1% measurement of h using trigonometry!

The program

- Find a tracer of the mass density field and compute its 2-point function.
- Locate the features in the above corresponding to the sound horizon, *s*.
- Measure the $\Delta\theta$ and Δz subtended by the sound horizon, *s*, at a variety of redshifts, *z*.
- Compare to the value at $z\sim10^3$ to get d_A and $H(z)$
- Infer expansion history, DE properties, modified gravity.

But ruler inconveniently large …

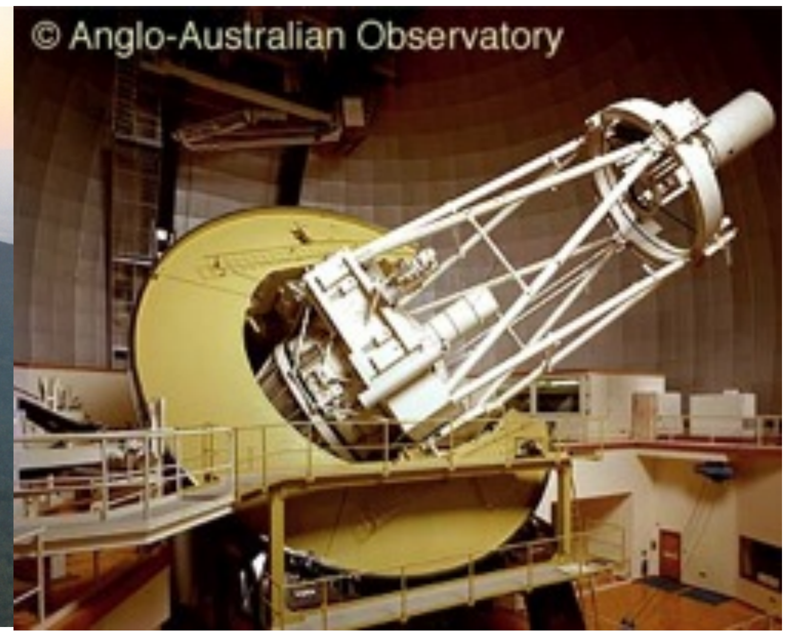
Early surveys too small



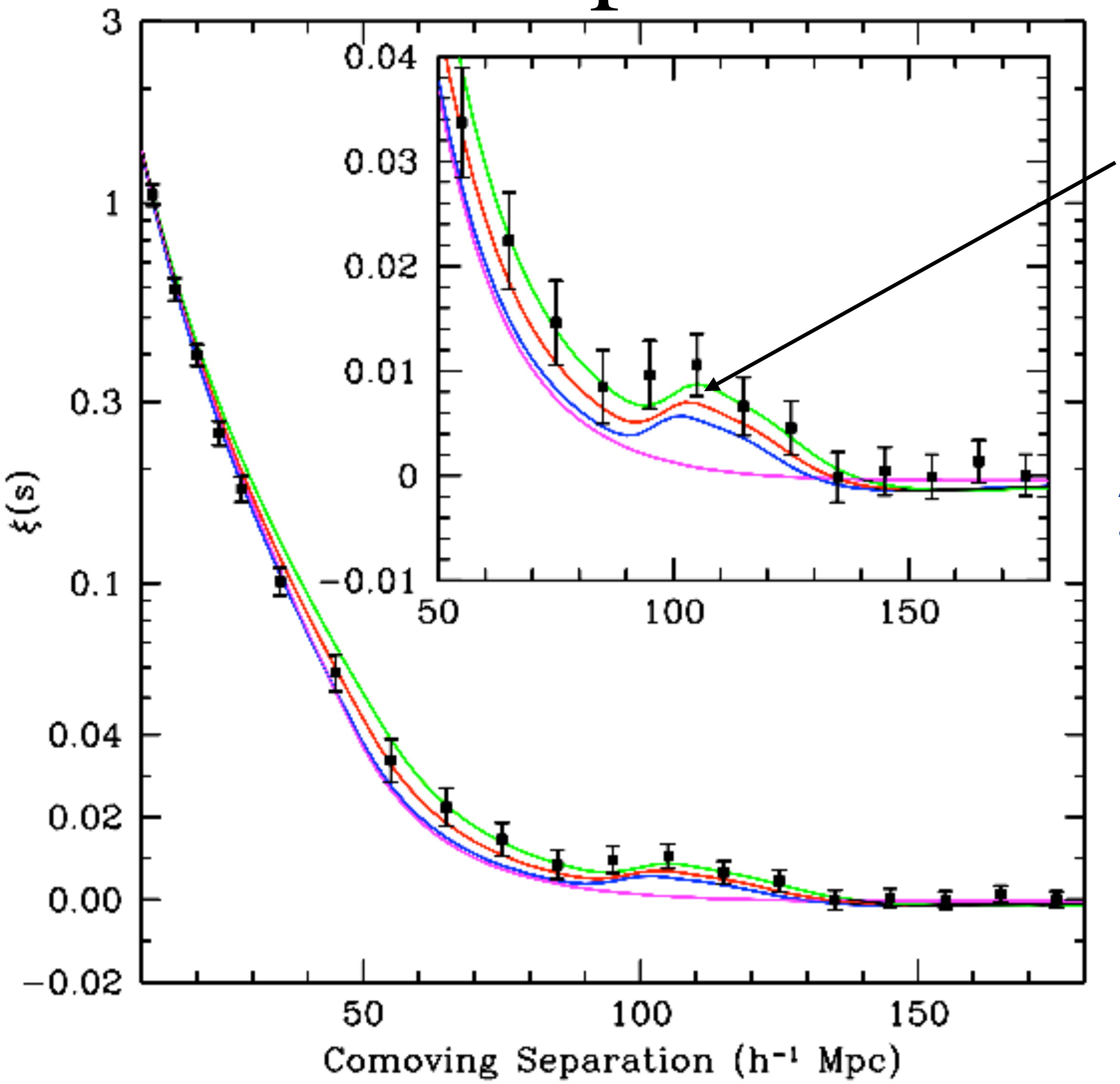
CfA2 redshift survey (Geller & Huchra 1989)
Formally, this could “measure” BAO with a ~0.05σ detection

Finally technically possible

SDSS and 2dF surveys allowed detection of BAO signal …



Another prediction verified!!



Eisenstein et al. (2005) detect oscillations in the SDSS LRG $\xi(r)$ at z~0.35! Knowing ***s*** determines D (*z*=0.35).

About 10% of the way to the surface of last scattering!

Constraints argue for the existence of DE, but do not strongly constrain its properties.

Current state of the art

1. Eisenstein et al 2005
   - 3D map from SDSS
   - 46,000 galaxies, 0.72 $(h^{-1}\ \mathrm{Gpc})^3$

   (spectro-z)
   4% distance measure

2. Cole et al 2005
   - 3D map from 2dFGRS at AAO
   - 221,000 galaxies in 0.2 $(h^{-1}\mathrm{Gpc})^3$

   (spectro-z)
   5% distance measure

3. Hutsi (2005ab)
   - Same data as (1).
4. Padmanabhan et al 2007
   - Set of 2D maps from SDSS
   - 600,000 galaxies in 1.5 $(h^{-1}\mathrm{Gpc})^3$

   (photo-z)
   6% distance measure

5. Blake et al 2007
   - (Same data as above)
6. Percival et al 2007
   - (Combination of SDSS+2dF)
7. Okumura et al 2007
   - (Anisotropic fits)

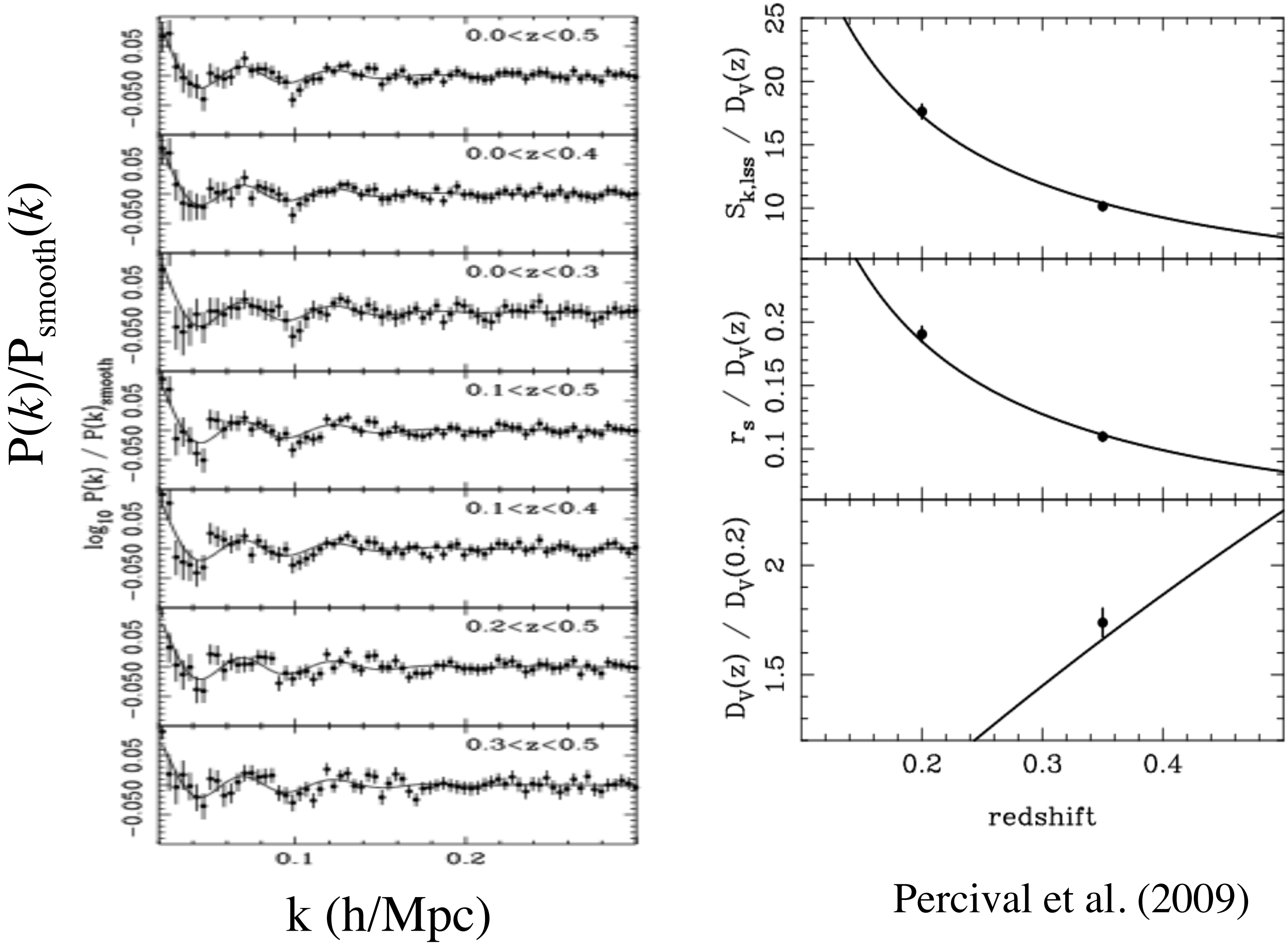
15. Gaztanaga et al. 2008a
    - (3pt function)
16. Gaztanaga et al. 2008b
    - (line-of-sight)

    (spectro-z)
    Detection

19. Percival et al. 2009
    - (DR7)
20. Kazin et al. (2010)
    - (DR7)

    (spectro-z)
    2.7%

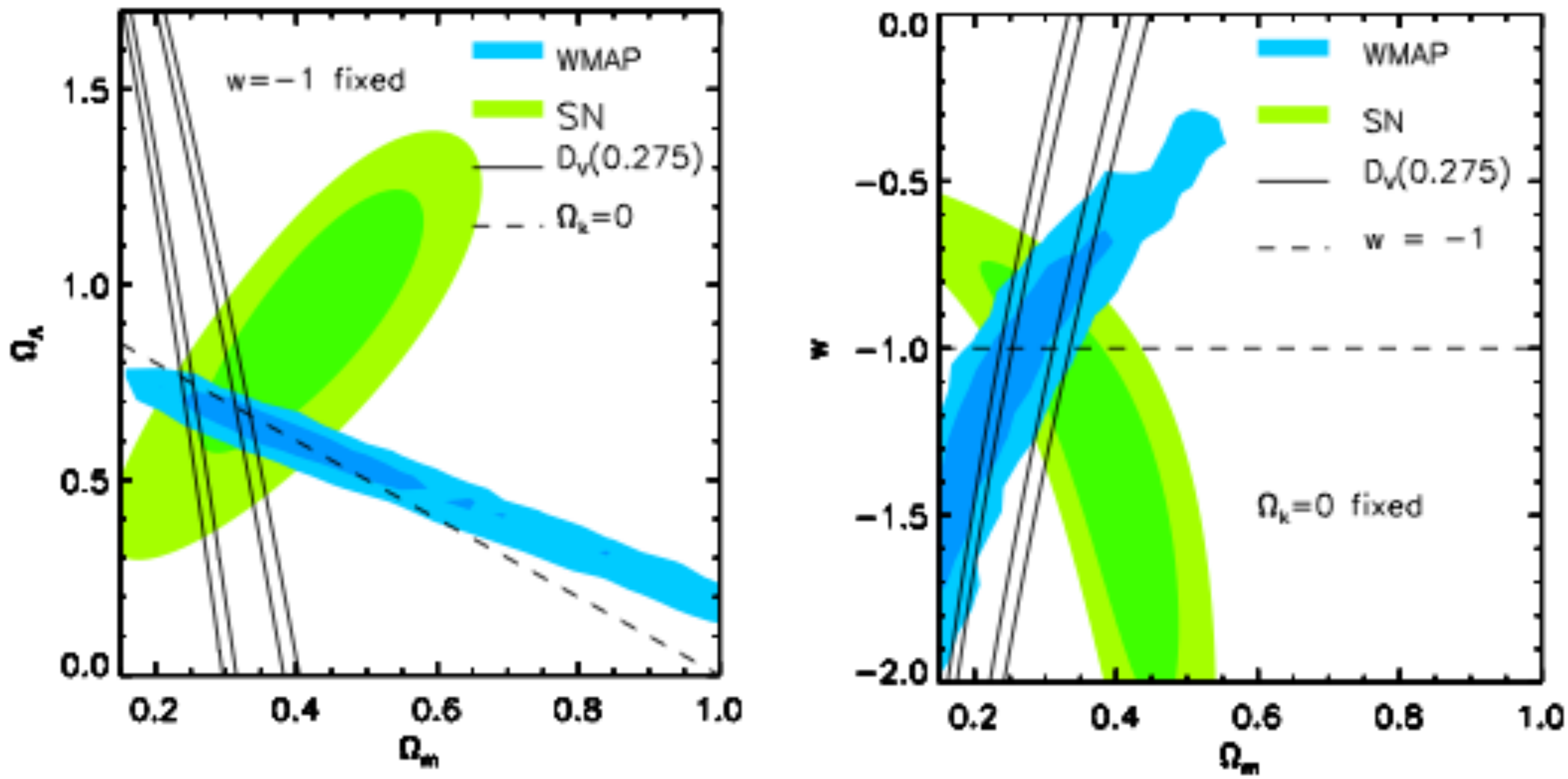
Current combined constraints



Percival et al. (2009)

… on cosmological parameters

Constraints on cosmological parameters from the distance to z=0.275.



From Percival et al. (2009); Reid et al. (2009)

The next step?

- We need a much more precise measurement of *s* at more redshifts to constrain DE.
- To measure *P(k)* or ξ(r) well enough to see such subtle features requires many well defined modes
  - a Gpc^3 volume.
  - Million(s) of galaxies.
  - Systematic errors need to be controlled to high precision.

The next generation

- There are now proposals for several next-generation BAO surveys, both spectroscopic and photometric.
  - Photometric surveys generally deeper and wider.
  - Not a requirements driver if already doing weak lensing.
  - More susceptible to systematic errors in *z* determination.
  - Generally takes 3-10x as much sky for same constraints as a spectro survey (# modes in 2D vs 3D).
  - Cannot make use of “reconstruction”.
- Future surveys should be able to measure d_A and H to ~1%, giving competitive constraints on DE
- Highly complementary to SNe surveys
  - Completes distance triangle, constrains Ω_K.
  - Locks SNe to absolute distance scale to CMB (in Mpc): h to ~1%.

The landscape

- It's difficult to do BAO at very low z, because you can't get enough volume.
- BAO surveys "turn on" around z~0.3 and can go as high as z~3.
- A point at high z constrains Ω_K
  - Allowing focus on w_0 and w_a at lower z.
- Lower z very complementary to SNe.
  - Completes distance triangle, constrains curvature.
  - Ground BAO+Stage IV SNe (opt), FoM ↑~6x.
- Tests of GR?
  - Can do lensing from BAO, but weak constraint.
  - Assuming GR, distances give $\delta(z\sim1)/\delta(z\sim10^3)$ to <1%.
  - A spectroscopic survey that does BAO can use redshift space distortions to measure the temporal metric perturbations (c.f. WL which measures sum of temporal and spatial) and hence constrain dD/dln(a).

Not-so-next-generation surveys

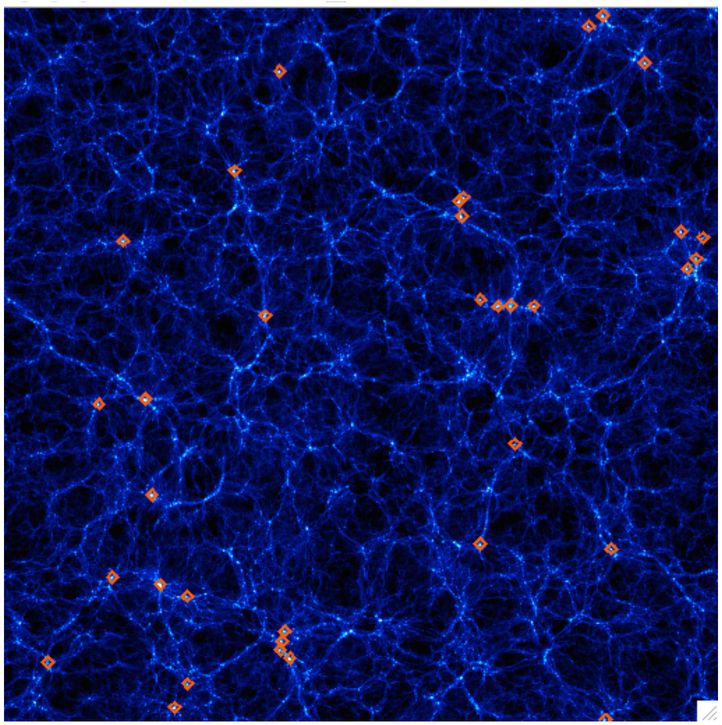
The final round of data (DR7) from SDSS-I & II has been analyzed -- the “next” generation of surveys is underway.

Project	Redshift	Area (sq. deg.)	n (10^{-4})
WiggleZ	0.4-1.0	1,000	3
HETDEX	2.0-4.0	350	4
SDSS-III (BOSS)	0.1-0.8 + 2.0-3.0	10,000 + 8,000	3
LAMOST	0-1	8,000	5
Pan-STARRS*	0-1	20,000	10

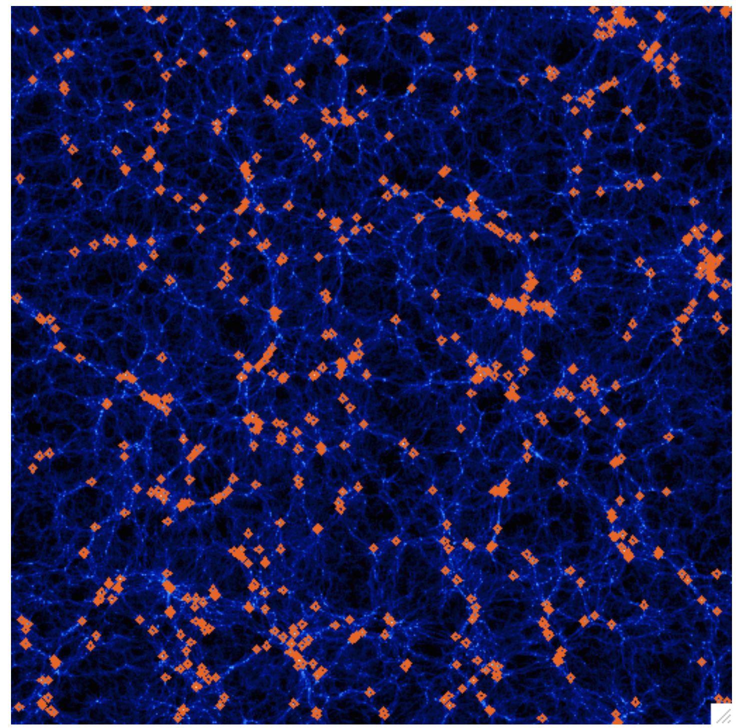
With more waiting in the wings …

Tracing large-scale structure

The cosmic web at z~0.5, as traced by luminous red galaxies

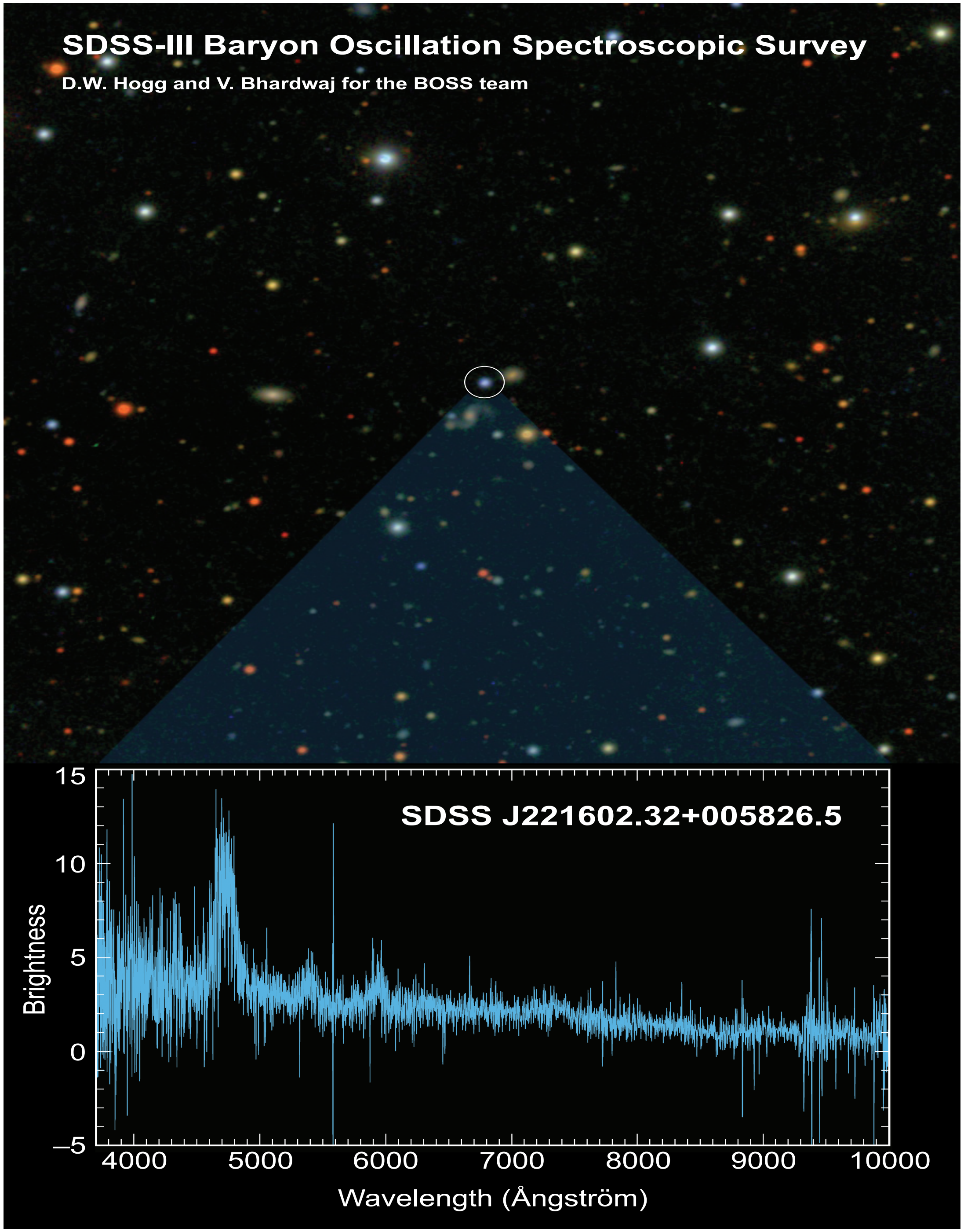


SDSS



BOSS

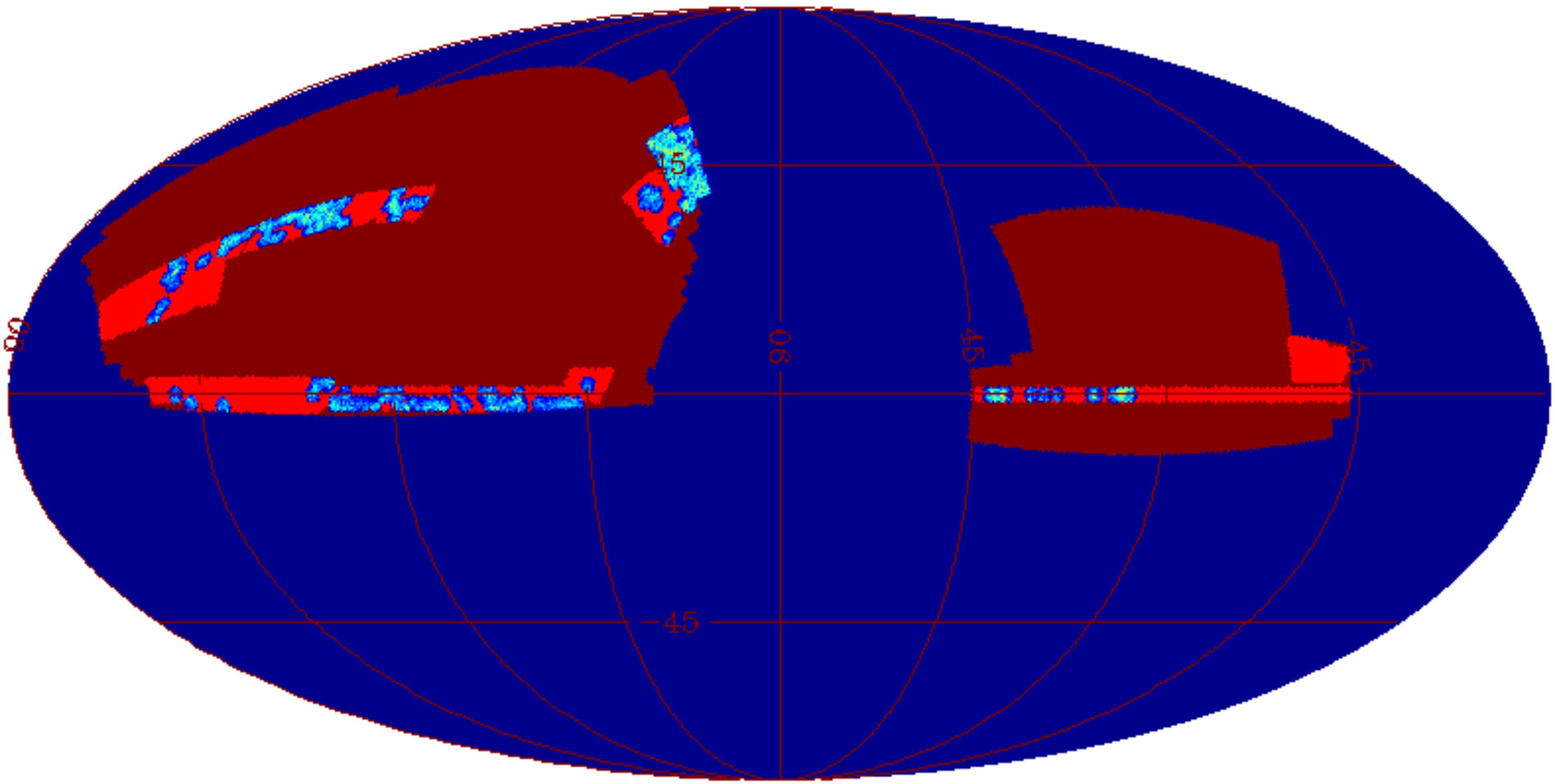
A slice 500h^{-1} Mpc across and 10 h^{-1} Mpc thick



The upgraded BOSS spectrographs achieved 1st light in Sep. 2009 and BOSS is currently taking data.

Spectroscopy will continue through 2014 with regular data releases to the public (starting in 2012).

BOSS: current status



Findings of the Dark Energy Task Force

(Reporting to DOE, NASA & NSF; chair Rocky Kolb)

- Four observational techniques for studying DE with baryon oscillations:
- “Less affected by astrophysical uncertainties than other techniques.”
- **BUT**
- “We need…Theoretical investigations of how far into the non-linear regime the data can be modeled with sufficient reliability and further understanding of galaxy bias on the galaxy power spectrum.”

Those pesky details …

- I have argued (convincingly?) that we understand and can calculate the real space, linear theory, matter power spectrum with exquisite accuracy and that it contains highly useful features for cosmology.
- Unfortunately we don’t measure the linear theory matter power spectrum in real space.
- We measure:
  - the non-linear
  - galaxy power spectrum
  - in redshift space
- How do we handle this?

Recent BAO "theory"

With the basic measurement demonstrated/validated, theoretical attention has been divided into four areas

1. Understanding the effects of non-linearity, bias & redshift space distortions.
2. Understanding how to perform "reconstruction".
3. Studying BAO in the IGM.
4. Looking at statistical estimators, covariance matrices, etc.

Effects of non-linearity: mass

As large-scale structure grows, neighboring objects "pull" on the baryon shell around any point. This causes a broadening of the peak and additional non-linear power on small scales. From simulations or PT (of various flavors) one finds:

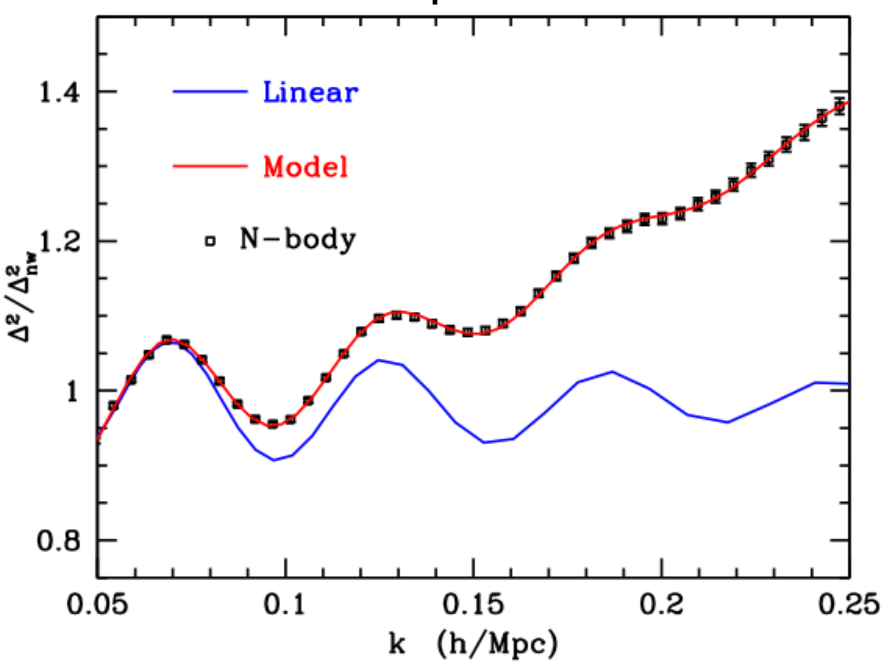
$$\Delta^2(k) = \left\{\Delta^2_{\rm lin}(k) + \cdots\right\} \exp\left[-k^2\Sigma^2/2\right] + \Delta^2_{22} + \cdots$$

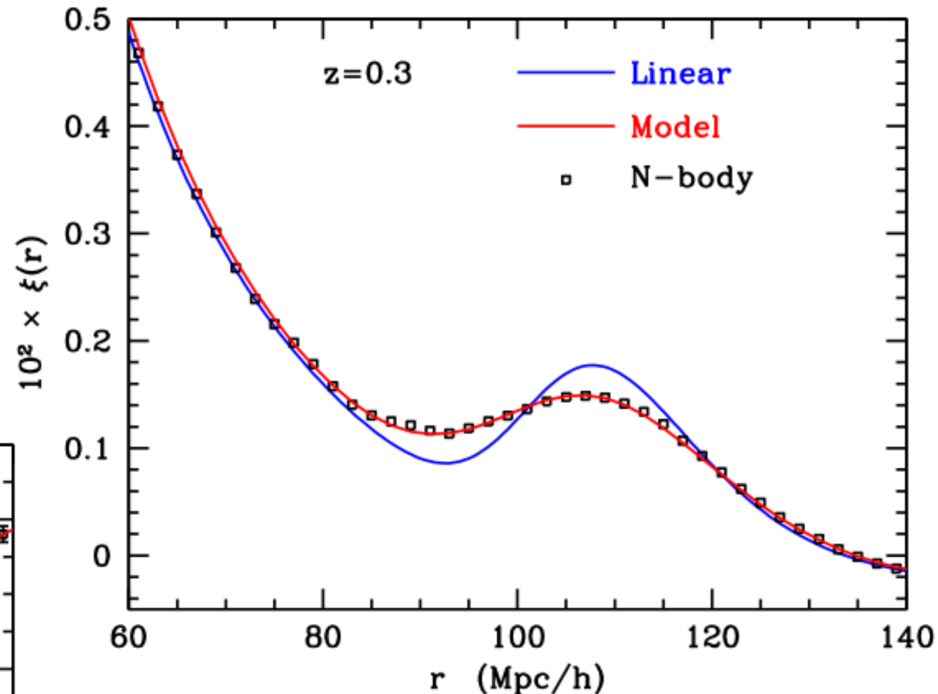
This does a reasonable job of providing a "template" low-z spectrum, and it allows us to understand where the information lives in Fourier space [forecasting].

Bharadwaj (1996); Eisenstein, Seo & White (2007); Smith, Scoccimarro & Sheth (2007); Eisenstein et al. (2007); Matsubara (2007); Padmanabhan, White & Cohn (2009); Padmanabhan & White (2009); Seo et al. (2009); Noh et al. (2009); Mehta et al. (2010); …

Non-linearities smear the peak

Loss of contrast and excess power from non-linear collapse.





Broadening of feature due to Gaussian smoothing and ~0.5% shift due to mode coupling.

Information on the acoustic scale

- For a Gaussian random field Var[x^2]=2Var[x]^2, so our power spectrum errors are go as the square of the (total) power measured.
  - Measured power is *P+1/n*
- For a simple 1D model

(Seo & Eisenstein 2006)

$$\sigma_{\ln s}^{-2} = \frac{V}{2} \int \frac{d^3k}{(2\pi)^3} \left(\frac{\partial P / \partial \ln s}{P + \bar{n}^{-1}} \right)^2$$

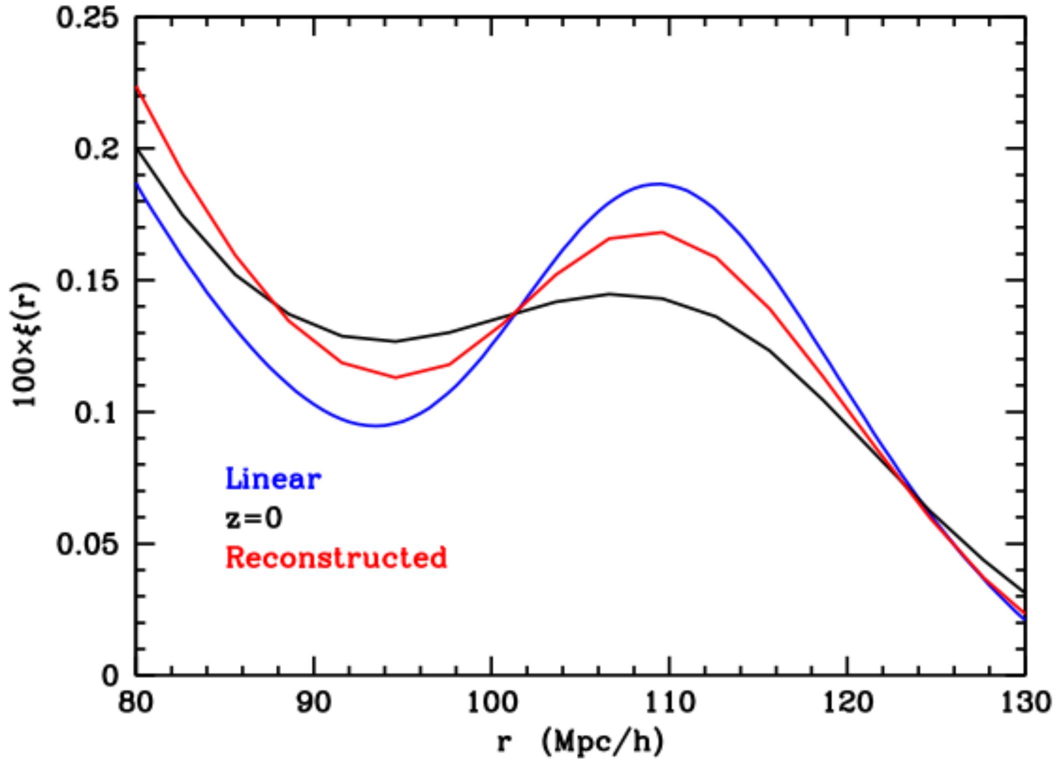
- Note that *δP/δlns* depends only on the wiggles while *P+1/n* depends on the whole spectrum.
- The wiggles are (exponentially) damped at high *k*.
- A more complete treatment keeps the angle-dependence due to redshift space distortions.
  - Such Fisher forecasts agree well with the results of numerical simulations.

Reconstruction

- The broadening of the peak comes from the "tugging" of large-scale structure on the baryon "shell".
- We measure the large-scale structure and hence the gravity that "tugged".
- Half of the displacement in the shell comes from "tugs" on scales > 100 Mpc/h
- Use the observations to "undo" non-linearity (Eisenstein++07)
  - Measure $\delta(x)$, infer $\phi(x)$, hence displacement.
  - Move the galaxies back to their original positions.
- Putting information from the phases back into $P(k)$.
- There were many ideas about this for measuring velocities in the 80's and 90's; but not much of it has been revisited for reconstruction (yet).

Eisenstein++07; Huff++07; Seo et al.++08,09; Wagner++08; Padmanabhan++09; Mehta++09; Noh++09; …

Reconstruction



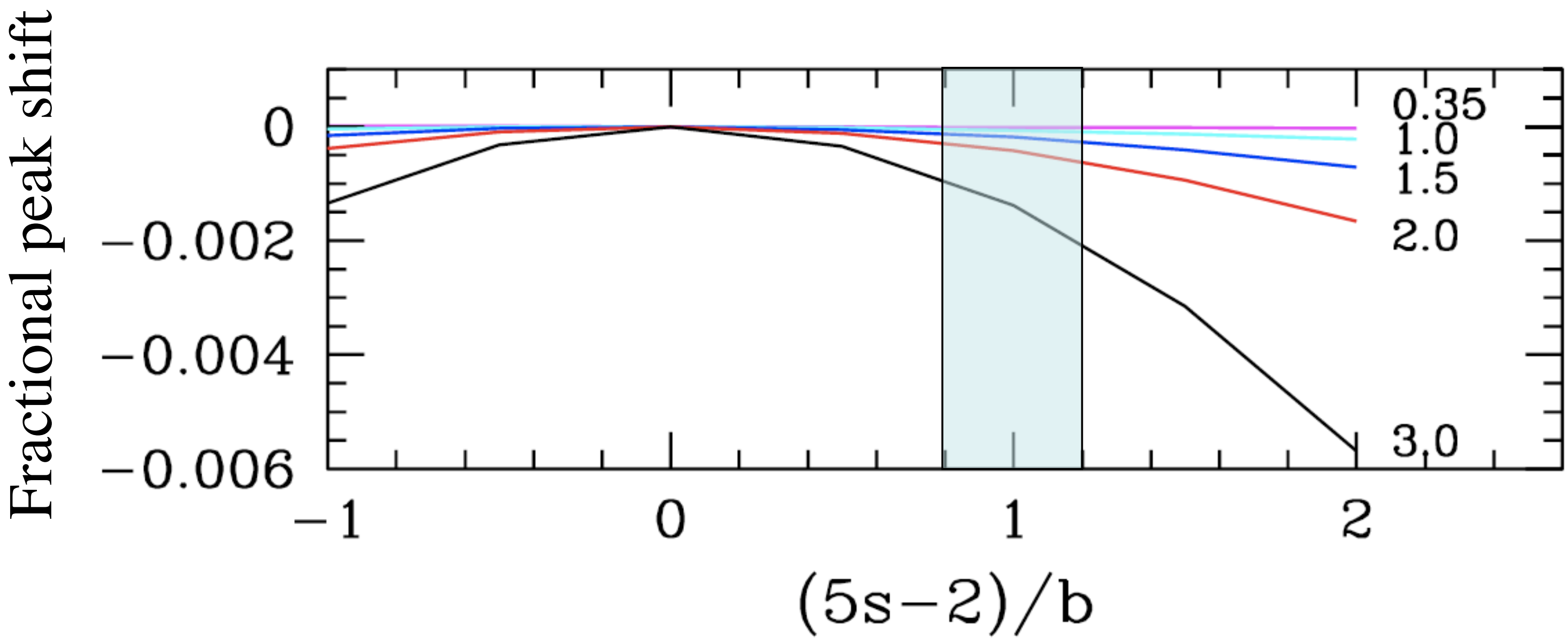
Reconstruction helps to sharpen the peak in the correlation function which is smeared by non-linear evolution.

This seems relatively "easy", **BUT**, to date reconstruction hasn't been demonstrated on non-simulated data.

Lensing

Hui, Gaztanaga & LoVerde: effects of lensing on the correlation function.
For next-generation experiments effect is small.
Eventually may be measurable: template known.

$$\xi_{obs}(R,z) = \xi\left(\sqrt{R^2+z^2}\right) + f(R)z + g(R)$$

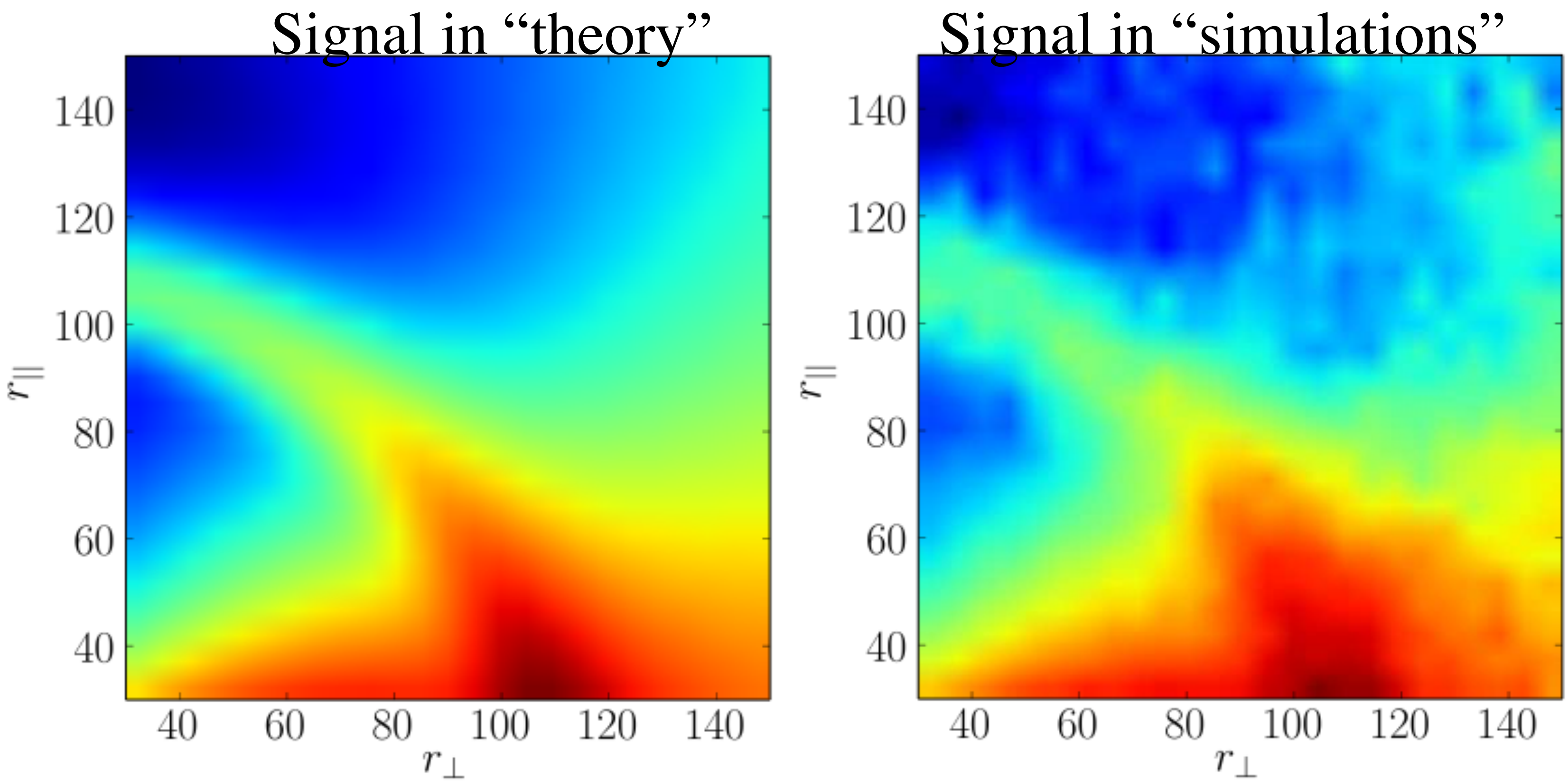


BAO and the IGM

- Distance constraints become tighter as one moves to higher *z*
  - More volume per sky area.
  - Less non-linearity.
- Expensive if use galaxies as tracers.
- Any tracer will do: HI
  - 21cm from HI in galaxies: SKA or custom expt.
  - Lyα from IGM as probed by QSOs.
    - If IGM is in photo-ionization equilibrium
      - Absorption traces mass in a calculable way.
      - Flux(λ) ~ $\exp[-A(1+\delta)^\beta]$ (Cen++94, Hui & Gnedin 97, Croft++98)
    - A dense grid of QSO sightlines could probe BAO
      - (White 2003, McDonald & Eisenstein 2007, Slosar++09, White++10)
- e.g. 8,000 deg^2 to g~22 gives 1.5% (d_A & H)
  - Comparable to other forecasts but with a 2.5m telescope!

BAO at high *z*

Slosar, Ho, White & Louis (2009)



BAO feature survives in the LyA flux correlation function, because on large scales flux traces density. Relatively insensitive to astrophysical effects.

(see also Norman++09, White++10)

New surveys, new statistics

- Estimating the 2-point function from survey data is an old problem.
  - Most techniques we use today were developed decades ago when surveys were in a very different regime.
    - Landy-Szalay: optimal for small N_{gal} in the no-clustering regime.
- New modes of operation.
  - Surveys are much larger, boundaries often less important, but
  - Signals are smaller and
  - Careful attention to errors is critical for proper statistical inference.
  - Frequently (always?) compare observations to simulations.
    - Does the statistic "play well" with periodic simulation boxes?
- Higher order statistics?
  - N-point functions.
    - On large scales structure is pretty Gaussian. Necessary?
    - Reconstruction??
  - Marked correlation functions with e.g. mark ρ.

Ongoing work

- Templates for fitting data, able to account for non-linearity, redshift space distortions and galaxy bias.
- New estimators optimized for large-scale signals calibrated by numerical simulations.
- Models for the covariance matrices, calibrated by simulations.
- More sophisticated reconstruction algorithms.
- Some “new” ideas, and experimental approaches …

Conclusions

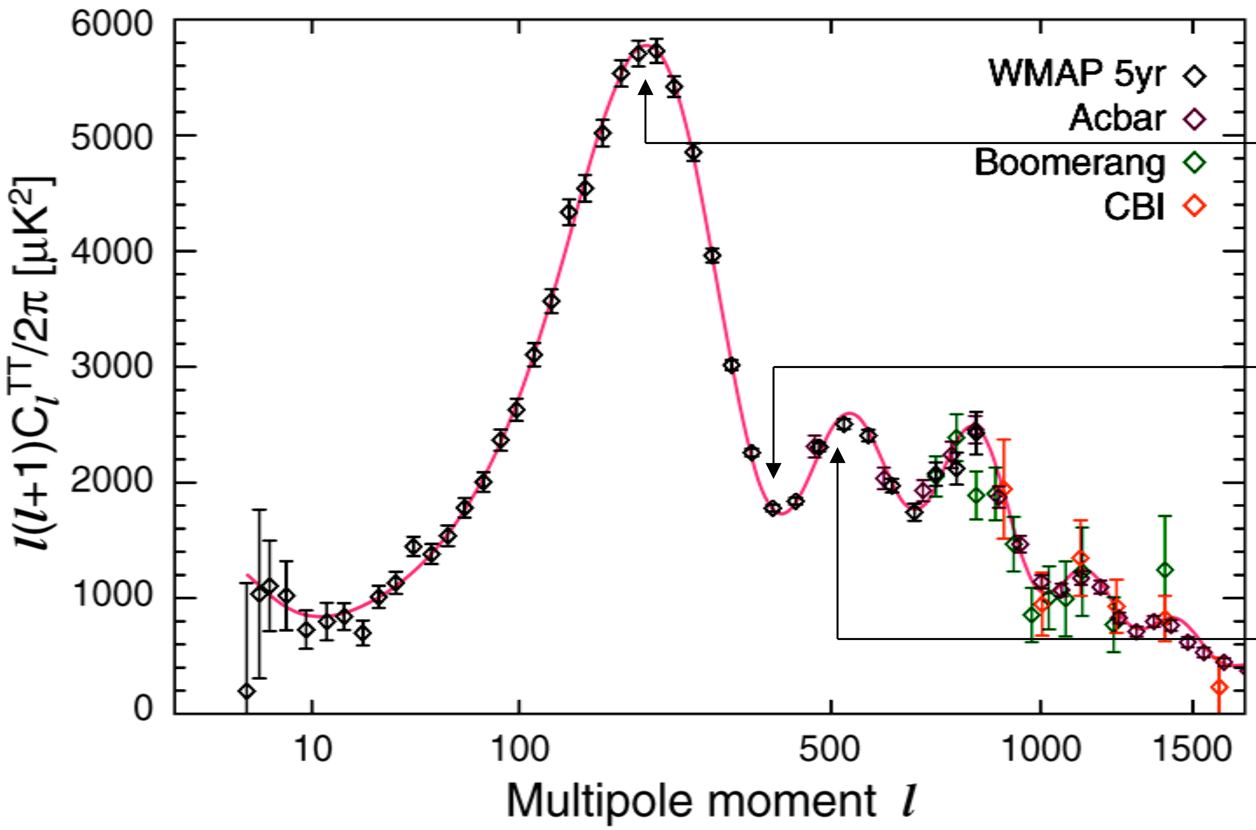
- Baryon oscillations are a firm prediction of CDM models.
- Method is "simple" geometry, with few systematics.
- The acoustic signature has been detected in the SDSS!
- With enough samples of the density field, we can measure $d_A(z)$ and $H^{-1}(z)$ to the percent level and thus constrain DE.
  - Was Einstein right?
- Require "only" a large redshift survey - we have >20 years of experience with redshift surveys.
- Exciting possibility of doing high z portion with QSO absorption lines, rather than galaxies.
- It may be possible to "undo" non-linearity.
- We will fill in more details next time …

The End

BAO in more detail

Linear theory revisited

Acoustic oscillations seen!



First "compression", at $kc_st_{ls}=\pi$. Density maxm, velocity null.

Velocity maximum

First "rarefaction" peak at $kc_st_{ls}=2\pi$

Acoustic scale is set by the *sound horizon* at last scattering: $s = c_s t_{ls}$

Beyond the cartoon

- In Newtonian gauge the evolution of the baryon and photon perturbations is governed by:
  - Continuity equation(s):

$$\begin{aligned} \dot{\delta}_\gamma &= -\tfrac{4}{3}kV_\gamma & -4\dot{\Phi} \\ \dot{\delta}_b &= -kV_b & -3\dot{\Phi} \end{aligned}$$

$$\Big(a \to a[1+\Phi]\Big)$$

  - Euler equation(s):

$$\dot{V}_\gamma = k\left[\tfrac{1}{4}\delta_\gamma + \Psi - \tfrac{1}{6}\Pi_\gamma\right] - \dot{\tau}\left(V_\gamma - V_b\right)$$

$$\dot{V}_b = -(\dot{a}/a)V_b + k\Psi + \dot{\tau}\left(V_\gamma - V_b\right)/R$$

Fluid equations

- These equations can be easily derived by stress-energy conservation, but physically:
  - Densities are enhanced/reduced by converging/ diverging flows and by the stretching of space.
  - Accelerations are sources by gradients of the potential, and comoving velocities decay due to the expansion.
- Scattering of photons off free electrons couples drags V_γ-V_b to zero, leading to a baryon-photon fluid.
  - The protons follow the electrons via electromagnetic interactions.

Acoustic oscillations: photons

- Ignore for now the τ and Π terms.

$$\dot{\delta} = -\tfrac{4}{3}kV - 4\dot{\Phi}$$

$$\dot{V} = k\left(\tfrac{1}{4}\delta + \Psi\right)$$

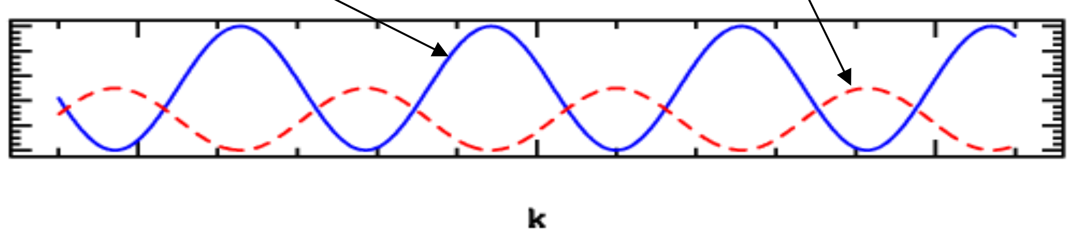
$$\Longrightarrow \quad \frac{\ddot{\delta}}{4} + \frac{k^2}{3}\frac{\delta}{4} = -\frac{1}{3}k^2\Psi - \ddot{\Phi}$$

- If Φ~const this becomes:

$$\frac{d^2}{d\eta^2}\left(\frac{\delta}{4} + \Psi\right) + k^2 c_s^2 \left(\frac{\delta}{4} + \Psi\right) = 0 \quad \Rightarrow \quad \left(\frac{\delta}{4} + \Psi\right) = A\cos\left(ks\right) + \cdots$$

Effective temperature

$$(\Delta T)^2_{\rm ls} \sim \cos^2(kc_s t_{\rm ls}) + \text{velocity terms}$$



Matter curves space

- The fluctuations in the matter/radiation generate spatial curvature:

$$k^2\Phi = 4\pi G a^2 \sum \rho_i \delta_i + 3\frac{\dot{a}}{a}\left(\rho_i + p_i\right) V_i / k$$

$$k^2\left(\Phi + \Psi\right) = -8\pi G a^2 \sum p_i \Pi_i$$

Tight coupling I

- At early times the density is high and the scattering is rapid compared with the "travel time" across a wavelength.
- To lowest order $V_\gamma=V_b=V$ and the continuity equation(s) give:

$$\begin{aligned}\dot{\delta}_\gamma &= -\tfrac{4}{3}kV_\gamma & -4\dot{\Phi}\\ \dot{\delta}_b &= -kV_b & -3\dot{\Phi}\end{aligned}$$

$$\begin{aligned}\frac{d}{d\eta}\left[(1+R)\dot{\delta}_b\right] &= \frac{d}{d\eta}\left[(1+R)\left\{-kV-3\dot{\Phi}\right\}\right]\\ &= -3\frac{d}{d\eta}\left[(1+R)\dot{\Phi}\right]-k\frac{d}{d\eta}\left[(1+R)V\right]\\ &= -3\frac{d}{d\eta}\left[(1+R)\dot{\Phi}\right]-k\dot{R}V-k(1+R)\dot{V}\end{aligned}$$

Tight coupling II

- Expand the Euler equation in powers of Compton mean-free-path over wavelength [or k/(dτ/dη)] to lowest order $V_\gamma = V_b = V$ and

$$\dot{V}_\gamma = k\left[\tfrac{1}{4}\delta_\gamma + \Psi - \tfrac{1}{6}\Pi_\gamma\right] - \dot{\tau}\left(V_\gamma - V_b\right)$$

$$\dot{V}_b = -(\dot{a}/a)V_b + k\Psi + \dot{\tau}\left(V_\gamma - V_b\right)/R$$

$$\begin{aligned}
\dot{V} + \tfrac{\dot{a}}{a}V &= k\Psi + R^{-1}\left[k(\delta_\gamma/4 + \Psi) - \dot{V}\right] \\
\left(\tfrac{1+R}{R}\right)\dot{V} + \tfrac{\dot{a}}{a}V &= \left(\tfrac{1+R}{R}\right)k\Psi + \tfrac{k}{R}\tfrac{\delta_\gamma}{4} \\
(1+R)\dot{V} &= (1+R)k\Psi + k(\delta_\gamma/4) - \tfrac{\dot{a}}{a}RV
\end{aligned}$$

Tight coupling III

- Combing these, and using $\delta_b=(3/4)\delta_\gamma$ for adiabatic fluctuations:

$$\frac{d}{d\eta}\left[(1+R)\dot{\delta}_b\right]+\frac{k^2}{3}\delta_b=-k^2(1+R)\Psi-\frac{d}{d\eta}\left[3(1+R)\dot{\Phi}\right]$$

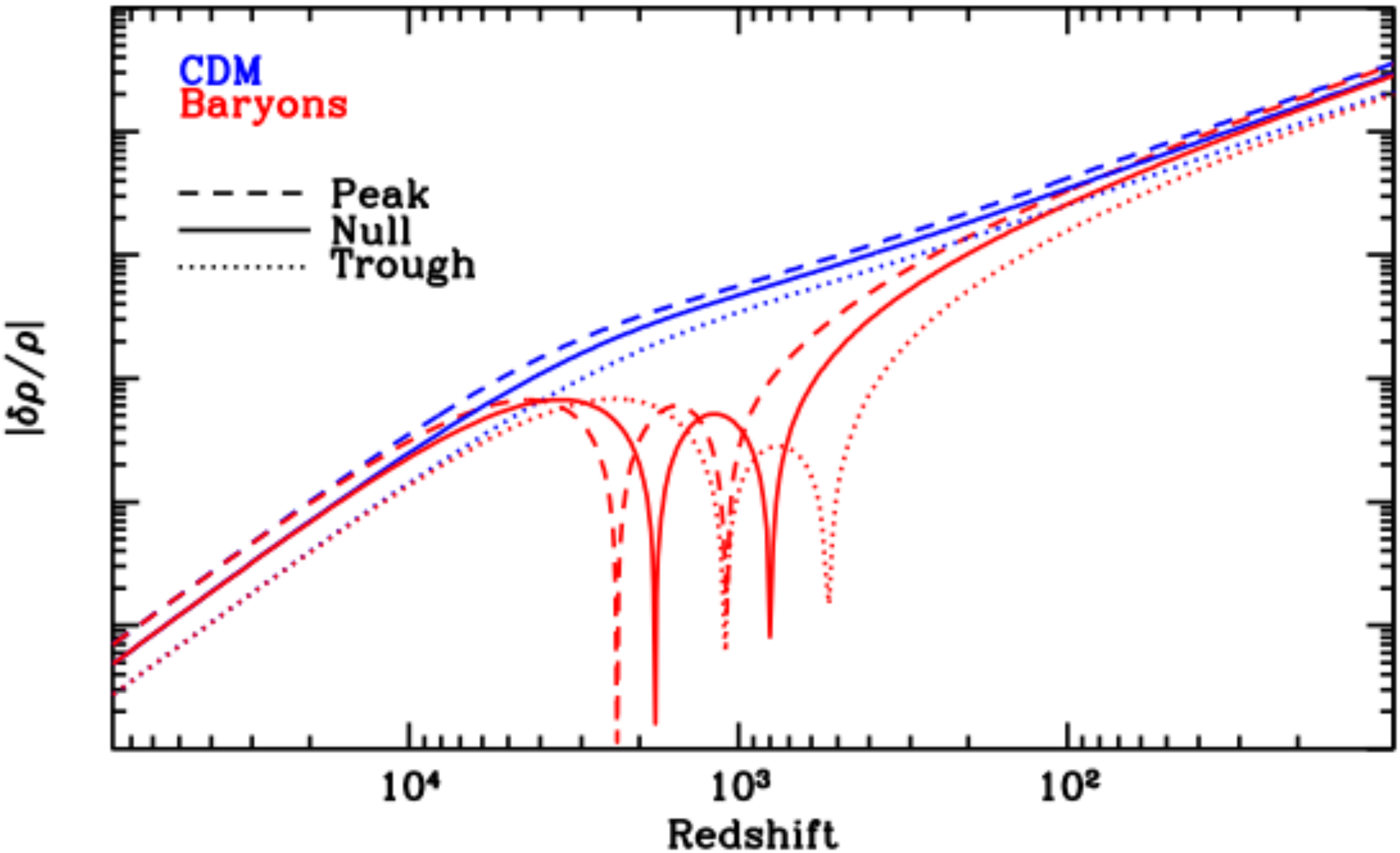
- A driven harmonic oscillator with natural frequency $c_s^{-2}=3(1+R)$.
- During tight-coupling the amplitude of the baryonic perturbation cannot grow
  - Harmonic motion with decaying amplitude $[(1+R)^{-3/4}$ in adiabatic limit.
- Baryons decouple when $\tau_b \sim 1$ $\left(\tau_b=\int\frac{\dot{\tau}\,d\eta}{1+R}\right)$

$$s=\int c_s(1+z)\,dt=\int\frac{c_s\,dz}{H(z)}=\frac{1}{\sqrt{\Omega_m H_0^2}}\frac{2c}{\sqrt{3z_{\rm eq}R_{\rm eq}}}\ln\frac{\sqrt{1+R_{\rm dec}}+\sqrt{R_{\rm dec}+R_{\rm eq}}}{1+\sqrt{R_{\rm eq}}}$$

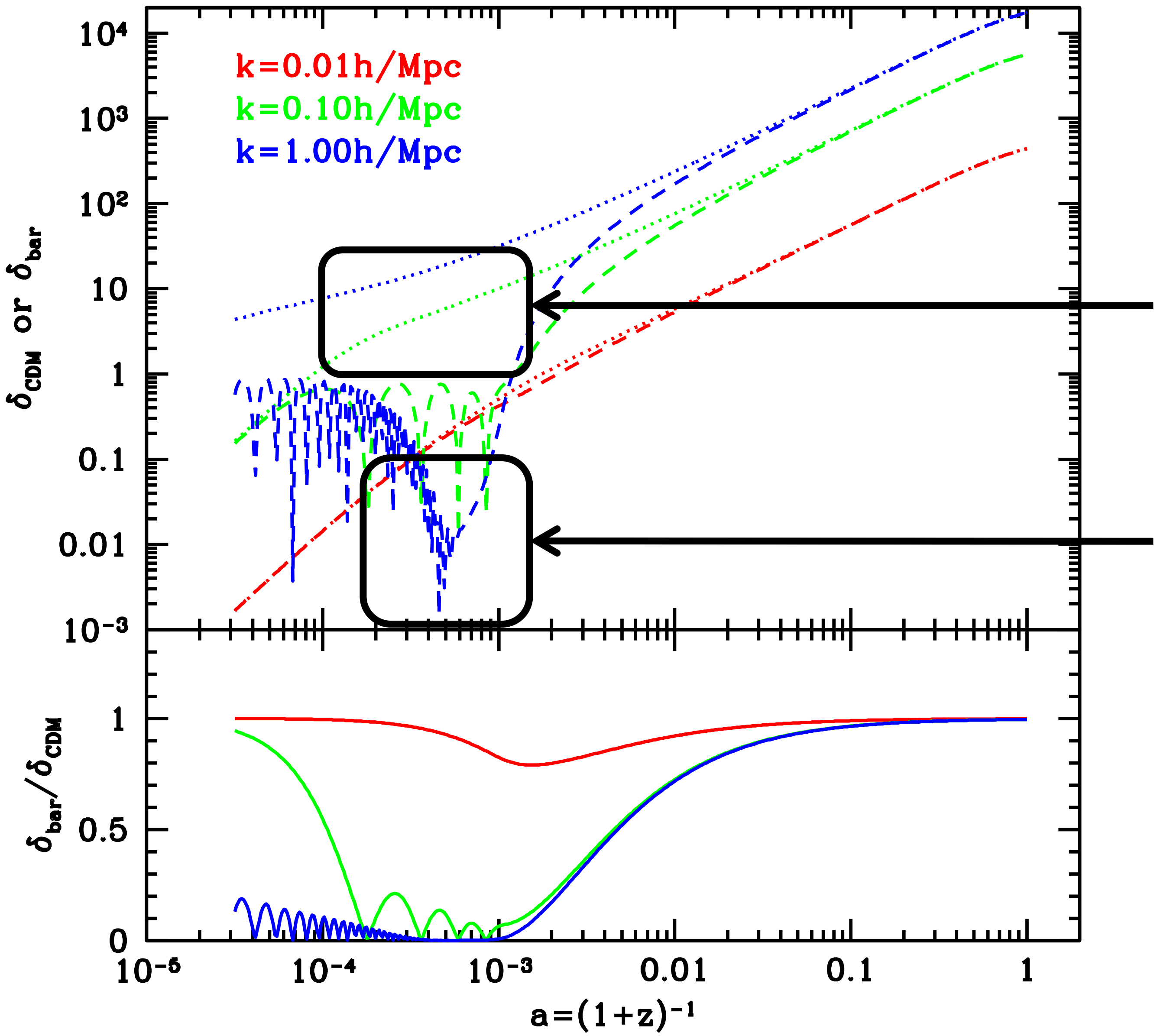
Post-decoupling

- Once the photons have released the baryons, both the CDM and baryon perturbations grow with $\delta \sim a$ (z>>1).
- Density and velocity perturbations from tight-coupling must be matched onto growing mode solution.
  - Velocity overshoot.
- Note: for the period between horizon entry and decoupling all perturbation growth is suppressed. Changes shape of *P(k)* near “peak”.
- Oscillations have larger amplitude for higher ω_B and lower ω_m

Evolution of perturbations



Perturbation evolution



Baryons slow the growth of the DM. “Stagflation”.

Oscillations at high *k* are damped.

Diffusion/Silk damping

- If we expand to next order in $k/[d\tau/d\eta]$ and assume R, Φ and Ψ are slowly varying we get a dispersion relation

$$\omega = \pm k c_s + \frac{ik^2}{6\dot{\tau}} \left[\frac{R^2}{(1+R)^2} + \frac{16}{15} \frac{1}{1+R} \right]$$

- which indicates (diffusion) damping of the oscillations with scale:

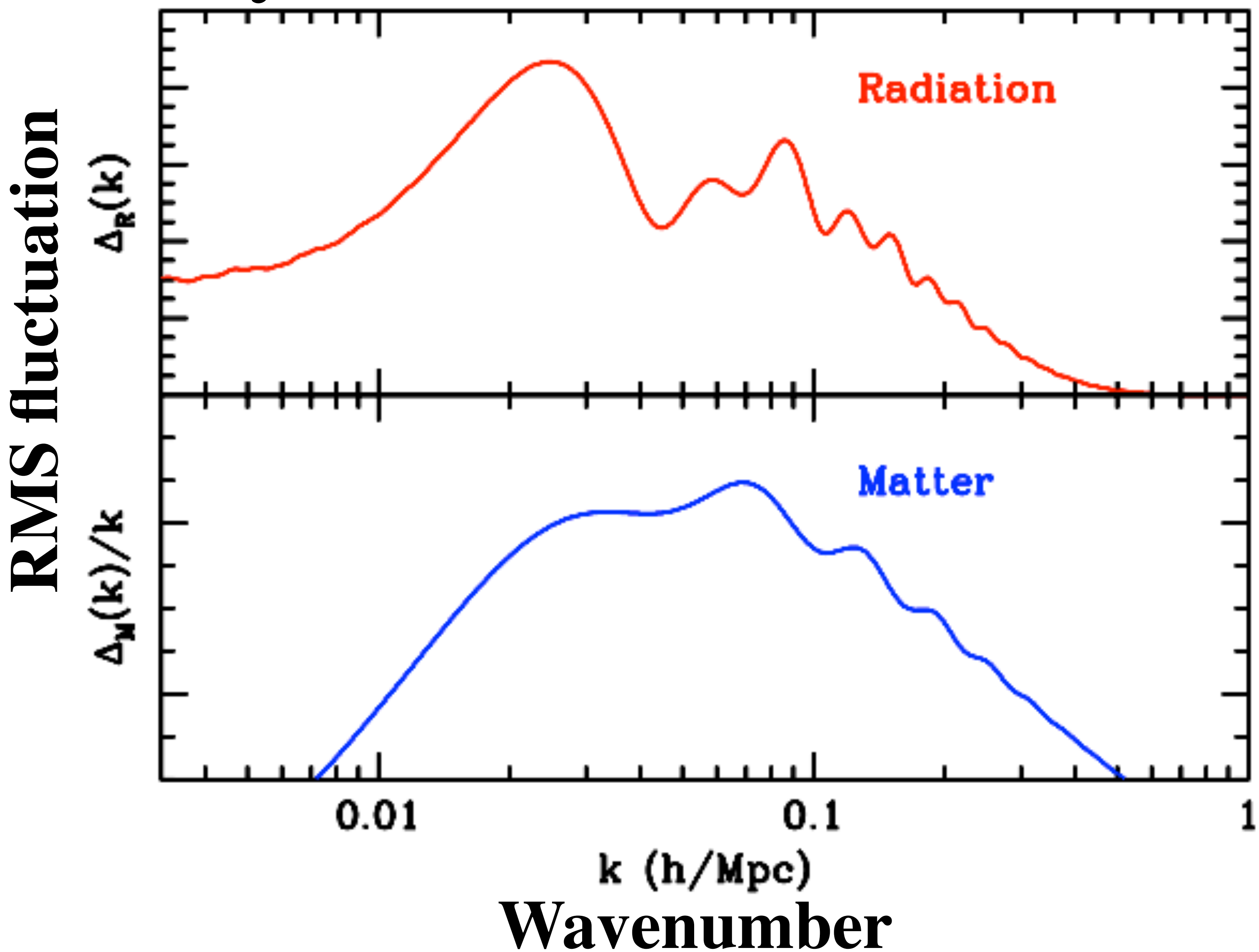
$$k_D^{-2} = \frac{1}{6} \int d\eta \; \frac{1}{\dot{\tau}} \; \frac{R^2 + 16(1+R)/15}{(1+R)^2}$$

Silk (1967),
Kaiser (1983),
Hu & White (1997)

  – Note $k_D \sim ([d\tau/d\eta]/\eta)^{1/2}$: geometric mean of mfp & horizon

- The acoustic signal is thus an (almost) harmonic series of peaks with a quasi-exponential damping at $k_D \sim 0.1$ *h*/Mpc.
  – True effect is more complicated due to rapid changes during recombination.

Baryon (acoustic) oscillations



DE or early universe weirdness?

- Key to computing ***s*** is our ability to model CMB anisotropies.
- Want to be sure that we don't mistake an error in our understanding of $z \sim 10^3$ for a property of the DE!
- What could go wrong in the early universe?
  - Recombination.
  - Misestimating c_s or ρ_B/ρ_γ.
  - Misestimating H(z>>1) (e.g. missing radiation).
  - Strange thermal history (e.g. decaying ν).
  - Isocurvature perturbations.
  - …
- It seems that future measurements of CMB anisotropies (e.g. with Planck) constrain ***s*** well enough for this measurement even in the presence of odd high-z physics.

Eisenstein & White (2004); White (2006)

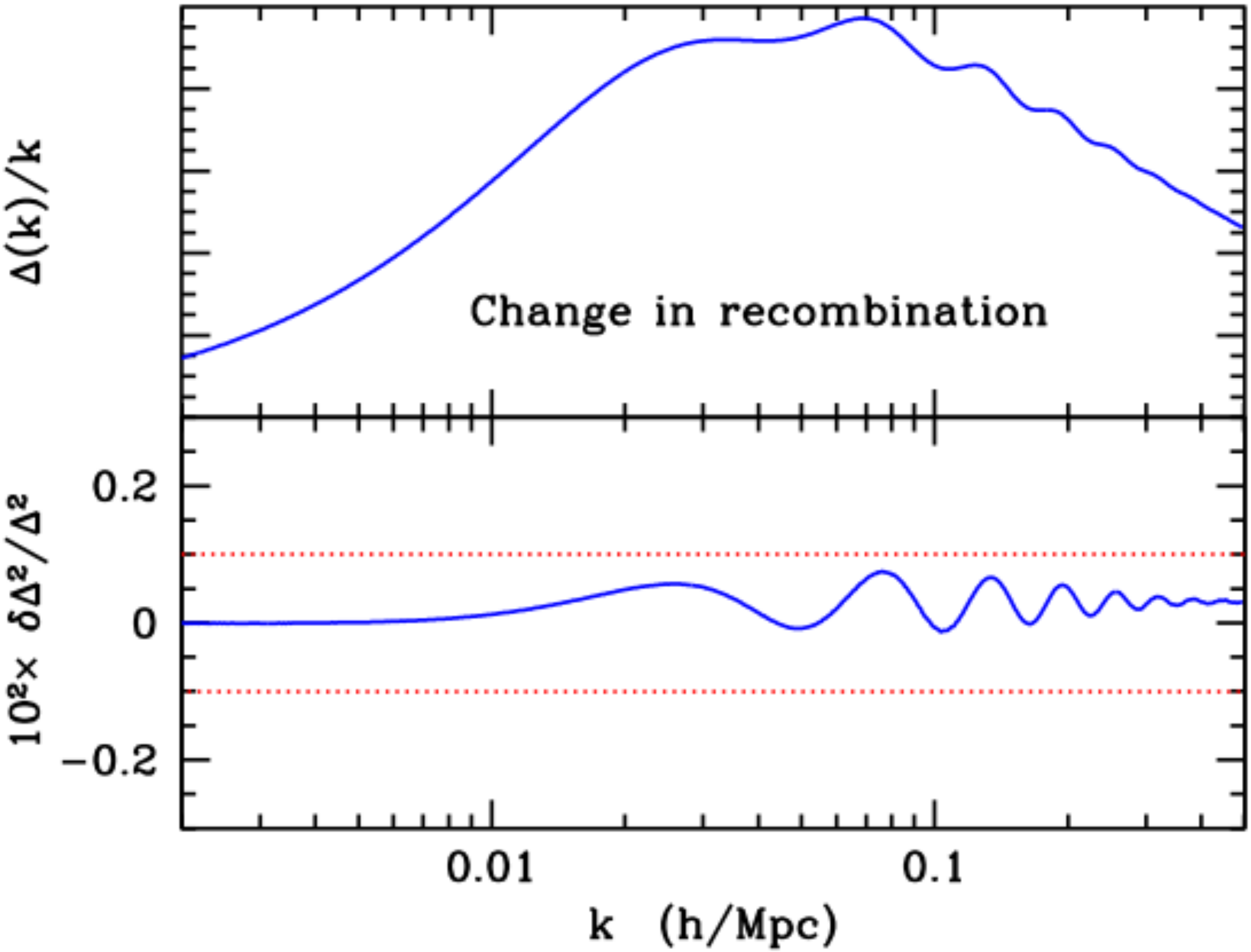
How well do we know *s*?

- The sound horizon is an integral, from early time to recombination, of the sound speed.

$$s = \int c_s(1+z)\,dt = \int \frac{c_s\,dz}{H(z)} = \frac{1}{\sqrt{\Omega_m H_0^2}} \frac{2c}{\sqrt{3 z_{\rm eq} R_{\rm eq}}} \ln \frac{\sqrt{1+R_{\rm dec}} + \sqrt{R_{\rm dec}+R_{\rm eq}}}{1+\sqrt{R_{\rm eq}}}$$

- Depends on
  - Details of recombination.
    - This is “just” atomic physics.
  - Sound speed.
    - The baryon-to-photon ratio, *R*.
  - Expansion rate [through *H(z)*]
    - Just matter and radiation at high *z*?

Recombination



What controls the sound horizon?

- Aside from the $\omega_m^{-1/2}$ prefactor, the sound horizon depends only on the baryon-to-photon ratio and equality*.
  - Robustly measured by CMB.
  - Actual densities of matter and radiation drop out of calculation.
  - Even if ω_m is misinterpreted, relative distances are unchanged, *h* is mis-measured.

*Neglecting early DE.

Eisenstein & White (2004)

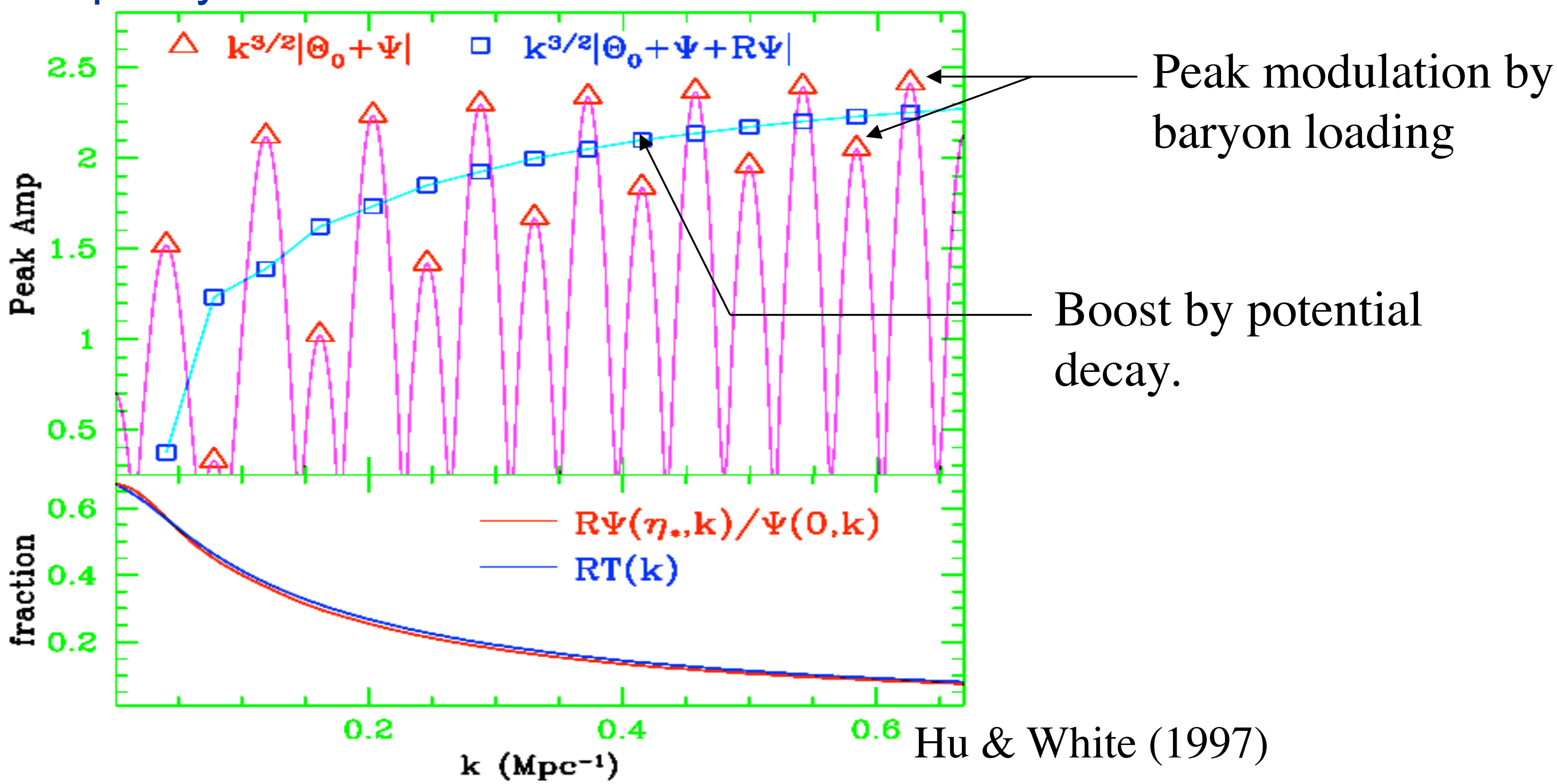
Baryon loading and the potential envelope

- Baryons give weight to the photon-baryon fluid. This makes it easier to fall into a potential well and harder to "bounce" to become a rarefaction.
  - Baryon loading enhances the compressions and weakens the rarefactions, leading to an alternating height of the peaks.
- At earlier times the baryon-photon fluid contributes more to the total density of the universe than the CDM. The effects of baryon-photon self-gravity enhance the fluctuations on small scales.
  - Since the fluid has pressure, it is hard to compress.
  - Infall into potentials is slower than free-fall.
  - Because the (over-)density cannot grow fast enough, the potential is forced to decay by the expansion of the universe.
  - The photons are then left in a compressed state with no need to fight against the potential as they leave -- enhancing small-scale power.

Measuring the higher peaks constrains the matter density!

Modulation and enhancement

- The acoustic oscillations are suppressed at small angular scales by diffusion/Silk damping. Well understood!
- Removing this shows the effects of baryons and the epoch of equality.



Hu & White (1997)

What do we measure?

- Epoch of recombination and ρ_b/ρ_γ are well measured, and influence *s* relatively little.
- Equality will (soon) be well measured, and also affects *s* relatively little.
- Thus we well constrain $\omega_m^{1/2}\, s$ at $z{\sim}10^3$.

$$s = \int c_s (1+z)\, dt = \int \frac{c_s\, dz}{H(z)} = \frac{1}{\sqrt{\Omega_m H_0^2}} \frac{2c}{\sqrt{3 z_{\rm eq} R_{\rm eq}}} \ln \frac{\sqrt{1+R_{\rm dec}} + \sqrt{R_{\rm dec}+R_{\rm eq}}}{1+\sqrt{R_{\rm eq}}}$$

- At lower z want to measure d_A/s and Hs.
- Actually get $\omega_m^{1/2}\, d_A$ and $\omega_m^{1/2}/H(z)$.
  - But these only contain terms depending on Ω_m, Ω_{DE}, w_{DE} etc.
  - Ratios of distances unaffected, only *h* mis-measured!

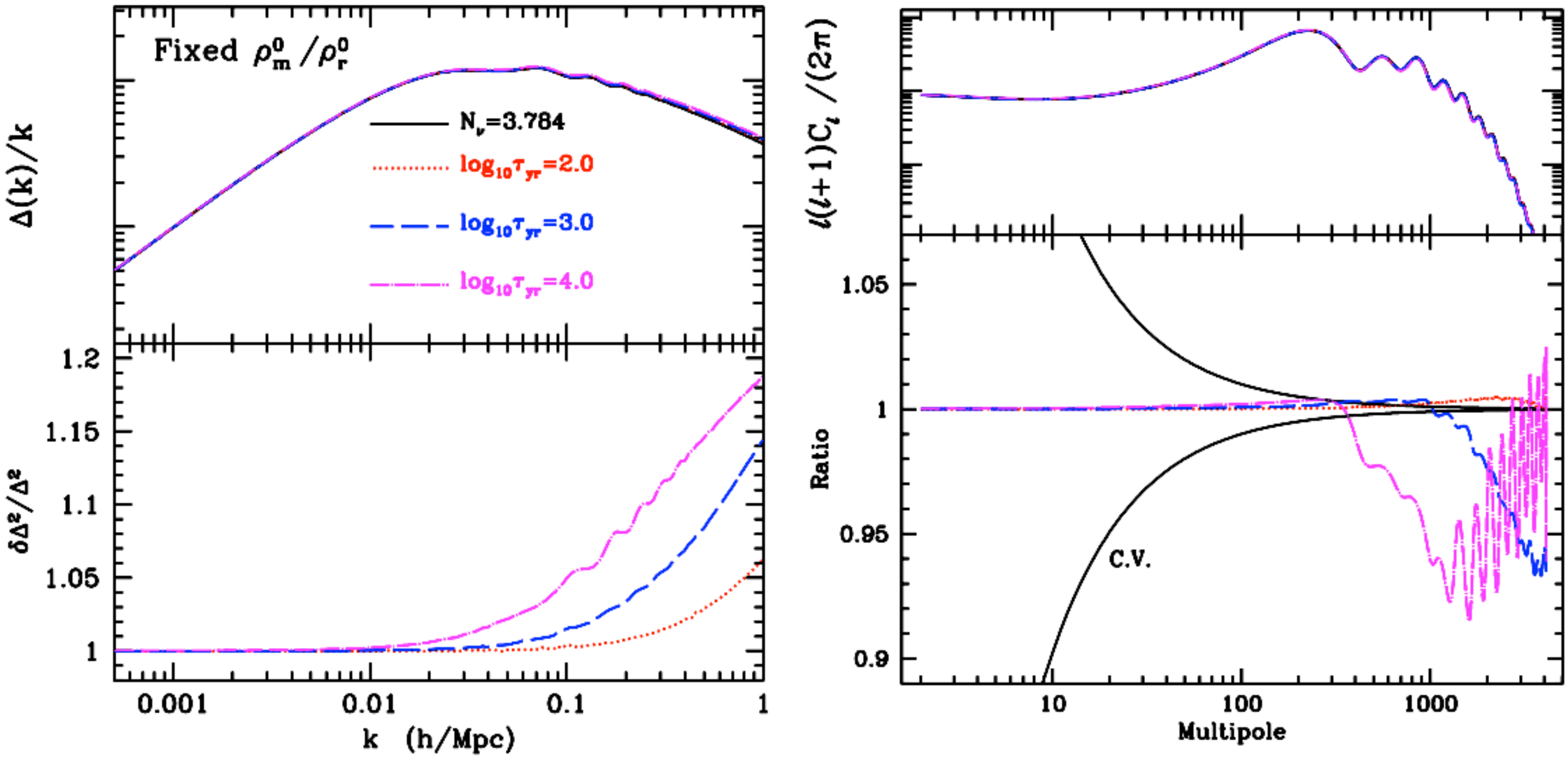
Massive neutrinos

- Massive neutrinos should be counted as radiation at z~10^3, but matter today.
  - Compute $\omega_{cdm}^{1/2}s$ from CMB, but low-z distance scale as $\omega_m^{-1/2}$.
- Leads to a $(1+\Omega_\nu/\Omega_{cdm})^{-1/2}$ or ~0.2-0.4% correction to the distances.
  - Want to constrain neutrinos with shape of power spectrum.

Decaying "X" ?

A non-relativistic (massive) particle which undergoes a momentum conserving decay into massless neutrinos with lifetime τ leads to excess small-scale power.

White (2006)



Beyond linear theory

Limited options

- Scale of non-linearity.
- Analytical models of non-linear growth.
  - Zel'dovich approximation.
  - Spherical top-hat collapse.
- Perturbation theory.
- Direct simulation.

Scale of non-linearity

- There are several ways to define a "scale" of non-linearity.
- Where $\Delta^2(k)=1$ (or ½, or …).
  - Dangerous when $\Delta^2(k)$ is very flat.
- By the rms linear theory displacement.

$$R_{\rm nl}^2 \propto \frac{1}{k_{\rm nl}^2} \propto \int \frac{dk}{k}\ \frac{\Delta^2(k)}{k^2}$$

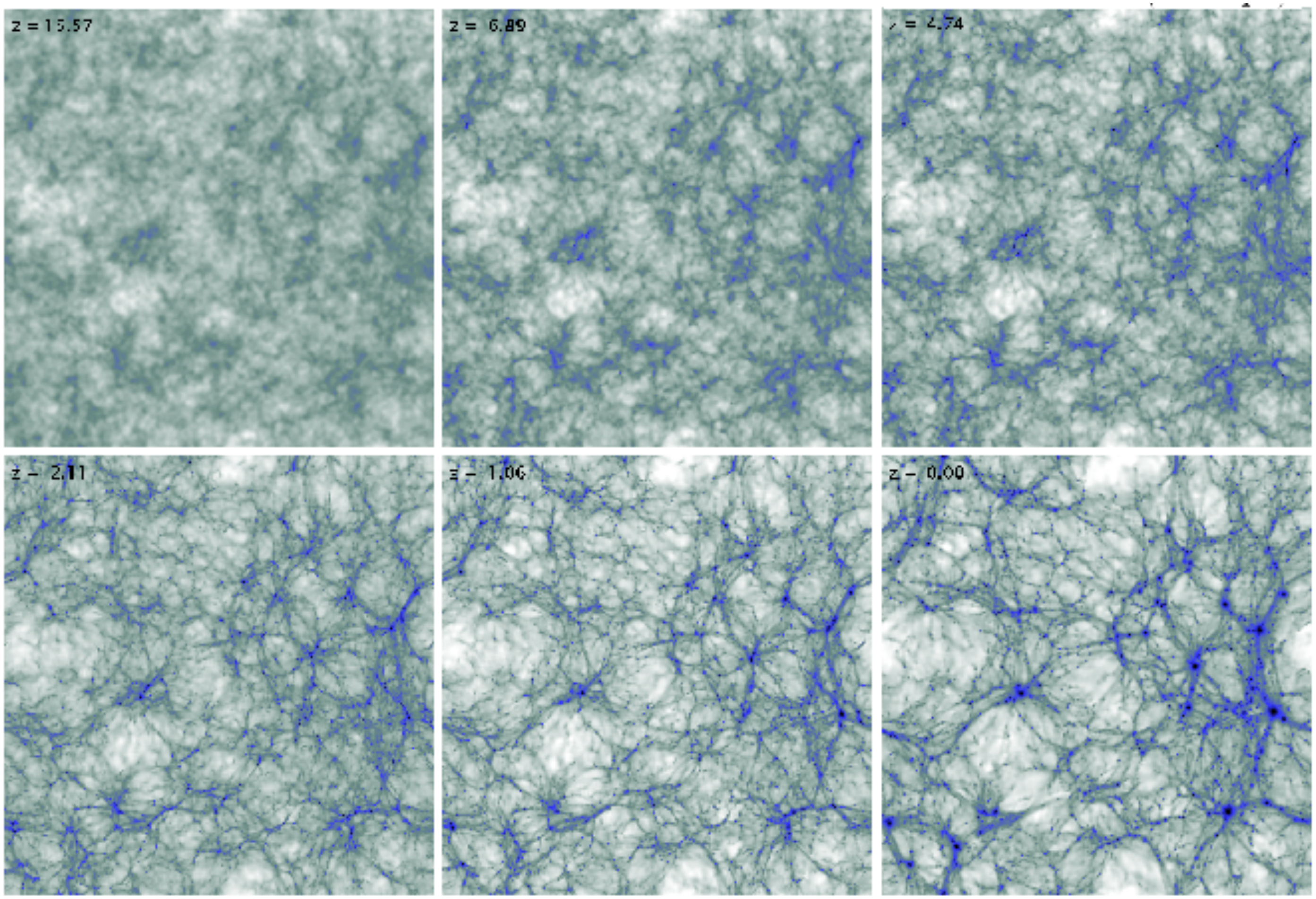
- Where the 2nd order correction to some quantity is 1% (10%) of the 1st order term.

Zel'dovich approximation

- Assume particles move in a straight line with their linear perturbation theory velocity.
- Defines a mapping from initial (Lagrangian) position, q, to final (Eulerian) position, x:
  - $x=q+\Psi$ with $\Psi(q,t)=D(t)\Psi(q)$ and $\Psi_i=d\Phi/dq_i$
  - $\Psi_k = -ik/k^2\,\delta_k$
- If the initial field is uniform, the final density is the Jacobian of this mapping.
  - $\rho\sim[(1-D\alpha)(1-D\beta)(1-D\gamma)]^{-1}$
  - α,β,γ e-values of $-d^2\Phi/dq_idq_j$
- Collapse takes place first along largest e-value (pancake/sheet), then middle (filament) then final (halo).

The cosmic web

The Zel'dovich approximation, plus the statistics of Gaussian fields, qualitatively describes large-scale structure.



Springel, Hernquist & White (2000)

Spherical top-hat collapse

Partridge & Peebles (1967), Gunn & Gott (1972), Peebles (1980).

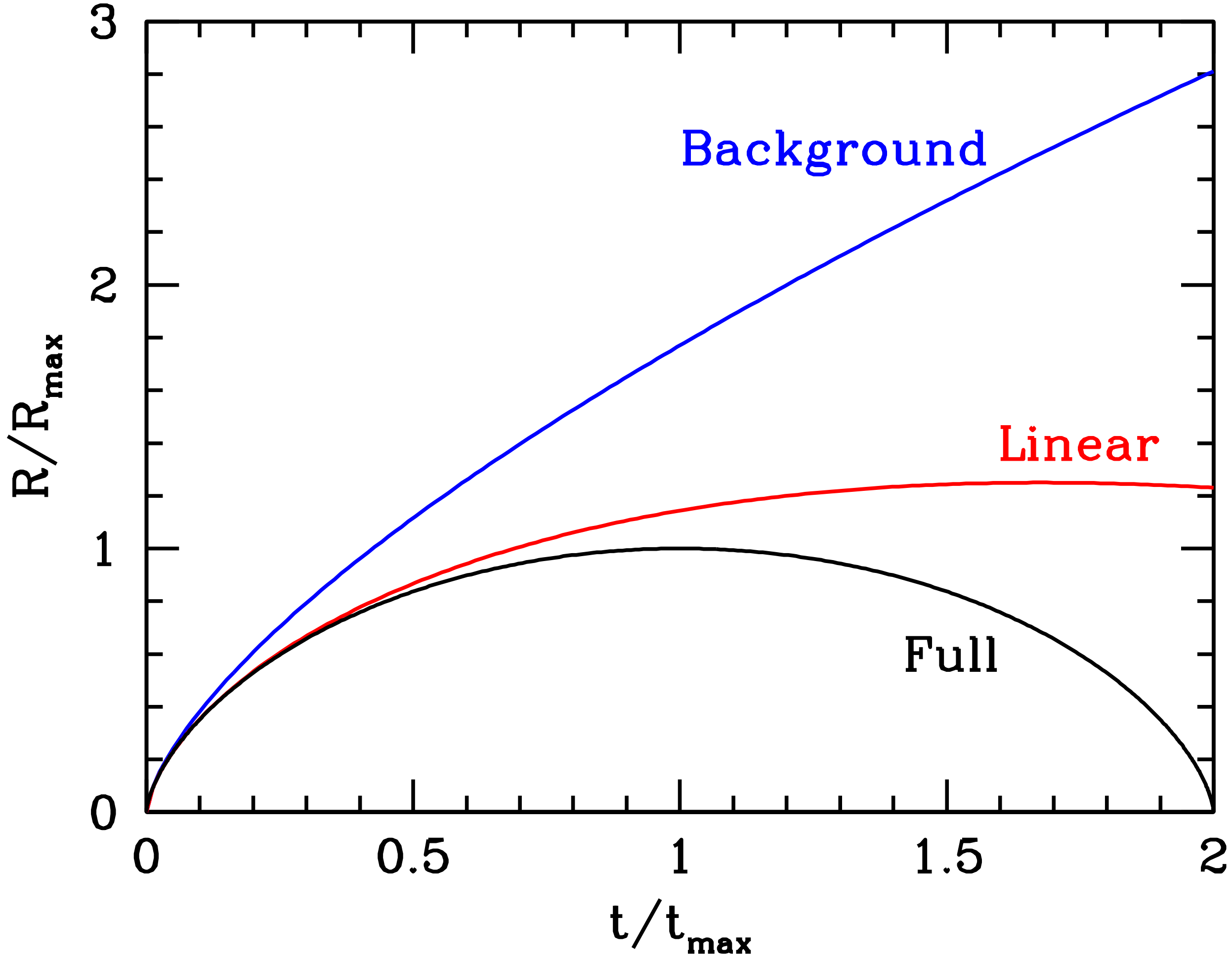
- Imagine a completely uniform, overdense sphere of radius R and overdensity δ, embedded in a completely uniform, Ω_m=1, FRW universe.
- By Birkhoff's theorem we can model the overdensity as a closed Friedmann universe.
  - $R/R_{max}=(1-\cos\theta)/2$ and $t/t_{max}=(\theta-\sin\theta)/\pi$
  - For small θ can expand $R(\tau=t/t_{max})$

$$\frac{R(\tau)}{R_{\max}} = \frac{1}{4}\,(6\pi\tau)^{2/3}\left[1-\frac{1}{20}\,(6\pi\tau)^{2/3}+\cdots\right]$$

↑

Background $R\sim t^{2/3}$

Spherical top-hat collapse



Note: expansion of Universe dominates collapse until "turn-around" at t/t_{max}=1. After this point, sphere begins to collapse under its own gravity.

Spherical top-hat collapse

- Note $\delta_{lin}=(R_{bkgnd}{}^3/R_{lin}{}^3)-1$
- At turnaround $\delta_{lin}=(3/20)(6\pi)^{2/3}\sim1.06$
- At collapse $\delta_{lin}=(3/20)(12\pi)^{2/3}\sim1.686$
- In this model collapse proceeds to $\rho=\infty$
- In real world collapse halts (aspherical).
- If object reaches virial equilibrium $R_{fin}=(1/2)R_{max}$
  - $1+\delta_{vir}=(6\pi)^2/2\sim178$.
- If $\Omega_m\neq1$ need to solve the equations numerically.
  - For $\Omega_m<1$ find δ_{vir} is increased.

Perturbation theory

- There is no reason (in principle) to stop at linear order in perturbation theory.
  - Can expand to all orders: $\delta=\delta^{(1)}+\delta^{(2)}+\delta^{(3)}+\ldots$
  - Can sum subsets of terms.
  - Usefulness/convergence of such an expansion not always clear.
- Consider **only** dark matter and **assume** we are in the single-stream limit.

Peebles (1980), Juszkiewicz (1981), Goroff++(1986), Makino++(1992), Jain&Bertschinger(1994), Fry (1994).
Reviews/comparison with N-body:
Bernardeau++(2002; Phys. Rep. 367, 1).
Carlson++(2009; PRD 80, 043531)

Equations of motion

Under these approximations, and assuming Ω_m=1

$$
\begin{aligned}
\frac{\partial \delta}{\partial \tau} + \vec{\nabla} \cdot \left[(1+\delta)\vec{v}\right] &= 0 \\
\frac{\partial \vec{v}}{\partial \tau} + \mathcal{H}\vec{v} + \left(\vec{v} \cdot \vec{\nabla}\right)\vec{v} &= -\vec{\nabla}\Phi \\
\nabla^2 \Phi &= \tfrac{3}{2}\mathcal{H}^2 \delta
\end{aligned}
$$

- Very familiar looking fluid equations
  - means we can borrow methods/ideas from other fields.
- Note the quadratic nature of the non-linearity.
- Since equations are now non-linear, can't use super-position of (exact) solutions even if they could be found!
- Proceed by perturbative expansion.

Go into Fourier space

Assume that *v* comes from a potential flow (self-consistent; curl[v]~a^{-1} at linear order) then it is totally specified by its convergence, θ, and …

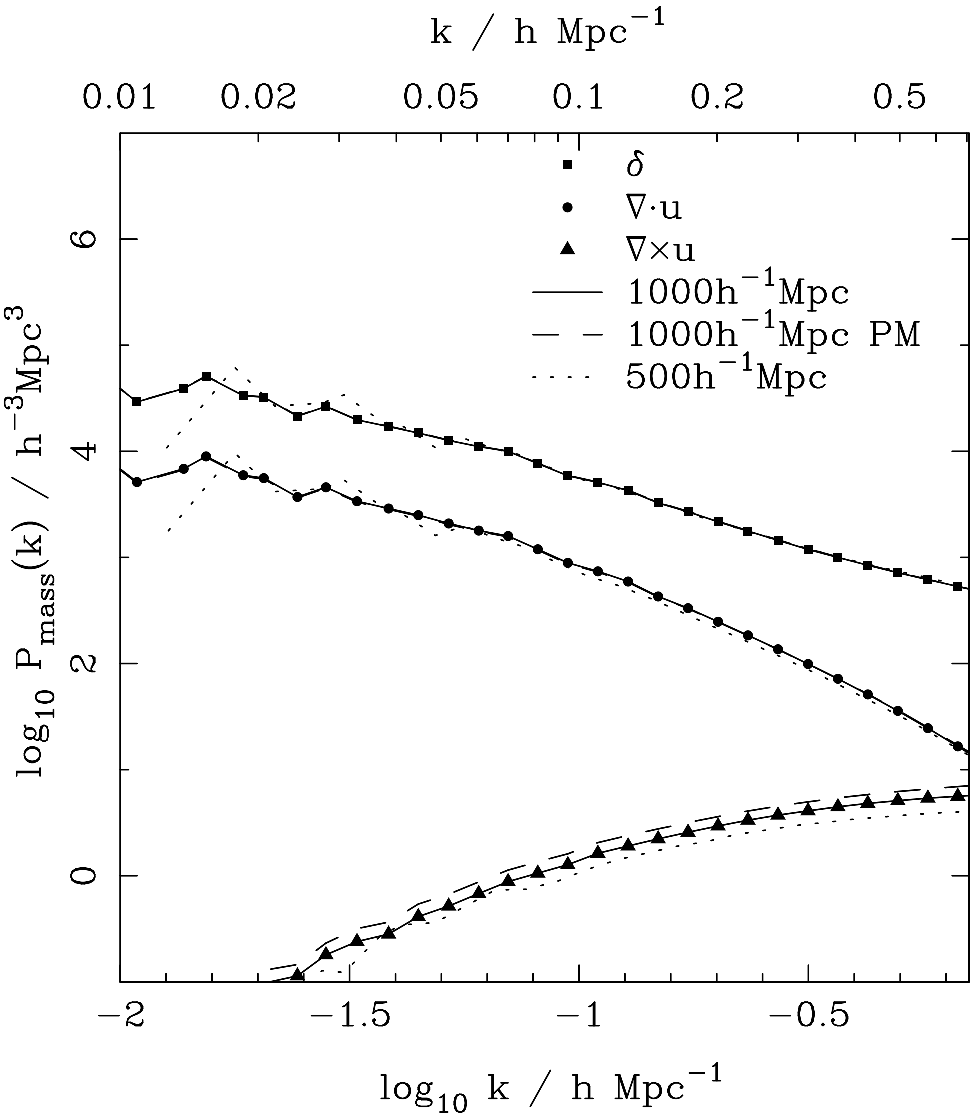
Div

$\mathbf{v} \sim (\mathbf{q}/q^2)\theta$

Product= Convolution

$$
\begin{aligned}
\frac{\partial \delta(\vec{k})}{\partial \tau} + \theta(\vec{k}) &= -\int \frac{d^3q}{(2\pi)^3} \frac{\vec{k}\cdot\vec{q}}{q^2}\, \theta(\vec{q})\delta(\vec{k}-\vec{q}), \\
\frac{\partial \theta(\vec{k})}{\partial \tau} + \mathcal{H}\theta(\vec{k}) + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta(\vec{k}) &= -\int \frac{d^3q}{(2\pi)^3} \frac{k^2\ \vec{q}\cdot(\vec{k}-\vec{q})}{2q^2|\vec{k}-\vec{q}|^2} \\
&\times \quad \theta(\vec{q})\theta(\vec{k}-\vec{q}).
\end{aligned}
$$

Velocities are a potential flow



Percival & White (2009)

N-body simulations validate this assumption for large scales.

Linear order

- To lowest order in δ and θ:

$$\delta_L(\mathbf{k}, z) = \frac{D(z)}{D(z_i)}\, \delta_i(\mathbf{k})$$

$$\theta_L(\mathbf{k}, z) = -f(z)\mathcal{H}(z)\, \frac{D(z)}{D(z_i)}\, \delta_i(\mathbf{k})$$

- with $f(z) \sim \Omega_m^{0.6} = 1$ for $\Omega_m = 1$ and $D(a) \sim a$.
- Decaying mode, $\delta \sim a^{-3/2}$, has to be zero for δ to be well-behaved as a->0.
- Define $\delta_0 = \delta_L(k, z=0)$.

Standard perturbation theory

- Develop δ and θ as power series:

$$\begin{aligned}\delta(\mathbf{k}) &= \sum_{n=1}^{\infty} a^n \delta^{(n)}(\mathbf{k}) \\ \theta(\mathbf{k}) &= -\mathcal{H} \sum_{n=1}^{\infty} a^n \theta^{(n)}(\mathbf{k})\end{aligned}$$

- then the $\delta^{(n)}$ can be written

$$\begin{aligned}\delta^{(n)}(\mathbf{k}) &= \int \frac{d^3q_1 d^3q_2 \cdots d^3q_n}{(2\pi)^{3n}} (2\pi)^3 \delta_D\left(\sum \mathbf{q}_i - \mathbf{k}\right) \\ &\times \quad F_n\left(\{\mathbf{q}_i\}\right) \delta_0(\mathbf{q}_1) \cdots \delta_0(\mathbf{q}_n)\end{aligned}$$

- with a similar expression for $\theta^{(n)}$.
- The F_n and G_n are just ratios of dot products of the ***q***s and obey simple recurrence relations.

Example: 2^{nd} order

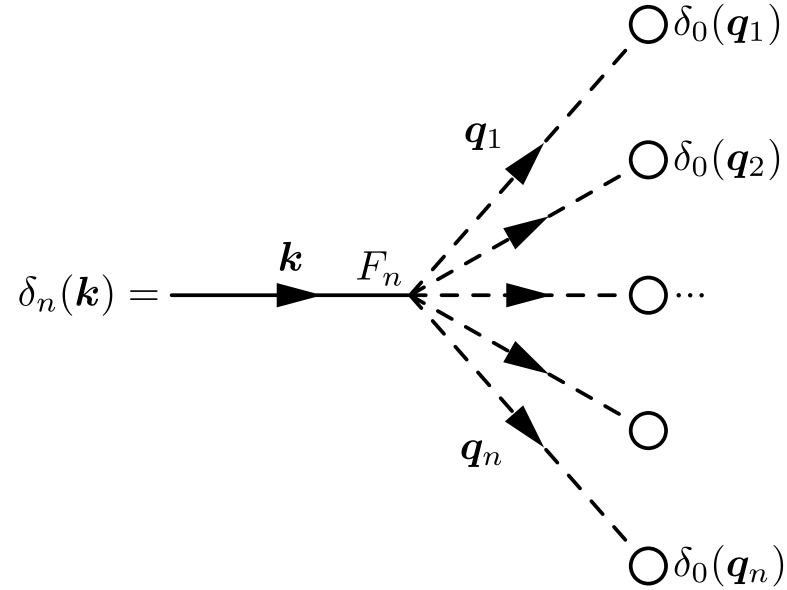
- The coupling function:

$$F_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{5}{7} + \frac{2}{7}\frac{(\mathbf{k}_1 \cdot \mathbf{k}_2)^2}{k_1^2 k_2^2} + \frac{(\mathbf{k}_1 \cdot \mathbf{k}_2)}{2}\left(k_1^{-2} + k_2^{-2}\right)$$

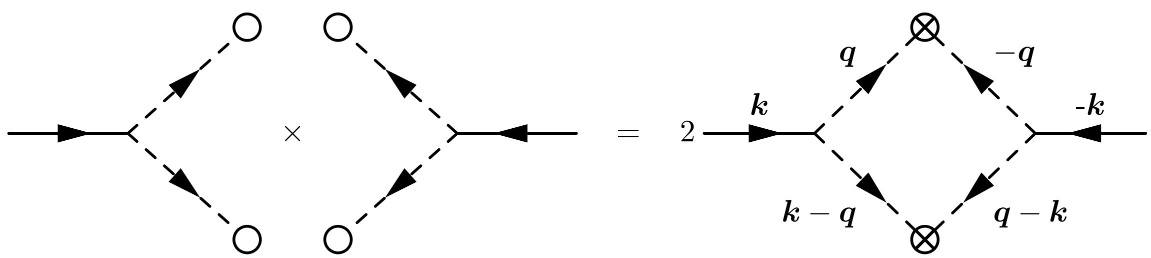
- If the initial fluctuations are Gaussian only expectation values even in δ survive:
  - $P(k) \sim \langle[\delta^{(1)}+\delta^{(2)}+\delta^{(3)}+\ldots][\delta^{(1)}+\delta^{(2)}+\delta^{(3)}+\ldots]\rangle$
  - $= P^{(1,1)} + 2P^{(1,3)} + P^{(2,2)}$

Perturbation theory: diagrams

Just as there is a diagrammatic short-hand for perturbation theory in quantum field theory, so there is in cosmology:



$\equiv \quad (2\pi)^3\delta_D(\boldsymbol{q}+\boldsymbol{q}')P_0(q),$



$$= 2\int \frac{d^3q}{(2\pi)^3} F_2(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})F_2(-\boldsymbol{q},\boldsymbol{q}-\boldsymbol{k})P_0(q)P_0(|\boldsymbol{k}-\boldsymbol{q}|)$$

Example: 2nd order

$$
\begin{aligned}
P^{(1,3)}(k) \quad &= \quad \frac{1}{252}\frac{k^3}{4\pi^2}P_L(k)\int_0^\infty dr\ P_L(kr)\left[\frac{12}{r^2} - 158 + 100r^2 - 42r^4\right. \\
&\quad \left. + \frac{3}{r^2}(r^2-1)^3(7r^2+2)\ln\left|\frac{1+r}{1-r}\right|\right],
\end{aligned}
$$

$$
\begin{aligned}
P^{(2,2)}(k) \quad &= \quad \frac{1}{98}\frac{k^3}{4\pi^2}\int_0^\infty dr\ P_L(kr)\int_{-1}^{1} dx\ P_L\left(k\sqrt{1+r^2-2rx}\right) \\
&\times \quad \frac{(3r+7x-10rx^2)^2}{(1+r^2-2rx)^2}.
\end{aligned}
$$

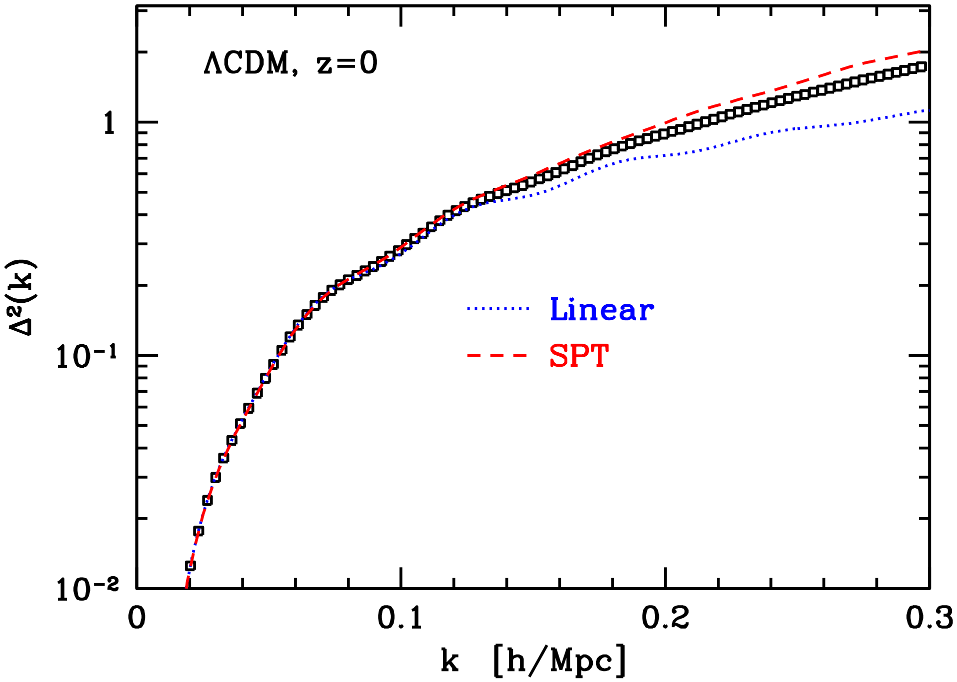
Perturbation theory enables the generation of truly impressive looking equations which arise from simple angle integrals. Like Feynman integrals, they are simple but look erudite!

Example: 2^{nd} order

- At low *k*, $P^{(2,2)}$ is positive and $P^{(1,3)}$ is negative
  - Large cancellation.
- For large *k* total contribution is negative:
  - $P^{(2,2)} \sim (1/4)\, k^2 \Sigma^2\, P_L(k)$
  - $P^{(1,3)} \sim -(1/2)\, k^2 \Sigma^2\, P_L(k)$
- Here Σ is the rms displacement (in each component) in linear theory.
  - It will come up again!!

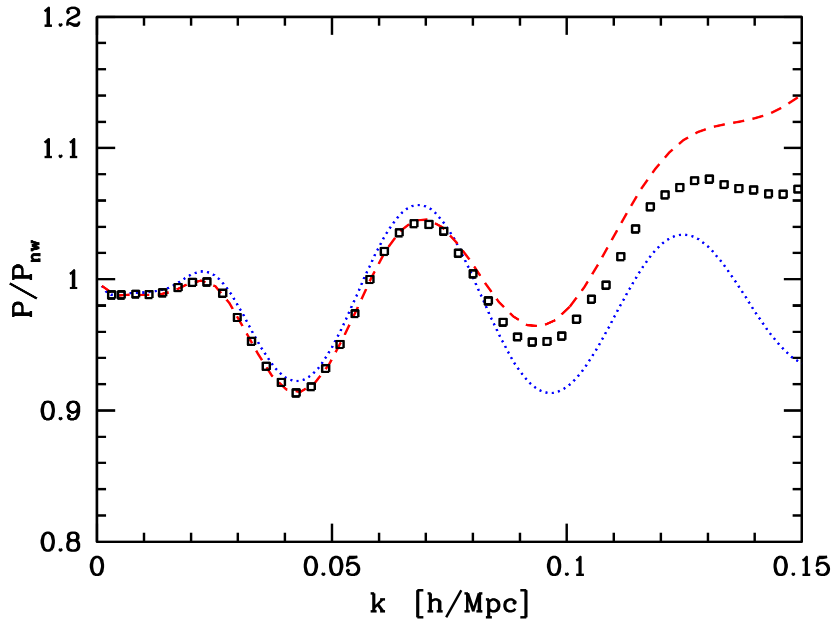
$$\Sigma^2 = \frac{1}{3\pi^2} \int_0^\infty dq\ P_L(q)$$

Example



The lowest order correction to the matter power spectrum at z=0 (1-loop SPT).

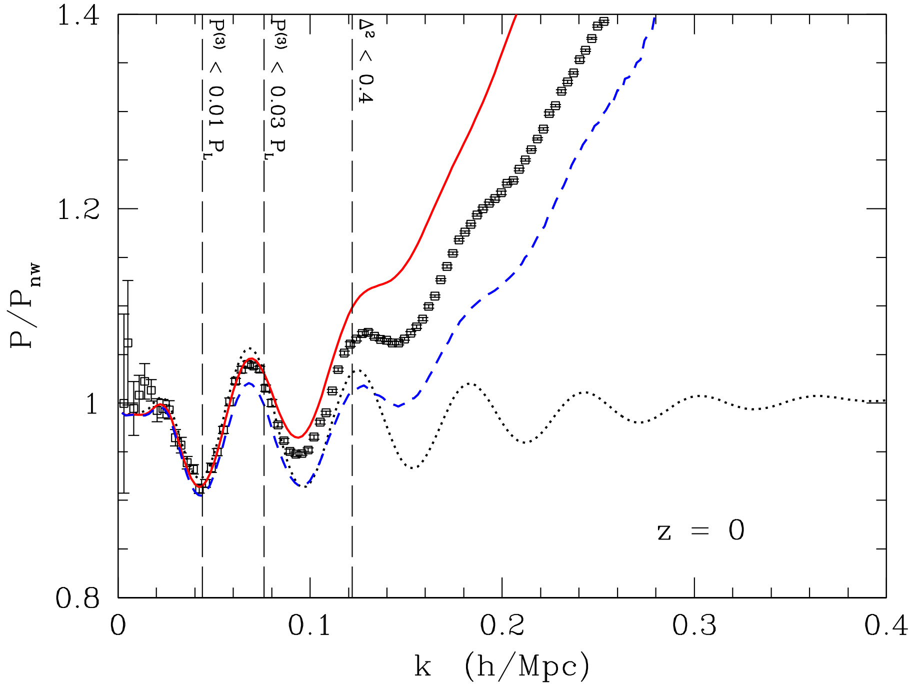
Note the improvement at low k where non-linear growth causes a <u>suppression</u> of power (pre-virialization).



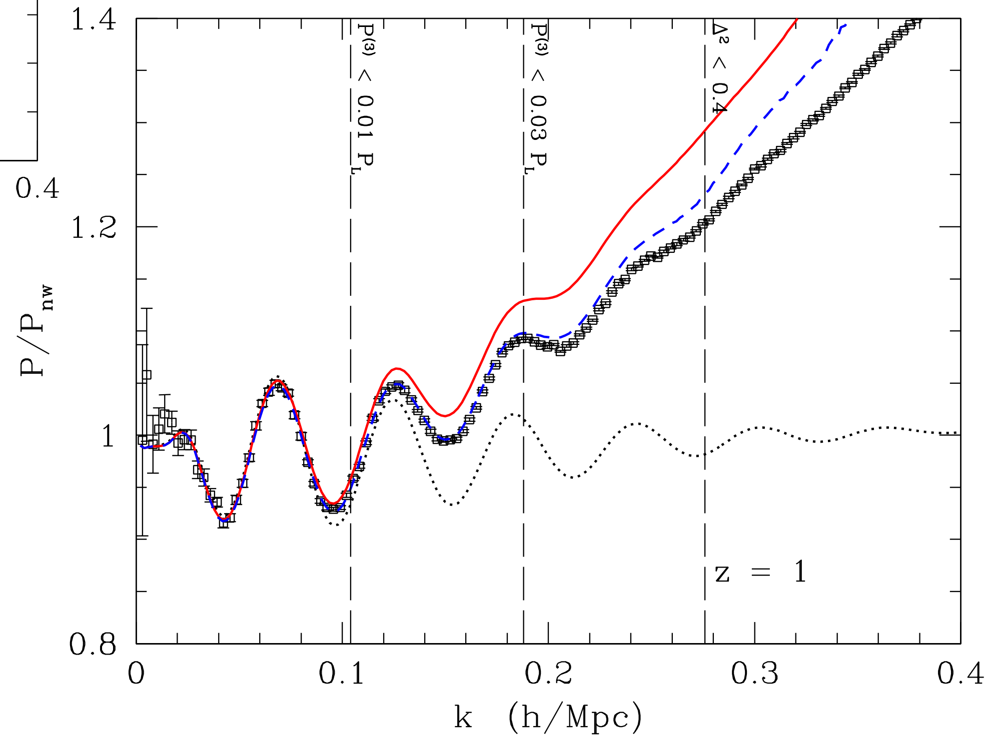
Beyond 2^{nd} order

- Expressions for higher orders are easy to derive, especially using computer algebra packages.
- Using rotation symmetry the N^{th} order contribution requires mode coupling integrals of dimension 3N-1.
  - Best done using Monte-Carlo integration.
  - Prohibitive for very high orders.
  - Not clear this expansion is converging!

Comparison with exact results



Broad-band shape of P_L has been divided out to focus on more subtle features.



Linear

1st order correction

2nd order correction

Carlson++09

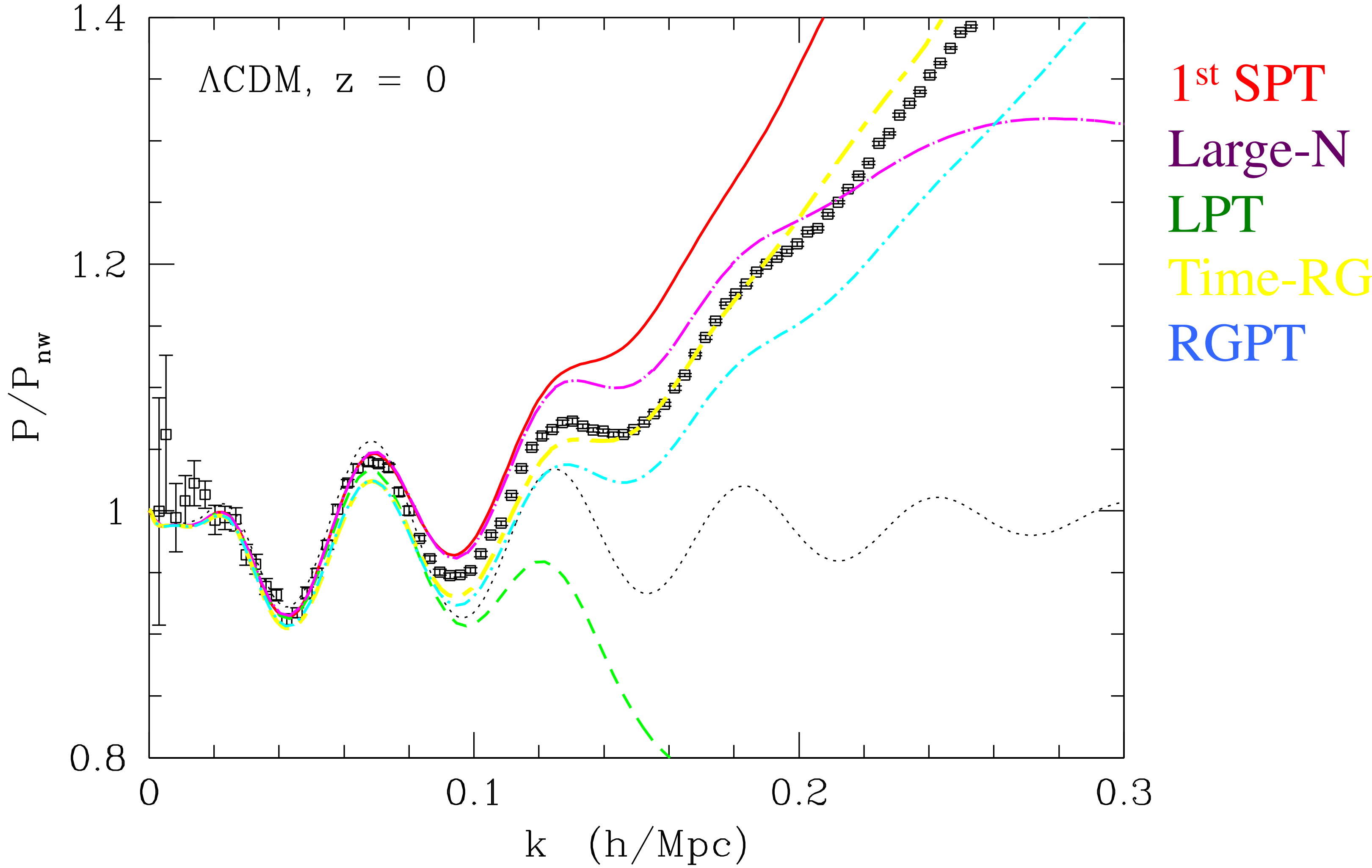
Including bias

- Perturbation theory clearly cannot describe the formation of collapsed, bound objects such as dark matter halos.
- We can extend the usual thinking about "linear bias" to a power-series in the Eulerian density field:
  - $\delta_{gal} = \Sigma\, b_n(\delta^n/n!)$
- The expressions for P(k) now involve b_1 to lowest order, b_1 and b_2 to next order, etc.
  - The physical meaning of these terms is actually hard to figure out, and the validity of the defining expression is dubious, but this is the standard way to include bias in Eulerian perturbation theory.

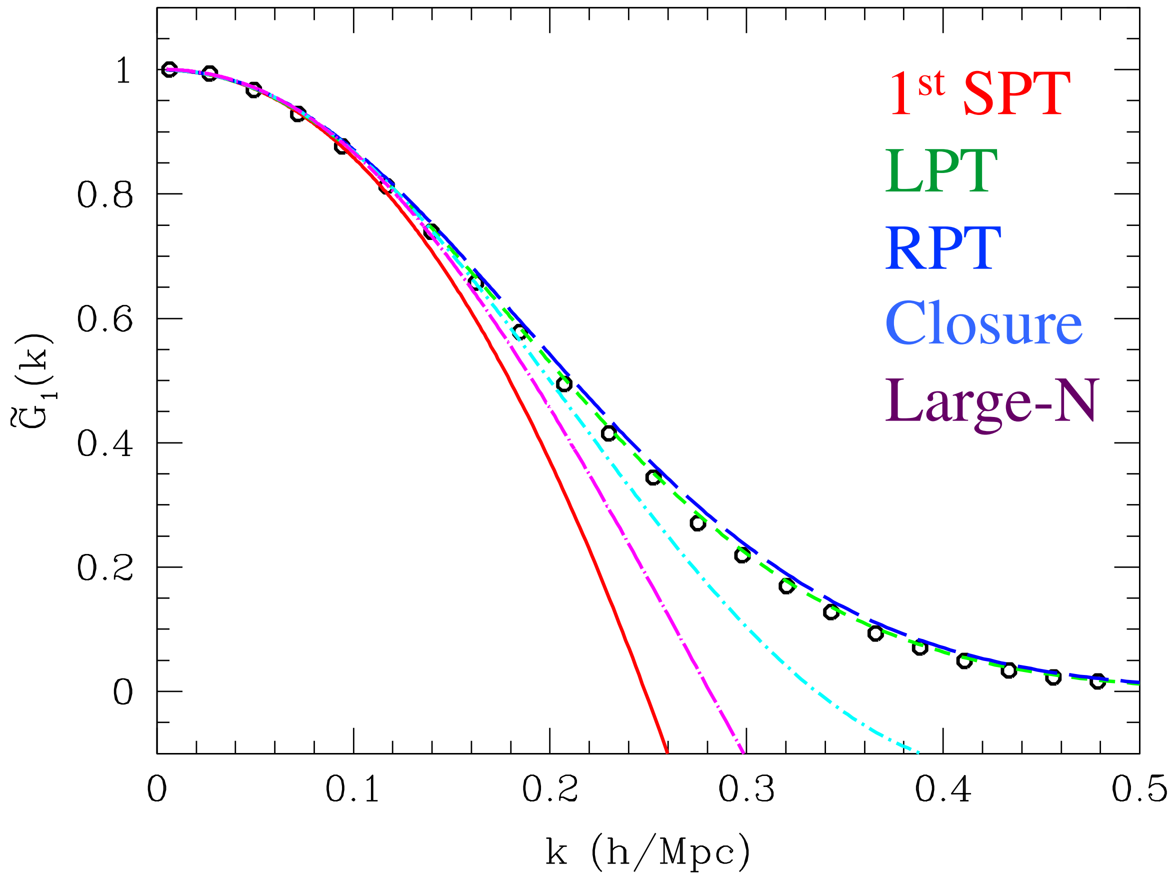
Other methods

- Renormalized perturbation theory
  - A variant of “Dyson-Wyld” resummation.
  - An expansion in “order of complexity”.
- Closure theory
  - Write expressions for $(d/d\tau)P$ in terms of P, B, T, …
  - Approximate B by leading-order expression in SPT.
- Time-RG theory (& RGPT)
  - As above, but assume B=0
  - Good for models with $m_\nu>0$ where linear growth is scale-dependent.
- Path integral formalism
  - Perturbative evaluation of path integral gives SPT.
  - Large N expansion, 2PI effective action, steepest descent.
- Lagrangian perturbation theory

Some other theories



Some other quantities



The propagator, or

$$G(k) \propto \frac{\langle \delta_{NL} \delta_L^* \rangle}{\langle \delta_L \delta_L^* \rangle}$$

which measures the decoherence of the final density field due to non-linear evolution.

Carlson++09

Lagrangian perturbation theory

- A very different approach, which has been radically developed recently by Matsubara and is *very* useful for BAO.
  - Buchert89, Moutarde++91, Hivon++95.
  - Matsubara (2008a; PRD, 77, 063530)
  - Matsubara (2008b; PRD, 78, 083519)
- Relates the current (Eulerian) position of a mass element, **x**, to its initial (Lagrangian) position, **q**, through a displacement vector field, Ψ.
  - Note **q** is a position, not a wave-vector!

Lagrangian perturbation theory

$$\begin{aligned}\delta(\mathbf{x}) &= \int d^3q\ \delta_D(\mathbf{x}-\mathbf{q}-\mathbf{\Psi})-1\\ \delta(\mathbf{k}) &= \int d^3q\ e^{-i\mathbf{k}\cdot\mathbf{q}}\left(e^{-i\mathbf{k}\cdot\mathbf{\Psi}(\mathbf{q})}-1\right)\ .\end{aligned}$$

$$\frac{d^2\mathbf{\Psi}}{dt^2}+2H\frac{d\mathbf{\Psi}}{dt}=-\nabla_x\phi\left[\mathbf{q}+\mathbf{\Psi}(\mathbf{q})\right]$$

$$\begin{aligned}\mathbf{\Psi}^{(n)}(\mathbf{k}) &= \frac{i}{n!}\int\prod_{i=1}^{n}\left[\frac{d^3k_i}{(2\pi)^3}\right]\ (2\pi)^3\delta_D\left(\sum_i\mathbf{k}_i-\mathbf{k}\right)\\ &\times\quad \mathbf{L}^{(n)}(\mathbf{k}_1,\cdots,\mathbf{k}_n,\mathbf{k})\delta_0(\mathbf{k}_1)\cdots\delta_0(\mathbf{k}_n)\end{aligned}$$

Standard LPT

- If we expand the exponential and keep terms consistently in δ_0 we regain a series $\delta=\delta^{(1)}+\delta^{(2)}+\ldots$ where $\delta^{(1)}$ is linear theory and e.g.

$$\begin{aligned}\delta^{(2)}(\mathbf{k}) &= \frac{1}{2}\int\frac{d^3k_1 d^3k_2}{(2\pi)^3}\delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\delta_0(\mathbf{k}_1)\delta_0(\mathbf{k}_2)\\ &\times \left[\mathbf{k}\cdot\mathbf{L}^{(2)}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k})+\mathbf{k}\cdot\mathbf{L}^{(1)}(\mathbf{k}_1)\mathbf{k}\cdot\mathbf{L}^{(1)}(\mathbf{k}_2)\right]\end{aligned}$$

- which regains “SPT”.

LPT power spectrum

- Alternatively we can use the expression for $\delta_{\mathbf{k}}$ to write

$$P(k) = \int d^3q\ e^{-i\vec{k}\cdot\vec{q}} \left(\left\langle e^{-i\vec{k}\cdot\Delta\vec{\Psi}} \right\rangle - 1 \right)$$

- where $\Delta\Psi=\Psi(\mathbf{q})-\Psi(0)$.
- Expanding the exponential and plugging in for $\Psi^{(n)}$ gives the usual results.
- **BUT** Matsubara suggested a different and very clever approach.

Cumulants

- The cumulant expansion theorem allows us to write the expectation value of the exponential in terms of the exponential of expectation values.
- Expand the terms $(\mathbf{k}\Delta\Psi)^N$ using the binomial theorem.
- There are two types of terms:
  - Those depending on Ψ at same point.
    - This is independent of position and can be factored out of the integral.
  - Those depending on Ψ at different points.
    - These can be expanded as in the usual treatment.

Example

- Imagine Ψ is Gaussian with mean zero.
- For such a Gaussian: $\langle e^X \rangle = \exp[\sigma^2/2]$.

$$P(k) = \int d^3q e^{-i\mathbf{k}\cdot\mathbf{q}} \left(\left\langle e^{-ik_i \Delta\Psi_i(\mathbf{q})} \right\rangle - 1 \right)$$

$$\left\langle e^{-i\mathbf{k}\cdot\Delta\boldsymbol{\Psi}(q)} \right\rangle = \exp\left[-\frac{1}{2} k_i k_j \left\langle \Delta\Psi_i(\mathbf{q}) \Delta\Psi_j(\mathbf{q}) \right\rangle \right]$$

$$k_i k_j \left\langle \Delta\Psi_i(\mathbf{q}) \Delta\Psi_j(\mathbf{q}) \right\rangle = 2k_i^2 \langle \Psi_i^2(\mathbf{0}) \rangle - 2k_i k_j \xi_{ij}(\mathbf{q})$$

Keep exponentiated. Expand

Resummed LPT

- The first corrections to the power spectrum are then:

$$P(k) = e^{-(k\Sigma)^2/2}\left[P_L(k) + P^{(2,2)}(k) + \widetilde{P}^{(1,3)}(k)\right],$$

- where $P^{(2,2)}$ is as in SPT but part of $P^{(1,3)}$ has been "resummed" into the exponential prefactor.

- The exponential prefactor is identical to that obtained from
  - The peak-background split (Eisenstein++07)
  - Renormalized Perturbation Theory (Crocce++08).
- Non-linearities, or mode coupling, erase the acoustic signature (Meiksin, White & Peacock 1999).
  - Fewer k-modes to measure.
  - Peak is "broadened" making it harder to centroid.
  - Much of the contribution to Σ comes from low k!

Beyond real-space mass

- One of the more impressive features of Matsubara's approach is that it can gracefully handle both biased tracers and redshift space distortions.
- In redshift space $\mathbf{\Psi} \to \mathbf{\Psi} + \frac{\widehat{\mathbf{z}} \cdot \dot{\mathbf{\Psi}}}{H} \widehat{\mathbf{z}}$
- For bias local in Lagrangian space:

$$\delta_{\rm obj}(\mathbf{x}) = \int d^3q \ F\left[\delta_L(\mathbf{q})\right] \delta_D(\mathbf{x} - \mathbf{q} - \mathbf{\Psi})$$

- we obtain

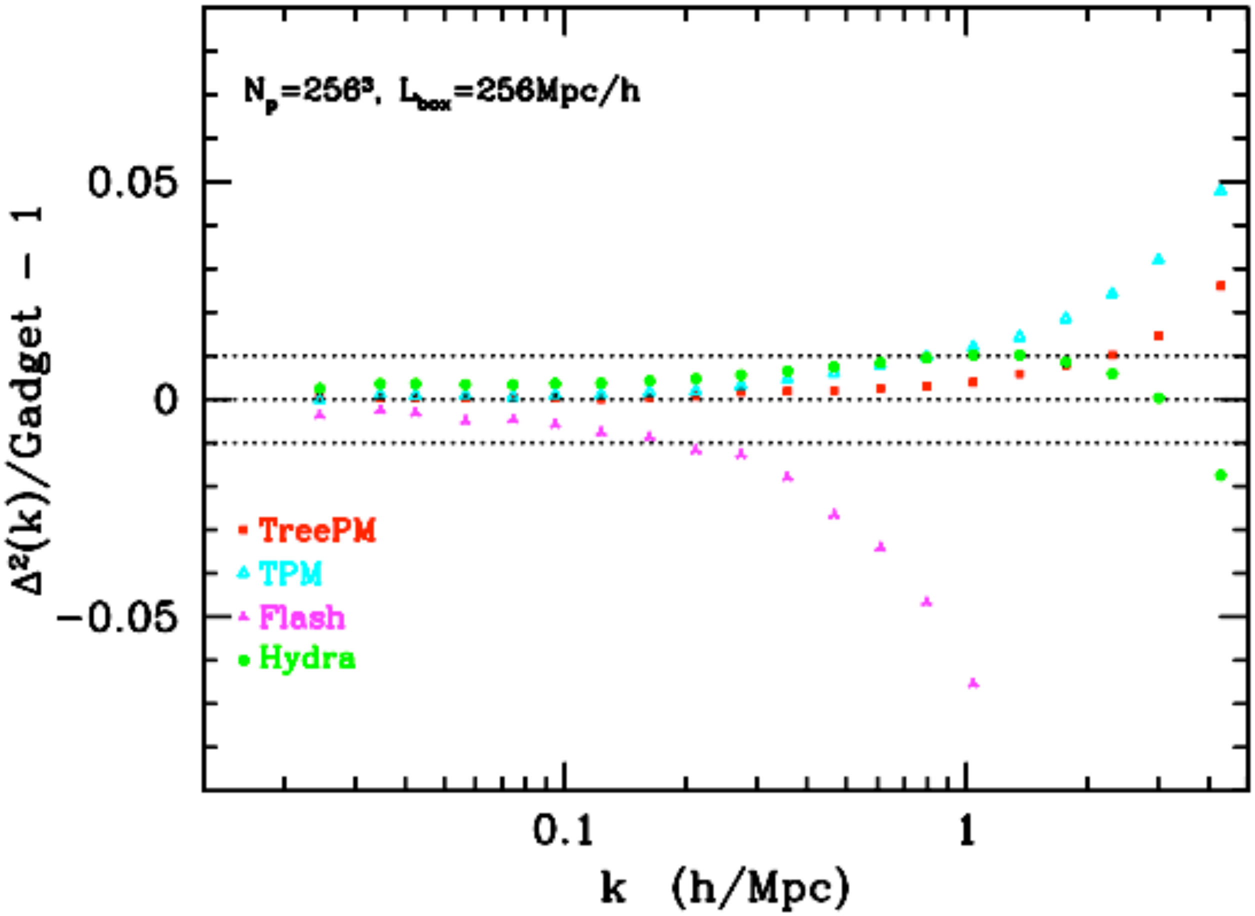
$$P(k) \quad = \quad \int d^3q \ e^{-i\mathbf{k}\cdot\mathbf{q}} \left[\int \frac{d\lambda_1}{2\pi} \frac{d\lambda_2}{2\pi} F(\lambda_1) F(\lambda_2) \left\langle e^{i[\lambda_1 \delta_L(\mathbf{q}_1) + \lambda_2 \delta_L(\mathbf{q}_2)] + i\mathbf{k}\cdot\Delta\mathbf{\Psi}} \right\rangle - 1 \right]$$

- which can be massaged with the same tricks as we used for the mass.
- If we assume halos/galaxies form at peaks of the initial density field ("peaks bias") then explicit expressions for the integrals of F exist.

Numerical simulations

- Our ability to simulate structure formation has increased tremendously in the last decade.
- Simulating the dark matter for BAO:
  - Meiksin, White & Peacock (1999)
    - 10^6 particles, 10^2 dynamic range, ~$1 Gpc^3$
  - Kim et al. (2009; the Horizon run)
    - 10^{11} particles, 10^5 dynamic range, ~$300 Gpc^3$
    - A similar sized simulation in Teyssier et al. (2009); smaller volume.
- Direct simulation of the N-body problem
  - Begin at early times, but during matter domination, by displacing particles from an initial grid using 1LPT or 2LPT.
  - Monte-Carlo integration of the Vlasov equation using “super-particles” which move along the characteristics.
  - Soften the forces to avoid particle-particle scattering or integrating unphysical, tight, orbiting particles.
- Our understanding of -- or at least our ability to describe -- galaxy formation has also increased dramatically.
  - Galaxies live in dark matter halos in ways we increasingly understand.

Accuracy - currently demonstrated



Updated from Heitmann et al. (2007)

Only a sub-sample of the codes are shown here.

All codes started from the same ICs and analyzed with the same *P(k)* codes.

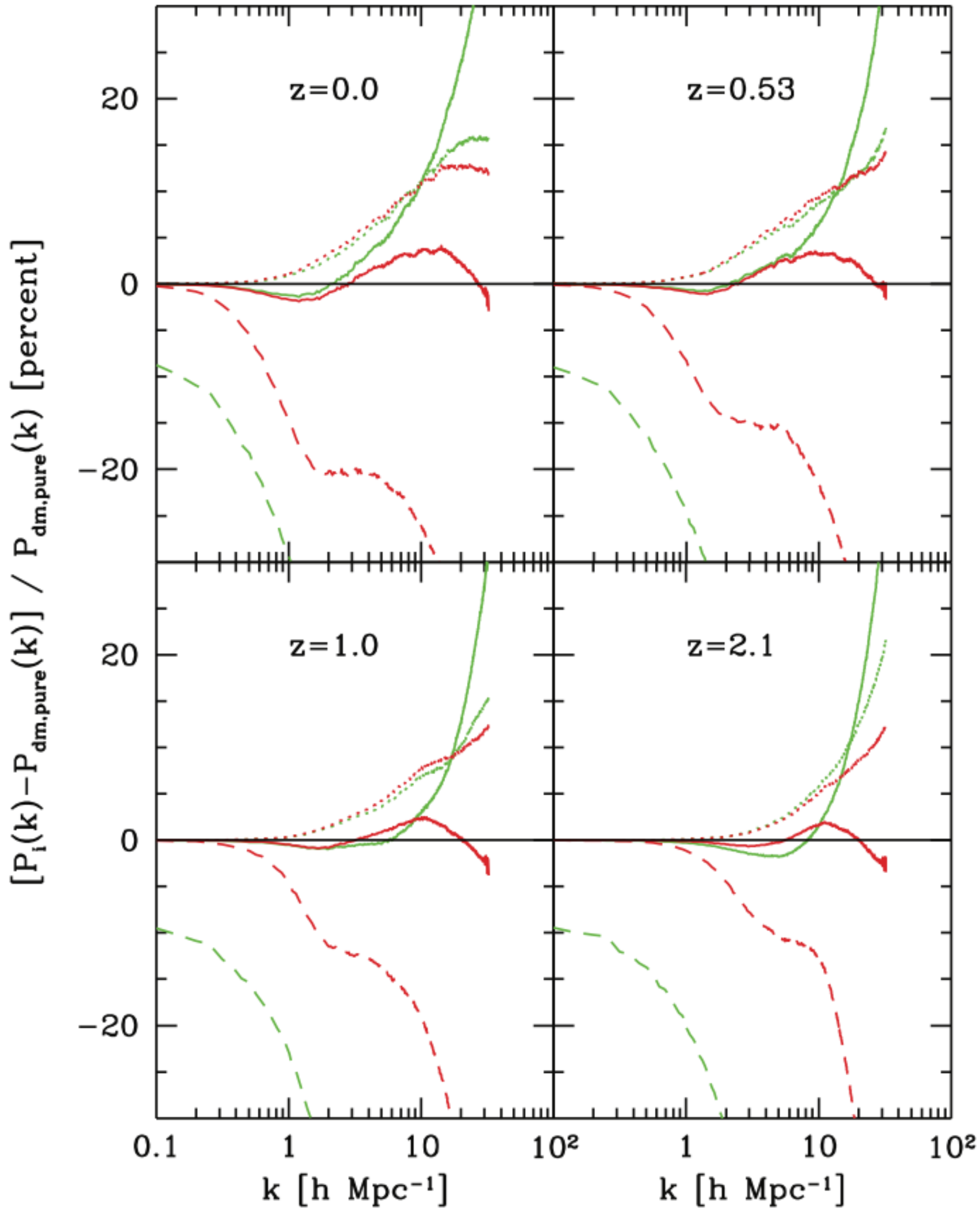
Numerical convergence

- Numerous tests of numerical convergence can be found in:
  - Heitmann et al. (2010; ApJ, 715, 104)
  - Heitmann et al. (2010; ApJ, 705, 156)
- Need to worry about
  - Starting redshift and method.
  - Force accuracy and softening.
  - Time stepping.
  - Box size.
  - Number of particles.
  - Method of computing statistic from particles.
  - How to choose *which* cosmologies to run.

Extra physics

- As we go to smaller scales, we must go beyond the “pure” N-body problem and include additional physics.
  - Hydrodynamics solvers well developed.
  - Gas cools dramatically in deep potential wells, reaching high densities in a clumpy, multiphase, turbulent, magnetized ISM where it can form stars, which give off winds and radiation and go supernova injecting momentum and energy into the surrounds and have active galactic nuclei which can impart energy to their enviroments, …
- There is little scale separation between including “gas” physics and including star formation, feedback, etc. so results typically depend on sub-grid models.

An example



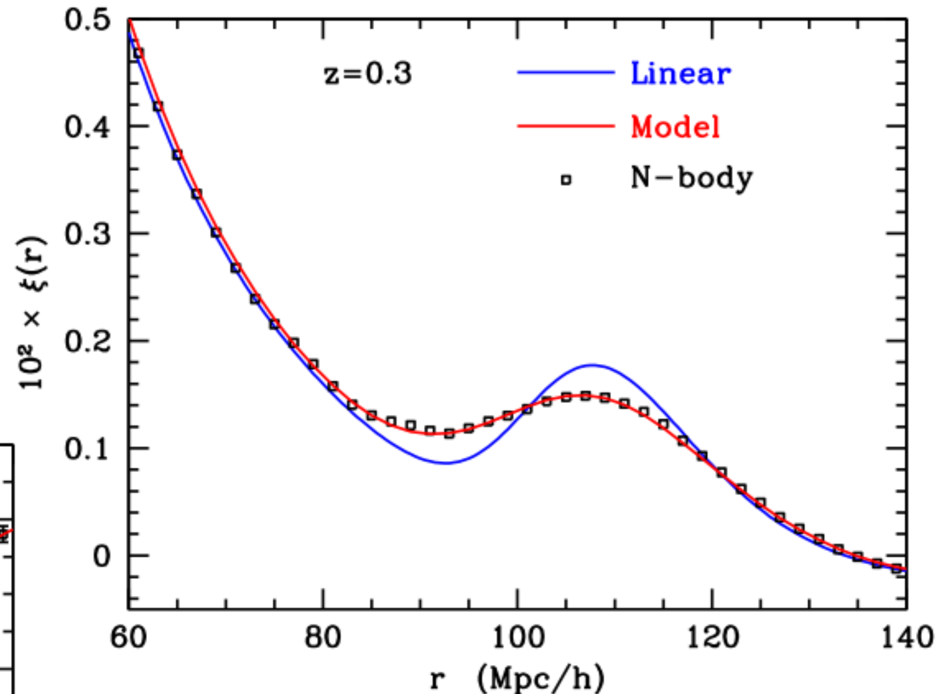
One possibility, from Jing et al. (2006), for the effects of baryons (red) and baryons including star-formation and feedback (green) on the total matter (solid), dark matter (dotted) and gas (dashed).

Non-linearities and BAO

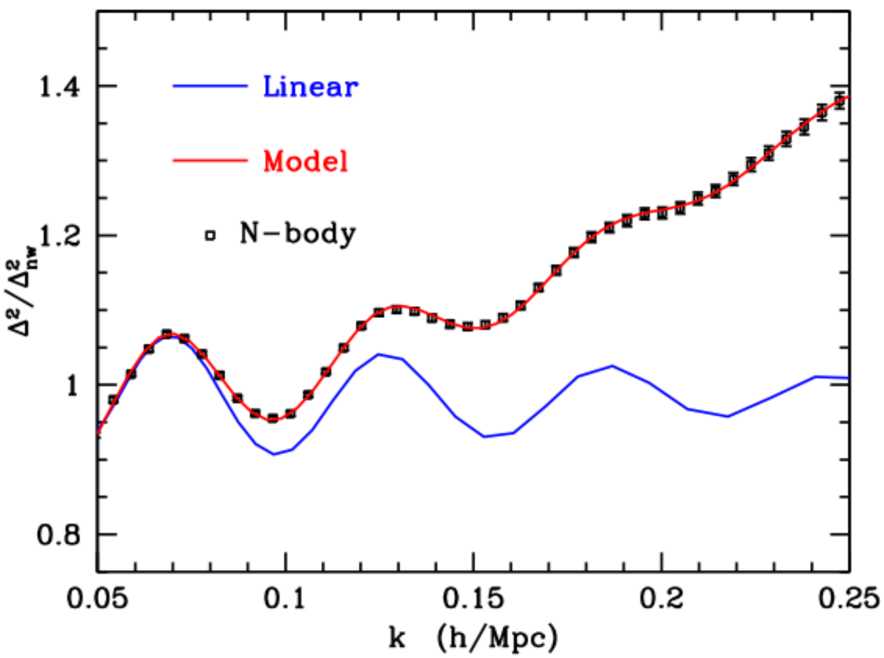
Effects of non-linearity on BAO

- Non-linear evolution has 3 effects on the power spectrum:
  - It generates “excess” high *k* power, reducing the contrast of the wiggles.
  - It damps the oscillations.
  - It generates an out-of-phase component.
- In configuration space:
  - Generates “excess” small-scale power.
  - Broadens the peak.
  - Shifts the peak.

Non-linearities smear the peak



Loss of contrast and excess power from non-linear collapse.



Broadening of feature due to Gaussian smoothing and ~0.5% shift due to mode coupling.

Understanding higher order

- We want to fit for the position of the acoustic feature while allowing for variations in the broadband shape (due e.g. to biasing).
  - $P_{fit}(k) = B(k)\, P_w(k,\alpha) + A(k)$
  - $B(k)$ and $A(k)$ are smooth functions.
    - Can take B(k)=const and A(k) as a spline, polynomial, Pade, ...
  - α measures shift relative to "fiducial" cosmology.
  - $P_w(k,\alpha)$ is a template.
    - Numerous arguments suggest $P_w(k,\alpha)=\exp[-k^2\Sigma^2/2]P_L(k/\alpha)$.
    - Take Σ *to be a free parameter, perhaps with a prior.*
- How does this do?

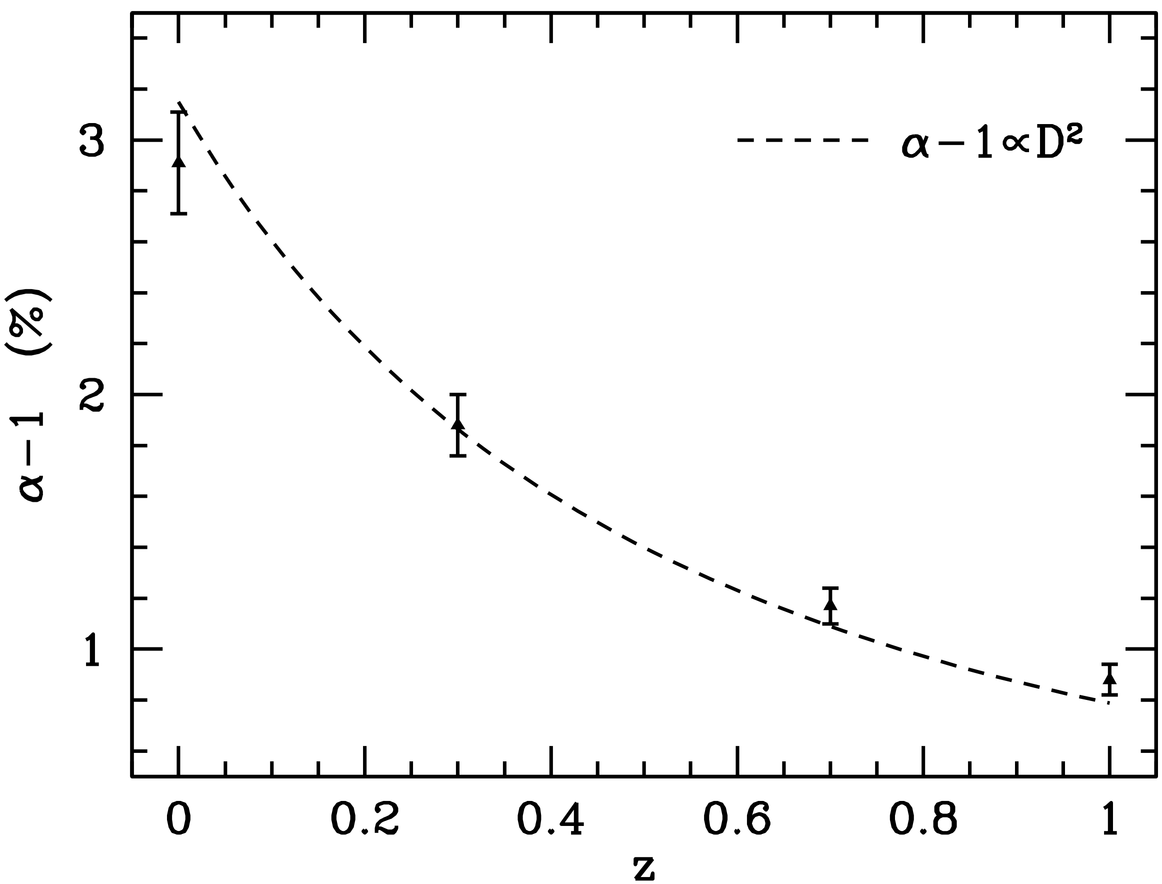
Argument from
Padmanabhan & White (2009)

Measuring shifts in cCDM

- Any "shift" in the acoustic scale is small in ΛCDM, and therefore hard to study.
- Work with a "crazy" cosmology
  - Ω_m=1, Ω_B=0.4, h=0.5, n=1, σ_8=1.
  - Sound horizon 50h^{-1}Mpc, not 100h^{-1}Mpc.
- The fitted shifts are (α-1 in percent):

z	DM	xδ_L	w/P_{22}
0.0	2.91 ± 0.20	-0.2 ±0.1	-0.03 ± 0.16
0.3	1.88 ± 0.12	-0.2 ±0.1	-0.38 ± 0.09
0.7	1.17 ± 0.07	-0.1 ±0.1	-0.12 ± 0.05
1.0	0.88 ± 0.06	-0.1 ±0.1	-0.04 ± 0.04

Shifts vs time



Amplitude of the shift vs. time (redshift) for the mass.

Shifts are consistent with D^2 scaling (dotted) suggesting an origin from 2^{nd} order terms …

Where do the shifts come from?

Recall in PT we can write $\delta=\delta^{(1)}+\delta^{(2)}+\ldots$ or
$P = \{P_{11} + P_{13} + P_{15} + \ldots\} + \{P_{22} + \ldots\} = P_{1n} + P_{mn}$.
We can isolate these two types of terms by considering the cross-spectrum of the final with the initial field, which doesn't contain P_{mn}.

z	DM	xδ_L	w/P_{22}
0.0	2.91 ± 0.20	-0.2 ±0.1	-0.03 ± 0.16
0.3	1.88 ± 0.12	-0.2 ±0.1	-0.38 ± 0.09
0.7	1.17 ± 0.07	-0.1 ±0.1	-0.12 ± 0.05
1.0	0.88 ± 0.06	-0.1 ±0.1	-0.04 ± 0.04

Shifts in the cross-spectrum are an order of magnitude smaller than shifts in the auto-spectrum!

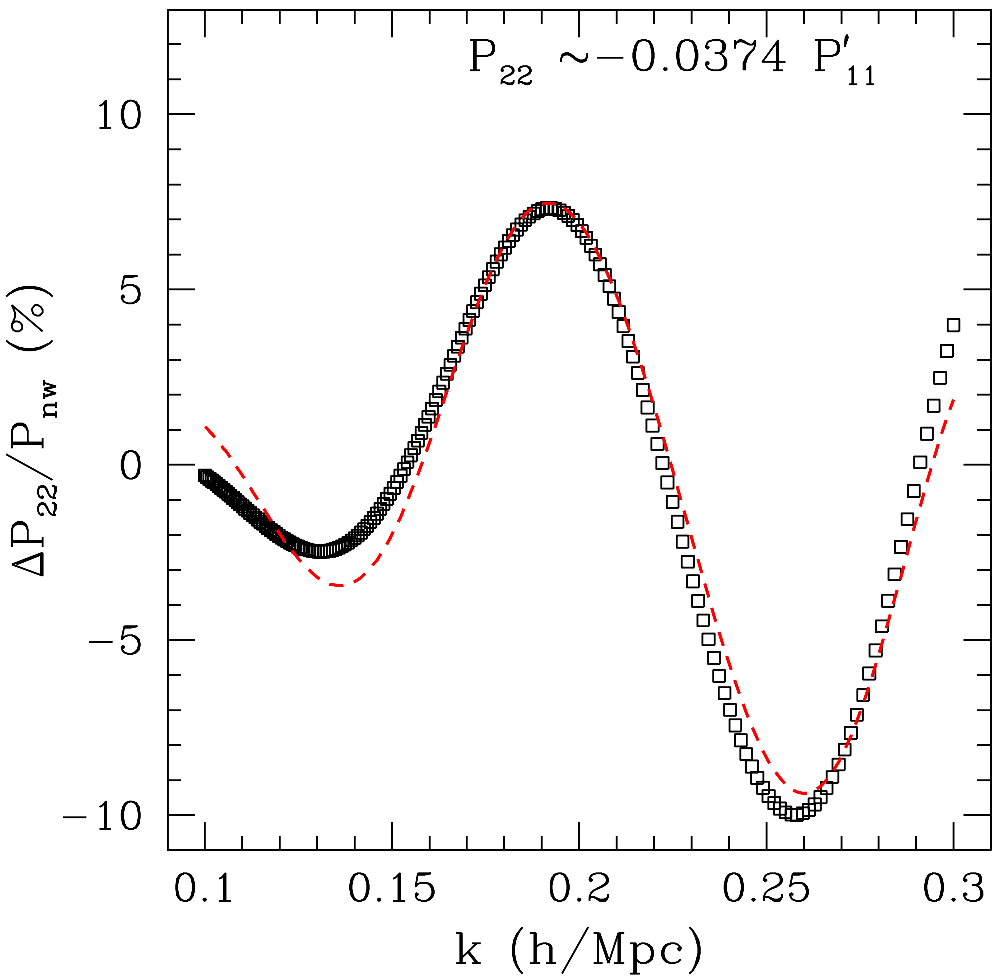
$$P_{1n}(k) \sim P_L(k) \int \prod_k \left[d^3 q_k P_L(q_k)\right] F_n(\cdots)$$

Broad kernel suppresses oscillations.

Mode-coupling

- By contrast the P_{mn} terms involve integrals of products of P_Ls times peaked kernels.
- Example: $P_{22} \sim \int P_L P_L\ F_2$ and F_2 is sharply peaked around $q_1 \approx q_2 \approx k/2$.
- Thus the $\int P_L P_L$ term contains an out-of-phase oscillation
  - $P_L \sim \ldots + \sin(kr)$: $P_L P_L F_2 \sim \sin^2(kr/2) \sim 1+\cos(kr)$
- Since $\cos(x) \sim d/dx \sin(x)$ this gives a “shift” in the peak
  - $P(k/\alpha) \sim P(k) - (\alpha-1)\ dP/d\ln k + \ldots$

Mode-coupling approximates derivative



Up to an overall factor the mode-coupling term, P_{22}, is well approximated by $dP_L/d\ln k$.

Modified template

- This discussion suggests a modified template, which has just as many free parameters as our old template:

$$P_{\rm w}(k,\alpha) = \exp\left(-\frac{k^2\Sigma^2}{2}\right) P_L(k/\alpha) + \exp\left(-\frac{k^2\Sigma_1^2}{2}\right) P_{22}(k/\alpha)\,.$$

- This removes most of the shift.

z	DM	$x\delta_L$	w/P_{22}
0.0	2.91 ± 0.20	-0.2 ±0.1	-0.03 ± 0.16
0.3	1.88 ± 0.12	-0.2 ±0.1	-0.38 ± 0.09
0.7	1.17 ± 0.07	-0.1 ±0.1	-0.12 ± 0.05
1.0	0.88 ± 0.06	-0.1 ±0.1	-0.04 ± 0.04

Biased tracers?

- In order to remove the shift we needed to know the relative amplitude of P_{11} and P_{22}.
- What do we do for biased tracers?
  - Eulerian bias

$$P_h = \left(b_1^E\right)^2 (P_{11} + P_{22}) + b_1^E b_2^E \left(\frac{3}{7} Q_8 + Q_9\right) + \frac{(b_2^E)^2}{2} Q_{13} + \cdots$$

  - Lagrangian bias

$$\begin{aligned} P_h &= \exp\left[-\frac{k^2\Sigma^2}{2}\right] \left\{ \left(1 + b_1^L\right)^2 P_{11} + P_{22} + b_1^L \left[\frac{6}{7} Q_5 + 2Q_7\right] + b_2^L \left[\frac{3}{7} Q_8 + Q_9\right] \right. \\ &+ \left. \left(b_1^L\right)^2 [Q_9 + Q_{11}] + 2 b_1^L b_2^L Q_{12} + \frac{1}{2} \left(b_2^L\right)^2 Q_{13} \right\} + \cdots \end{aligned}$$

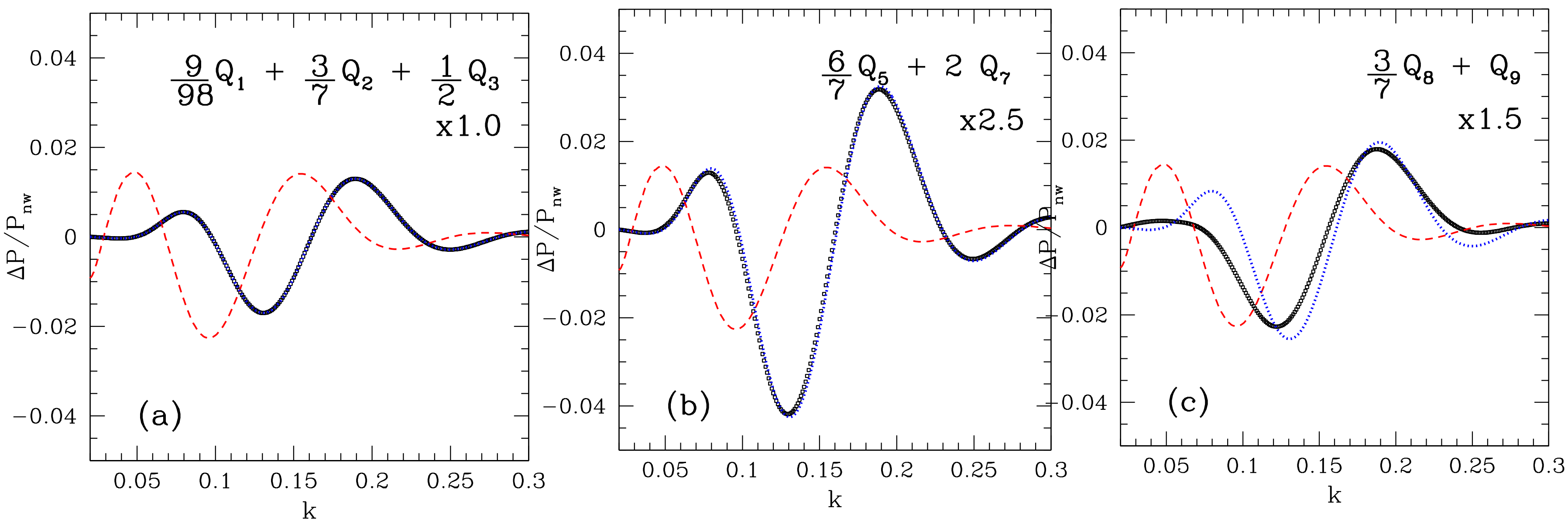
Mode-coupling integrals

$$Q_n(k) = \frac{k^3}{4\pi^2} \int_0^\infty dr\, P_L(kr) \int_{-1}^1 dx\, P_L(k\sqrt{1+r^2-2rx})\widetilde{Q}_n(r,x)$$

$$\begin{aligned}
\widetilde{Q}_1 &= \frac{r^2(1-x^2)^2}{y^2}, \quad \widetilde{Q}_2 = \frac{(1-x^2)rx(1-rx)}{y^2}, \\
\widetilde{Q}_3 &= \frac{x^2(1-rx)^2}{y^2}, \quad \widetilde{Q}_4 = \frac{1-x^2}{y^2}, \\
\widetilde{Q}_5 &= \frac{rx(1-x^2)}{y}, \quad \widetilde{Q}_6 = \frac{(1-3rx)(1-x^2)}{y}, \\
\widetilde{Q}_7 &= \frac{x^2(1-rx)}{y}, \quad \widetilde{Q}_8 = \frac{r^2(1-x^2)}{y}, \\
\widetilde{Q}_9 &= \frac{rx(1-rx)}{y}, \quad \widetilde{Q}_{10} = 1-x^2, \\
\widetilde{Q}_{11} &= x^2,\ \widetilde{Q}_{12} = rx,\ \widetilde{Q}_{13} = r^2
\end{aligned}$$

(Matsubara 2008)

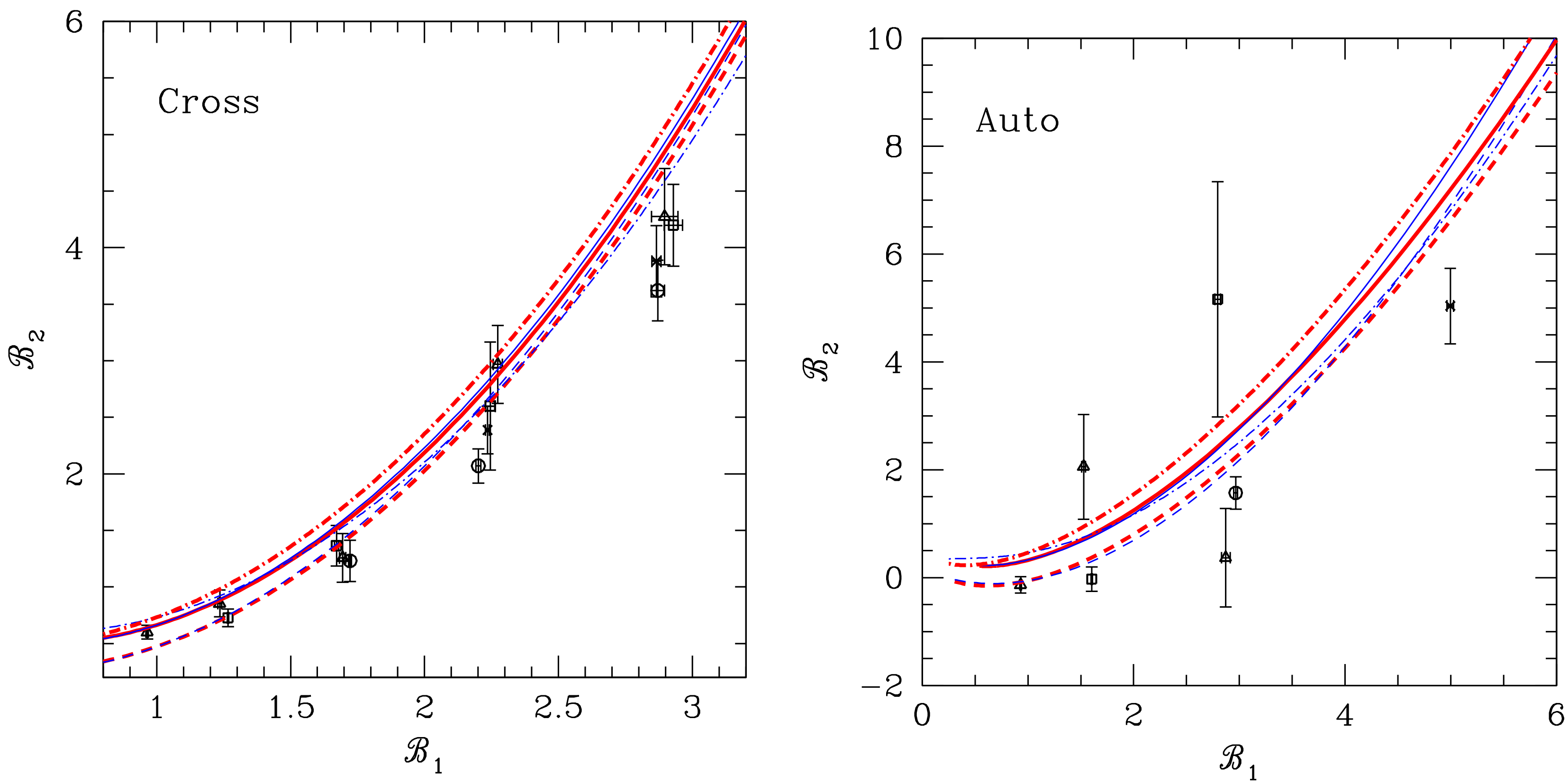
Out-of-phase?



The numerous combinations that come in are also well approximated by the (log-)derivative of P_{11}! All of these terms can be effectively written as:

$$P_h = \exp\left(-\frac{k^2\Sigma^2}{2}\right)\left[\mathcal{B}_1 P_L + \mathcal{B}_2 P_{22}\right].$$

Models do lie on a narrow line

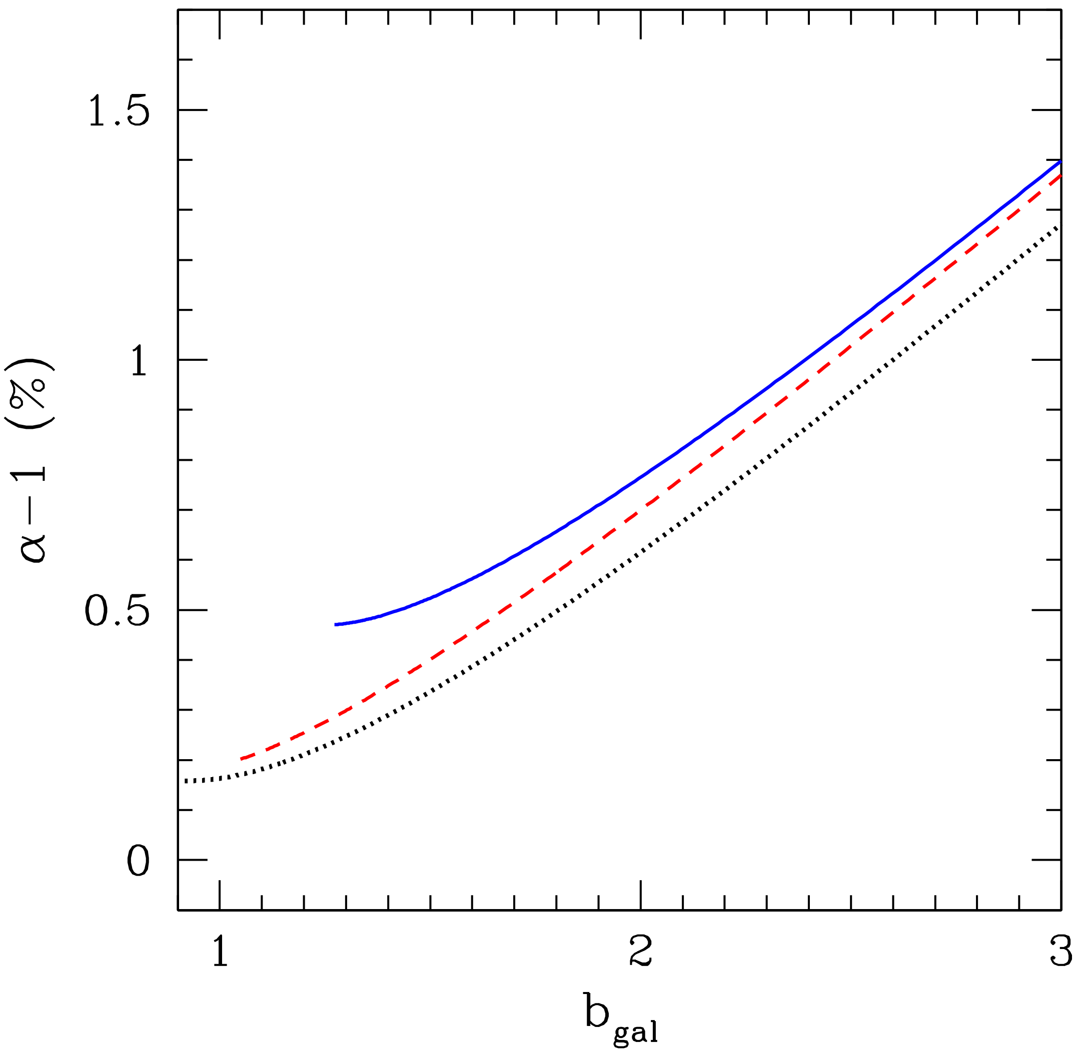


$$P_{\mathrm{w}}(k,\alpha) = b_1\left[\exp\left(-\frac{k^2\Sigma^2}{2}\right)P_L(k/\alpha) + \exp\left(-\frac{k^2\Sigma_1^2}{2}\right)\frac{\mathcal{B}_2}{\mathcal{B}_1}P_{22}(k/\alpha)\right]$$

Implications for ΛCDM?

- Shifts caused by P_{22}, well approximated by $dP_L/d\ln k$.
  - True also for ΛCDM, same scaling coeff.
- Additional shifts for biased tracers approximate $dP_L/d\ln k$.
  - True also for ΛCDM, same scaling coeff.
- Simple model explains B_1-B_2 relation.
  - True also for ΛCDM.
  - Can also be measured from simulations.
- For ΛCDM the shifts are an order of magnitude smaller than for cCDM.
  - α-1~0.5% x D^2 x B_2/B_1

Shifts for galaxies



Shifts at z=0 for

Halos of mass M
Halos above M
N~[1+M/M$_1$]

At higher z the shift decreases as D^2.

Recall, the final error in BAO scale is the *uncertainty* in this correction, not the size of the correction itself!

Redshift space

- In resummed LPT we can also consider the redshift space power spectrum for biased tracers.
- For the isotropic P(k) find a similar story though now the scaling coefficients depend on f~dD/dln$a$.
  – Expressions become more complex, but the structure is unchanged.
- The amplitude of the shift increases slightly.

Perturbation theory & BAO

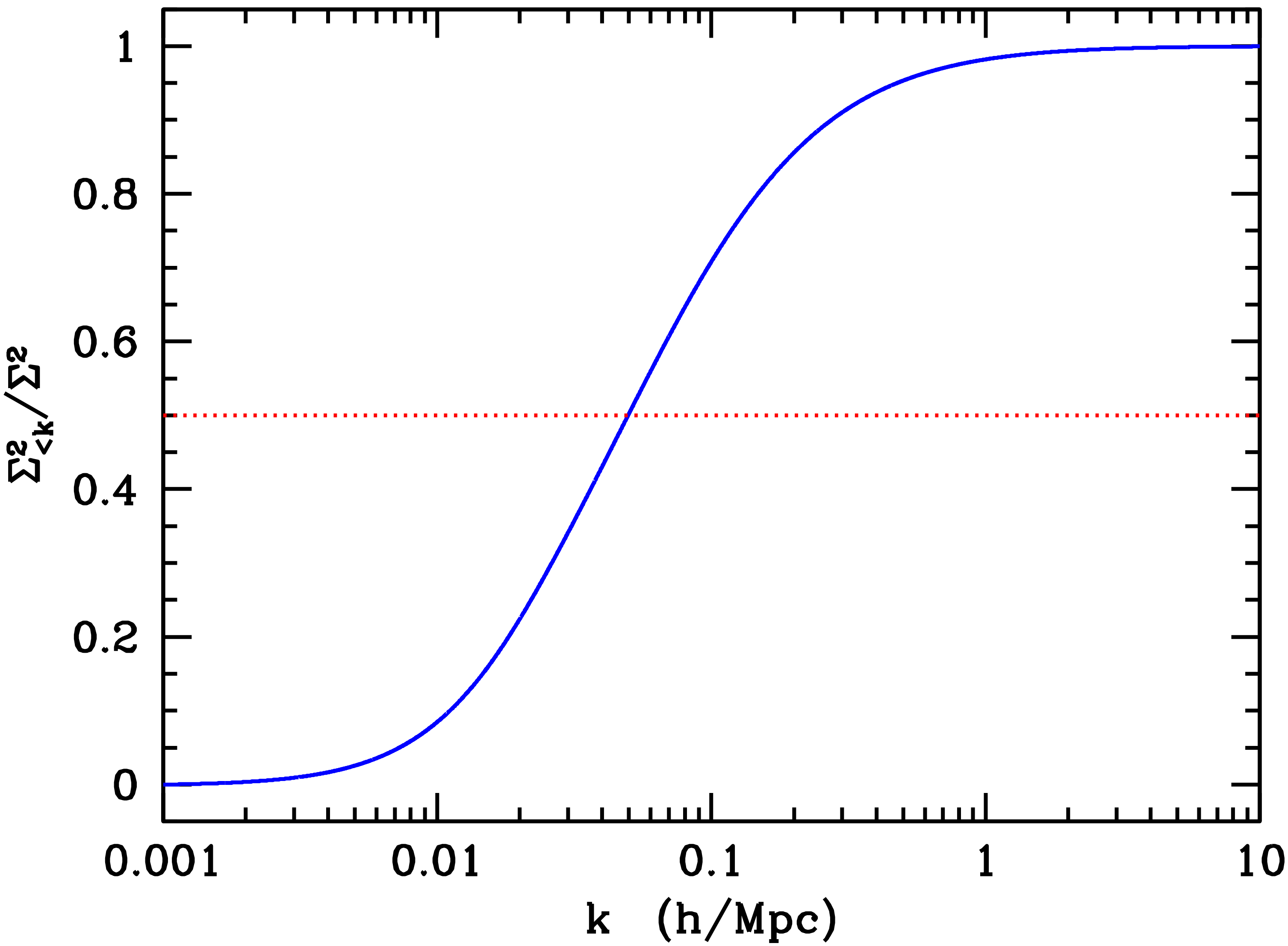
- Meiksin, White & Peacock, 1999
  – Baryonic signatures in large-scale structure
- Crocce & Scoccimarro, 2007
  – Nonlinear Evolution of Baryon Acoustic Oscillations
- Nishimichi et al., 2007
  – Characteristic scales of BAO from perturbation theory
- Matsubara, 2007, 2008
- Jeong & Komatsu, 2007, 2008
  – Perturbation theory reloaded I & II
- Pietroni, 2008
  – Flowing with time
- Padmanabhan et al., 2009; Noh et al. 2009
  – Reconstructing baryon oscillations: A Lagrangian theory perspective
  – Reconstructing baryon oscillations.
- Taruya et al., 2009
  – Non-linear Evolution of Baryon Acoustic Oscillations from Improved Perturbation Theory in Real and Redshift Spaces

Reconstruction
an analytic understanding?

Reconstruction and LPT

- Recall that the effect of non-linearity was to broaden (and slightly shift) the acoustic peak.
- The broadening was equal to the Zel'dovich displacement.
  - Much of the broadening comes from large scales.
- Since those scales are measured by the survey, one could hope to "reconstruct" the initial, unbroadened feature.
  - Eisenstein, Seo, Sirko & Spergel (2007).
- What does this procedure do?
  - Lagrangian perturbation theory is almost perfectly suited to studying reconstruction.

Contributions to the displacement

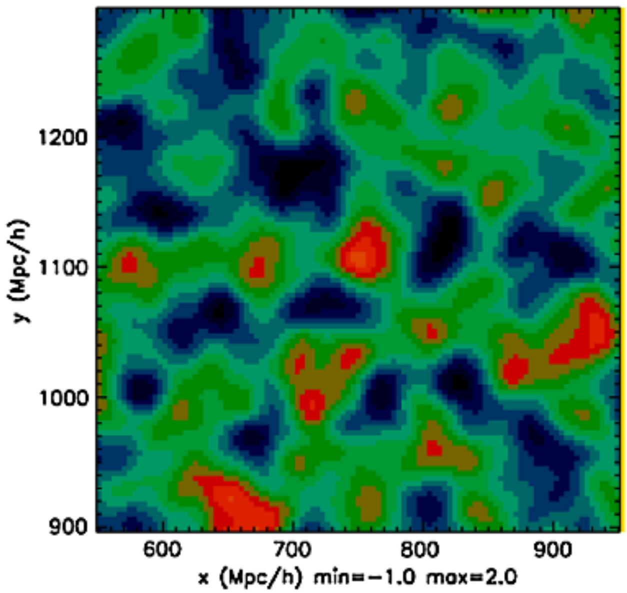


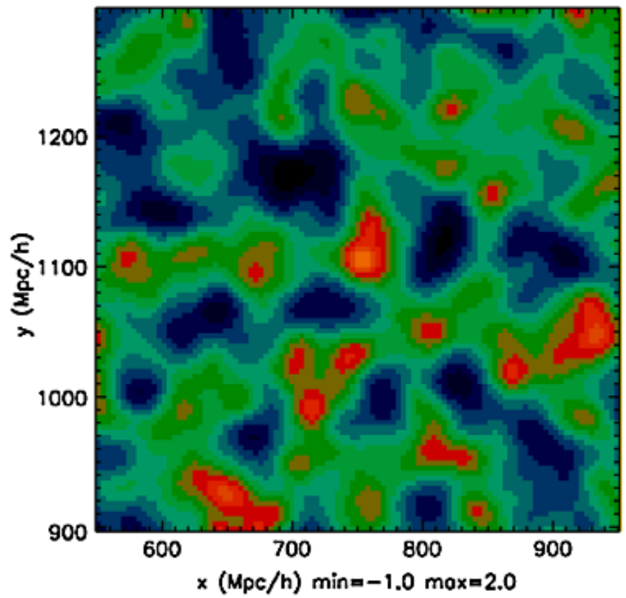
Reconstruction procedure

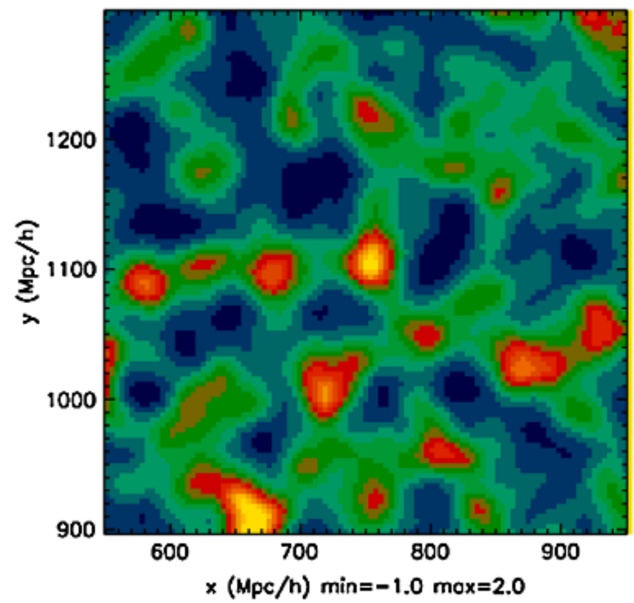
1. Smooth the density field
   - $\delta(k) \rightarrow \delta(k)\, S(k)$
2. Compute the negative Zel'dovich displacement, s, from the smooth field.
   - $\boldsymbol{s}(k) = (-i\boldsymbol{k}/k^2)\, S(k)\, \delta(k)$
3. Shift particles by s to generate "displaced" field, δ_d.
   - In linear theory $\delta_d=0$.
4. Shift spatially uniform grid of points by s to give "shifted" field, δ_s.
   - In linear theory $\delta_s=-\delta_d$.
5. Define $\delta_r=\delta_d-\delta_s$ (equals δ in linear theory).
6. Note: $S \rightarrow 0$ is equivalent to no reconstruction.

In pictures

Noh++09



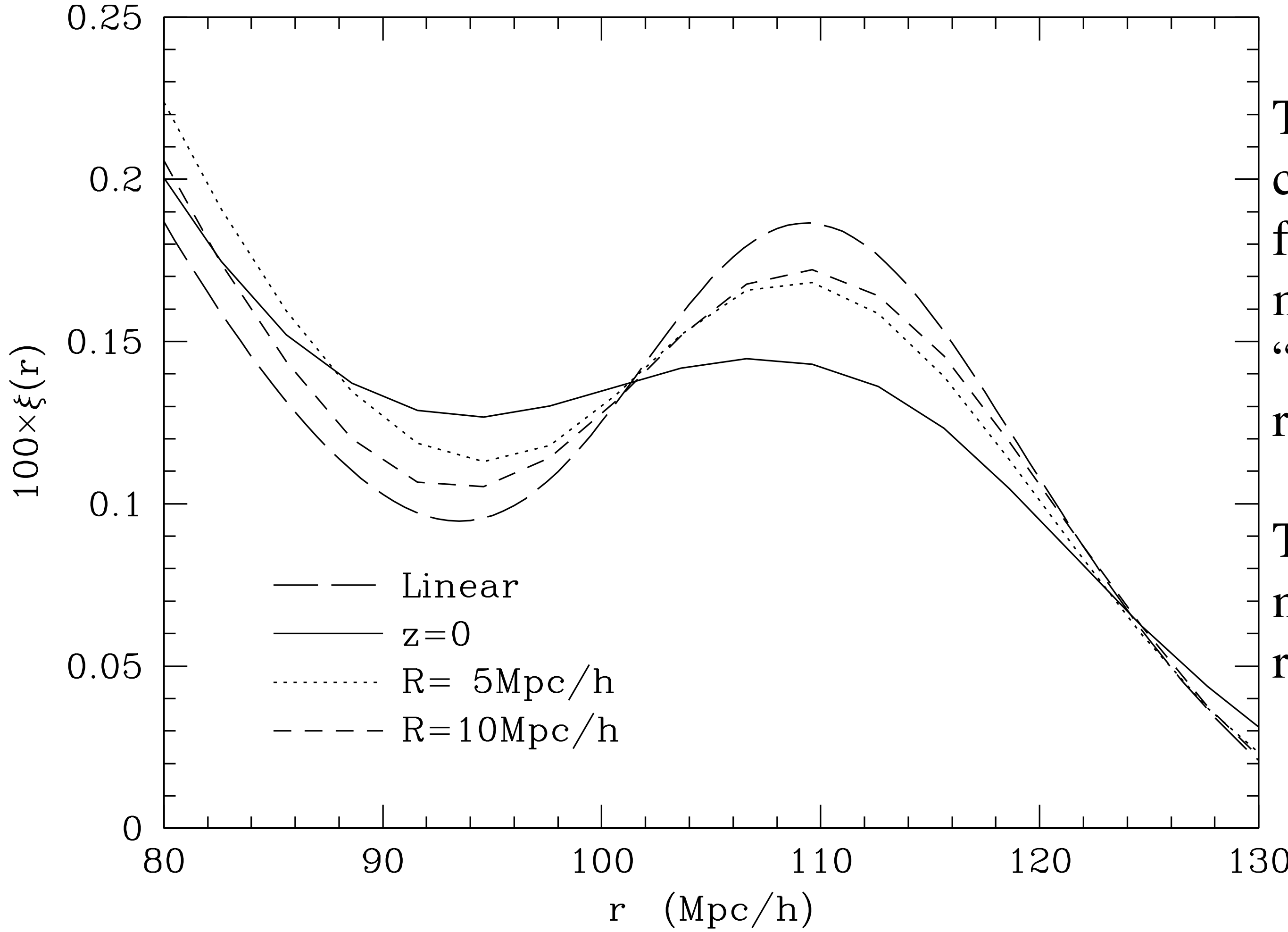




Initial Recon Final/NL

Note: the final field has sharper, more pronounced peaks than either the initial or reconstructed density fields.

Sharpens the peak



The z=0 correlation function of the mass in ΛCDM is "sharpened" by reconstruction.

The linear field is not fully recovered.

LPT

- Recall in LPT $\delta(\mathbf{k}) = \int d^3q\ e^{-i\mathbf{k}\cdot\mathbf{q}} \left(e^{-i\mathbf{k}\cdot\mathbf{\Psi}(\mathbf{q})} - 1\right)$
- The displaced field is generated by **Ψ+s**
- The shifted field is generated by **s**.
- To lowest order $\delta_r = \delta_L$.
- To next order

$$\begin{aligned}
\delta_r^{(2)} &= \delta^{(2)} - \frac{1}{2}\int \frac{d^3k_1 d^3k_2}{(2\pi)^3} \delta^{(D)}\left(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}\right) \\
&\times \delta_l(\mathbf{k}_1)\delta_l(\mathbf{k}_2)\ \mathbf{k}\cdot\mathbf{L}^{(1)}(\mathbf{k}_1)\mathbf{k}\cdot\mathbf{L}^{(1)}(\mathbf{k}_2) \\
&\times [\mathcal{S}(\mathbf{k}_1) + \mathcal{S}(\mathbf{k}_2)]
\end{aligned}$$

Why does reconstruction help?

A toy model

- Imagine $\Psi=\Psi_L+\Psi_H$ both Gaussian and uncorrelated.
  - Ψ_L is generated by δ_{lin},
  - Ψ_H contains no BAO.

$$P(k) = \int d^3q e^{-i\mathbf{k}\cdot\mathbf{q}} \left(\left\langle e^{-ik_i \Delta\Psi_i(\mathbf{q})} \right\rangle - 1 \right)$$

$$\left\langle e^{-i\mathbf{k}\cdot\Delta\boldsymbol{\Psi}(q)} \right\rangle = \exp\left[-\frac{1}{2} k_i k_j \left\langle \Delta\Psi_i(\mathbf{q}) \Delta\Psi_j(\mathbf{q}) \right\rangle \right]$$

$$k_i k_j \left\langle \Delta\Psi_i(\mathbf{q}) \Delta\Psi_j(\mathbf{q}) \right\rangle = 2k_i^2 \langle \Psi_i^2(\mathbf{0}) \rangle - 2k_i k_j \xi_{ij}(\mathbf{q})$$

A toy model

- $\xi_{ij}(0)=(\delta_{ij}/2)\ \Sigma^2$, and $\Sigma^2 \approx \Sigma_L{}^2$

$$P(k) = e^{-k^2\Sigma_L^2/2} \int d^3q\ e^{-ik_iq_i}\ e^{k_ik_j\xi_{ij}(q)} .$$

- Leave zero-lag piece exponentiated:

$$P_{\rm obs}(k) = e^{-\frac{1}{2}k^2\Sigma_L^2} P_L(k) + \underset{\substack{\uparrow \\ O(\Psi_H{}^2) \text{ and } O(\Psi_L{}^4)}}{P_{\rm mc}(k)} + \cdots$$

- Now $\mathbf{s}(k)=-S(k)\Psi_L(k)$, so the displaced and shifted fields are generated by $[1-S]\Psi_L+\Psi_H$ and $-S\Psi_L$.

A toy model

- The reconstructed power spectrum is
  - $P_r=(\delta_s-\delta_d)^2=P_{ss}+P_{dd}-2P_{sd}$
- with:
  - $P_{ss}=\exp[-k^2\Sigma_{ss}^2/2]S^2(k)\, P_L(k)+\ldots$
  - $P_{dd}=\exp[-k^2\Sigma_{dd}^2/2][1-S(k)\,]^2P_L(k)+\ldots$
  - etc.
- And modified damping terms (e.g.):

$$\Sigma^2_{ss} = \frac{1}{3\pi^2}\int dp\ \mathcal{S}^2(p)P_L(p)$$

- The effect of the S and [1-S] terms and the structure of the damping is to "effectively" reduce Σ to ~0.5 Σ.

LPT

- A very similar calculation carries through in the full LPT, except you have to keep more terms in the exponential if things aren't all Gaussian.
- The damping turns out to be the same.
  - We were working to lowest order in Σ, so this is not surprising.
- You additionally get the mode-coupling terms.
  - Slightly painful since you need to redo all of Matsubara with 3 different spectra.
- Find that the mode-coupling term is suppressed.

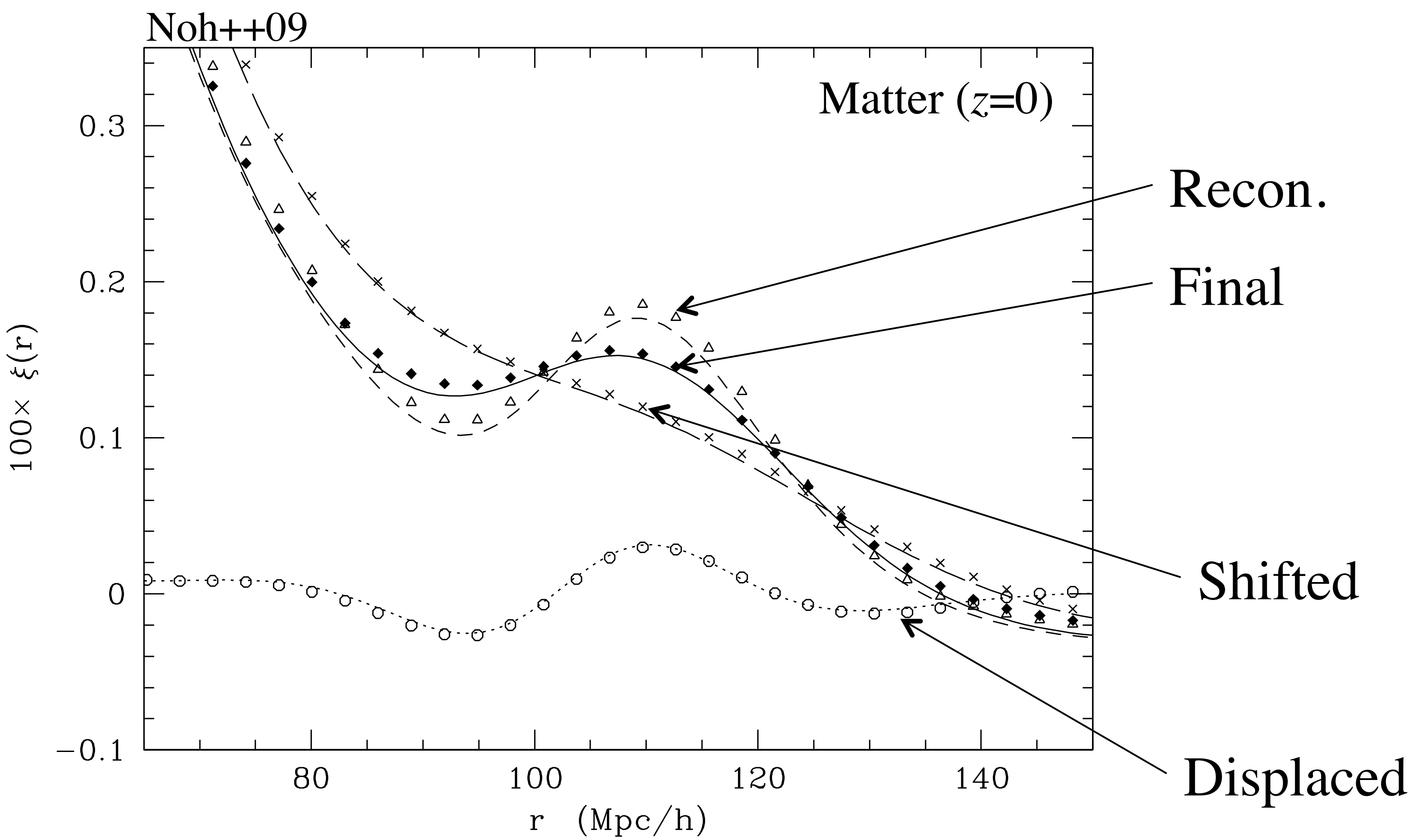
The details

$$
\begin{aligned}
P^{dd} \quad \propto \quad & P_L \bar{\mathcal{S}}^2 + \frac{9}{98} Q_1 + \frac{3}{7} Q_2^{(1d1d)} + \frac{1}{2} Q_3^{(dddd)} \\
+ \quad & \bar{\mathcal{S}} \left[\frac{10}{21} R_1 + \frac{6}{7} R_2^{(d)} \right] \\
+ \quad & \langle F' \rangle \left[2 P_L \bar{\mathcal{S}} + \frac{6}{7} Q_5^{(1d11)} + \boxed{2 Q_7^{(1ddd)}} + \right. \\
& \left. \frac{10}{21} R_1 + \frac{6}{7} R_2^{(d)} + \frac{6}{7} \bar{\mathcal{S}} (R_1 + R_2) \right] \\
+ \quad & \langle F'' \rangle \left[\frac{3}{7} Q_8 + Q_9^{(1d1d)} \right] \\
+ \quad & \langle F' \rangle^2 \left[P_L + \frac{6}{7} (R_1 + R_2) + Q_9^{(1d1d)} + Q_{11}^{(11dd)} \right] \\
+ \quad & 2 \langle F' \rangle \langle F'' \rangle Q_{12}^{(111d)} + \frac{1}{2} \langle F'' \rangle^2 Q_{13} \qquad (1)
\end{aligned}
$$

Noh++09; based on Matsubara 07 & 08

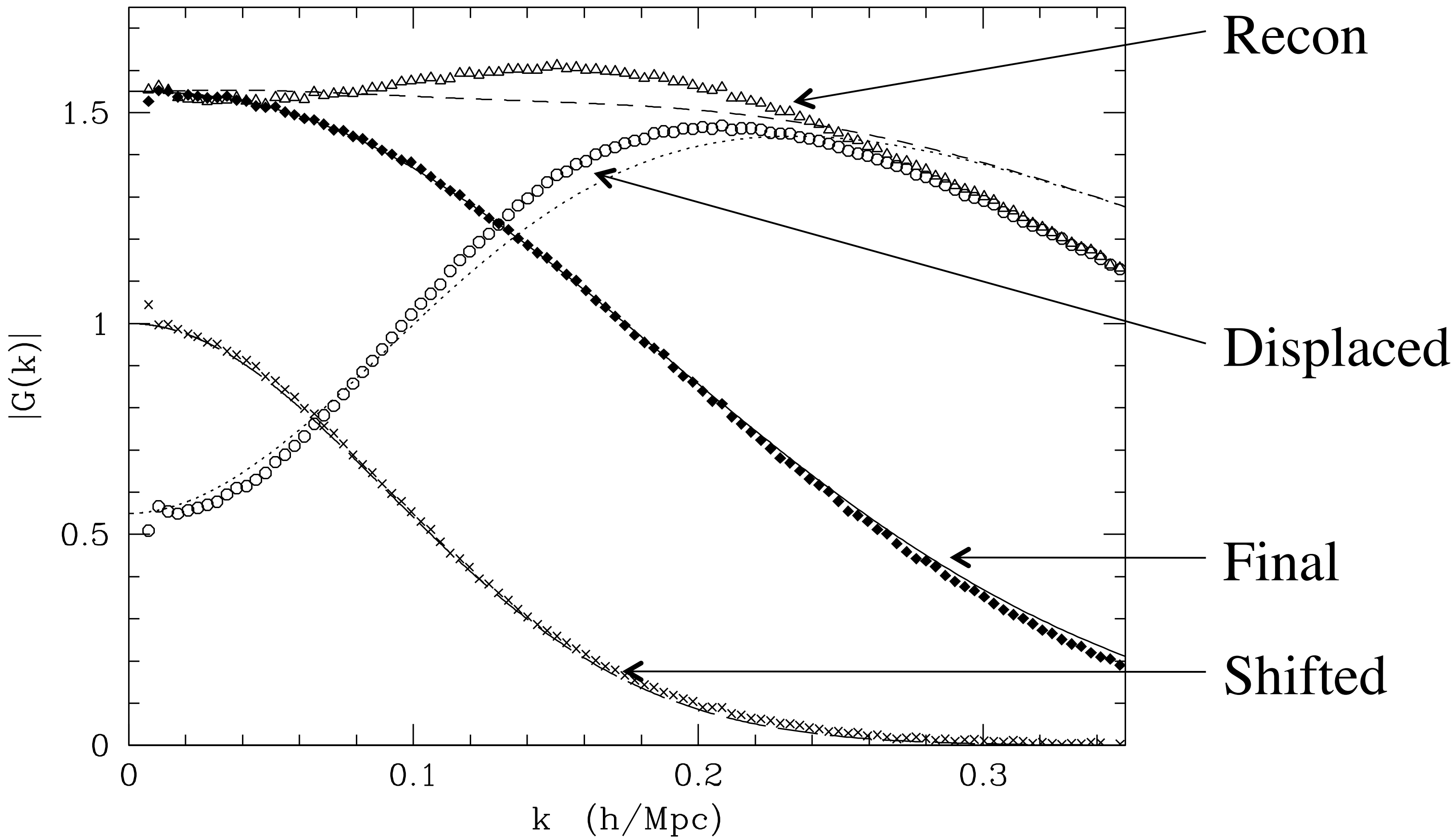
$$
Q_7^{(1ddd)}(k) = \frac{k^3}{(2\pi)^2} \int_0^\infty dr \; P_L(kr) \bar{\mathcal{S}}(kr) \int_{-1}^{+1} d\mu \; P_L(ky) \bar{\mathcal{S}}(ky) \bar{\mathcal{S}}(ky) \widetilde{Q}_7(r, \mu)
$$

LPT agrees with simulations



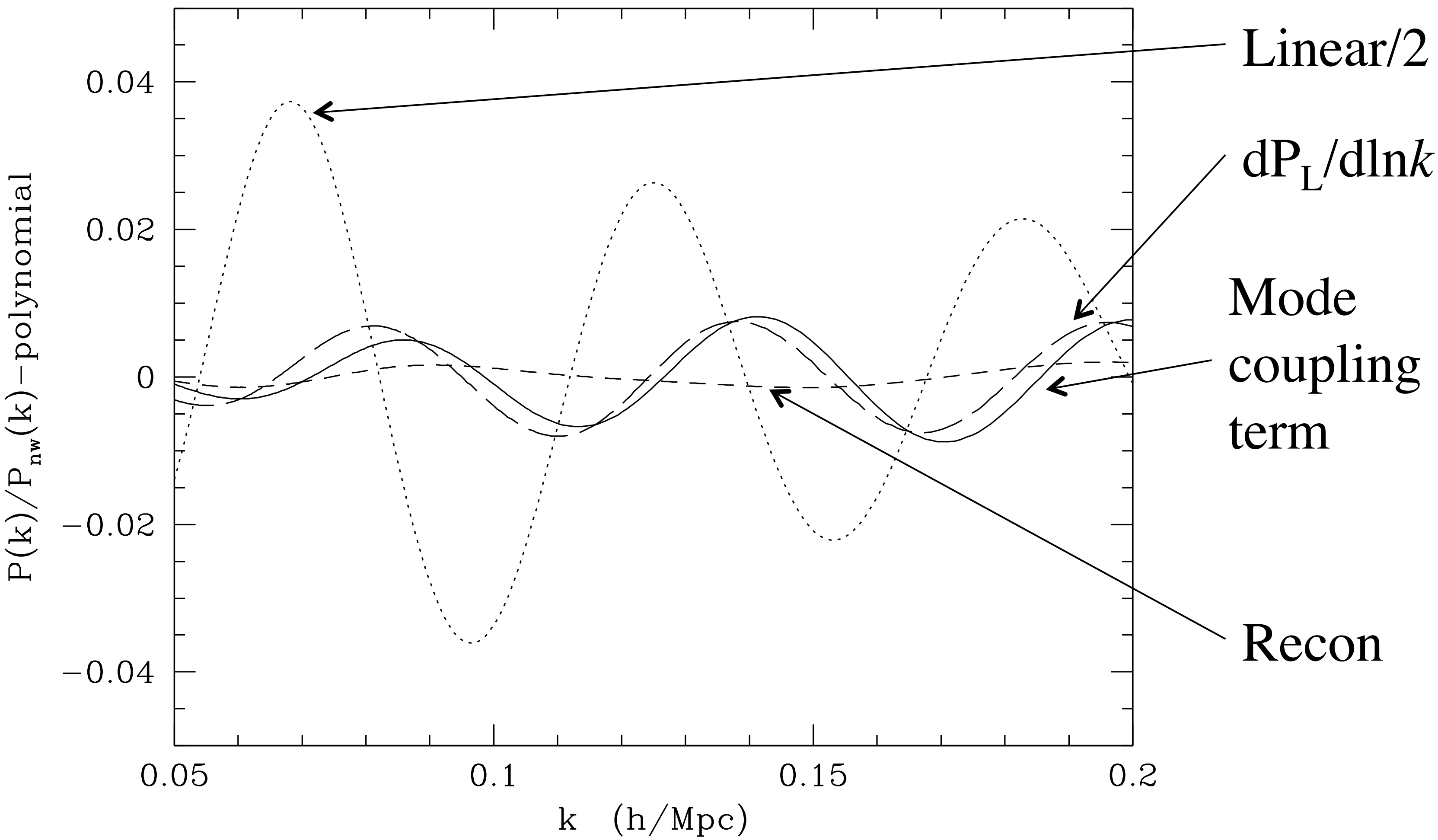
Coherence regained

Noh++09



The cross-correlation between the initial field and the other fields for halos above 10^{13}.

Out-of-phase term reduced



Out-of-phase terms in P(k) for halos more massive than 10^{13}.

Effects of shot-noise

- Within the LPT formalism the effects of shot-noise from finite galaxy number density are easy to include.
- The largest effect is a change in the damping scale:

$$\Sigma_{ss}^2 \quad \to \quad \frac{1}{3\pi^2} \int dp\ \mathcal{S}^2(p) \left[P_L(p) + P_N(p) \right]$$

$$\Sigma_{dd}^2 \quad \to \quad \frac{1}{3\pi^2} \int dp\ \left[1 - \mathcal{S}(p) \right]^2 P_L(p) + \mathcal{S}^2(p) P_N(p)\,,$$

- where $P_N=1/(b^2 n)$ is the shot-noise power.
- Gains saturate around $n\sim10^{-4}$ $(h/\mathrm{Mpc})^3$.

White (2010)

Other complications

Galaxy bias

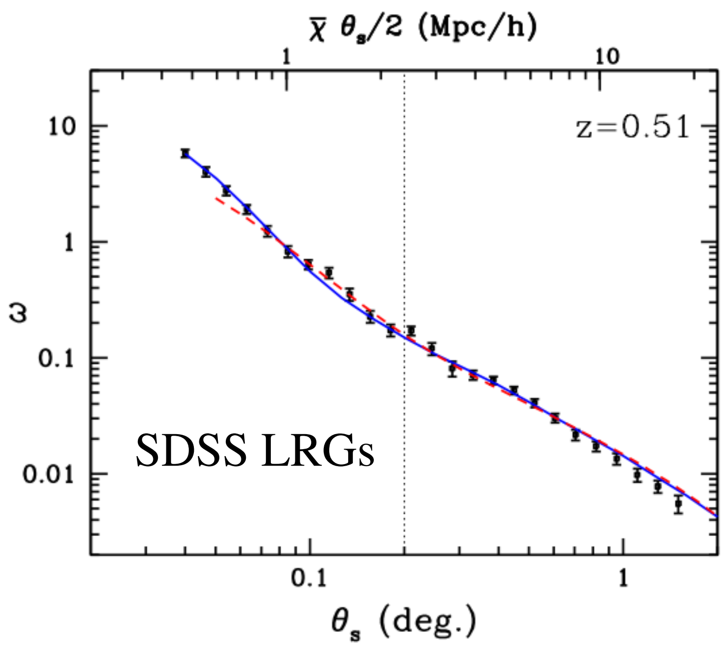
- The hardest issue is galaxy bias.
  - Galaxies don't faithfully trace the mass
- ... but galaxy formation "scale" is << 100Mpc so effects are "smooth".
  - In $P(k)$ effect of bias can be approximated as a smooth multiplicative function and a smooth additive function.
- Work is on-going to investigate these effects:
  - Seo & Eisenstein (2005)
  - White (2005)
  - Schulz & White (2006)
  - Eisenstein, Seo & White (2007)
  - Percival et al. (2007)
  - Huff et al. (2007)
  - Angulo et al. (2007)
  - Smith et al. (2007)
  - Padmanabhan et al. (2008, 2009)
  - Seo et al. (2008)
  - Matsubara (2008)
  - Noh et al. (2009)

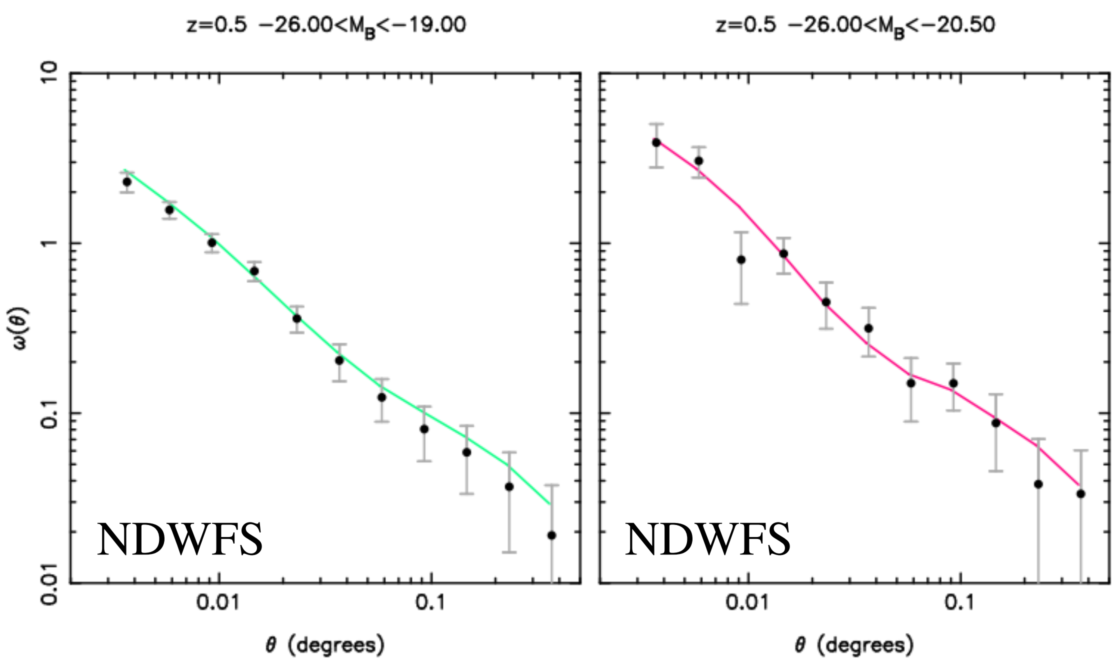
$$\Delta^2_g(k)=B^2(k)\ \Delta^2(k) + C(k)$$

Rational functions or polynomials or splines.

Modeling red galaxies

Recent advances in our ability to model (understand?) red galaxies as a function of luminosity in the range 0<*z*<1:





Padmanabhan et al. (2008); Brown et al. (2008); …

This small-scale understanding aids our models of large-scale effects.

Anisotropic clustering

- We have mostly talked about angle-averaged statistics and a single scale.
- Because peculiar velocities introduce anisotropies in the observed clustering we have the ability to measure not only d_A but also H.
  - Isotropic statistics constrain d_A^2/H.
- Expand P in Legendre polynomials:

$$P(k,\mu) = \sum_\ell P_\ell(k)\mathcal{P}_\ell(\mu)$$

Dilations and Warps

- Imagine that you were fitting the data, but you had chosen a wrong cosmology hence a wrong d_A and H.
- We can model this as a deformation from the "true" to the "observed" k:

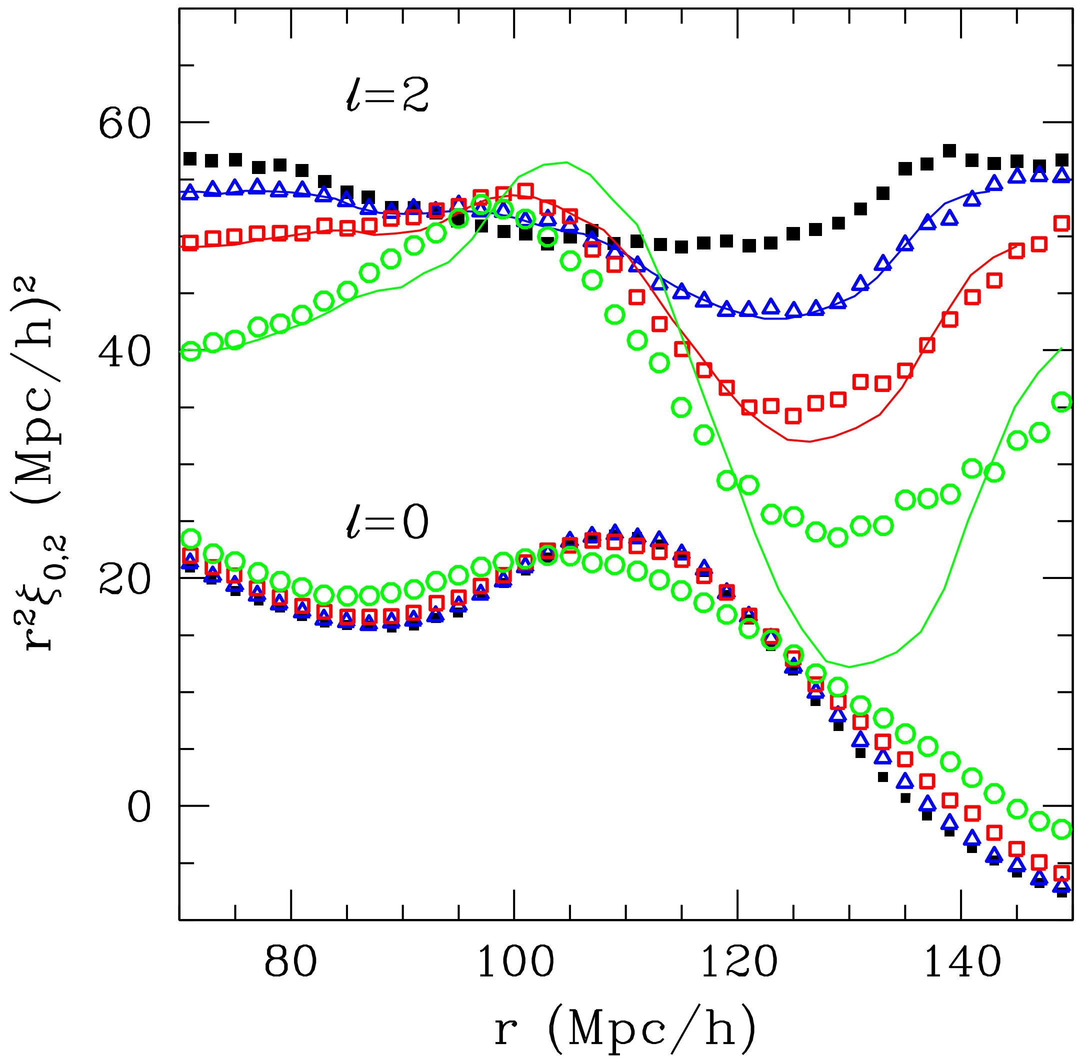
$$k_\perp \to \alpha^{-1}(1+\epsilon)k_\perp \quad , \quad k_{||} \to \alpha^{-1}(1+\epsilon)^{-2}k_{||}$$

- where α constrains d_A^2/H and ε constrains $(d_A H)^{-1/3}$.
- If we approximate $P_{>2}\approx 0$ then

$$\begin{aligned} P_0 &\to P_0 - \frac{2\epsilon}{5}\frac{d\,P_2}{d\ln k} - \frac{6\epsilon}{5}P_2 \ , \\ P_2 &\to \left(1-\frac{6\epsilon}{7}\right)P_2 - \frac{4\epsilon}{7}\frac{d\,P_2}{d\ln k} - 2\epsilon\frac{d\,P_0}{d\ln k} \ , \end{aligned}$$

- At large scales the two corrections to P_0 almost cancel.
- The change in scale of P_2 is effectively unobservable due to bias.
- The $dP_2/d\ln k$ term shifts the feature, but is very small.
- The $dP_0/d\ln k$ remains to be handled.

In simulations



0% warp
2% warp
5% warp
10% warp

Model fits “small” warps. Would need to iterate to handle larger warps.

A possible procedure

- Measure P_0, assuming it is unaffected by the warping.
  - Use this to correct for isotropic shifts (α).
- Fit P_2 using a template for P_2 and the derivative of the measured P_0 in the correction term.
  - Want to marginalize out “smooth” terms at this stage.
- Iterate the fit until convergence.
- This works in simulations, …

Padmanabhan & White (2008)

Estimators of clustering

- Estimators of the 2-point function are subject to a number of difficulties.
  - Large scales, integral constraint.
  - Survey geometry, window function.
  - Binning in k or r.
- Can we develop new estimators which have advantages over the “old” standards?
- Fourier methods nicely isolate different modes
  - Correlations only arise due to $W(k)$.
  - Can be hard to implement efficiently on large surveys.
- Configuration space methods deal well with complex/ irregular survey boundaries.
  - Lead to correlated estimates and are sensitive to uncertain mean-density of sample.

Low-*k* power and the integral constraint

- Correlation function has support to *k*~0.

$$\xi(r) = \int \frac{dk}{k}\,\Delta^2(k) j_0(kr) \simeq \int \frac{dk}{k}\,\Delta^2(k)\left[1 - \frac{(kr)^2}{6} + \cdots\right]$$

- Problematic when running simulations or measuring from survey data.
- Can generate a "bandpower" estimate, e.g.

Huff++07

$$\Delta\xi(r) \equiv \bar{\xi}(<r) - \xi(r) = \frac{3}{r^3}\int_0^r x^2 dx\, \xi(x) - \xi(r)$$

$$\Delta\xi(r) = \int \frac{dk}{k}\,\Delta^2(k) j_2(kr) \simeq \int \frac{dk}{k}\,\Delta^2(k)\left[\frac{(kr)^2}{15} - \frac{(kr)^4}{210} + \cdots\right]$$

- but this requires lags near r~0

Basic relations

$$\Delta^2(k, \hat{k}\cdot\hat{z}) \equiv \frac{k^3 P(k,\mu)}{2\pi^2} = \sum_\ell \Delta_\ell^2(k) L_\ell(\mu)$$

$$\xi(r, \hat{r}\cdot\hat{z}) \equiv \sum_\ell \xi_\ell(r) L_\ell(\hat{r}\cdot\hat{z}) \quad , \quad \xi_\ell(r) = i^\ell \int \frac{dk}{k}\ \Delta_\ell^2(k) j_\ell(kr)$$

$$\begin{aligned}
\omega_\ell(r_s) &\equiv i^\ell \int d^3r\ \xi_s(r,\mu) W_\ell(r, r_s) L_\ell(\mu) \\
&= \frac{4\pi i^\ell}{2\ell+1} \int r^2\, dr\ \xi_\ell(r) W_\ell(r, r_s) \\
&= \int \frac{dk}{k}\ \Delta_\ell^2(k) \widetilde{W}_\ell(k, r_s) \\
\widetilde{W}_\ell(k, r_s) &\equiv (-1)^\ell \frac{4\pi}{2\ell+1} \int r^2\, dr\ W_\ell(r, r_s) j_\ell(kr)
\end{aligned}$$

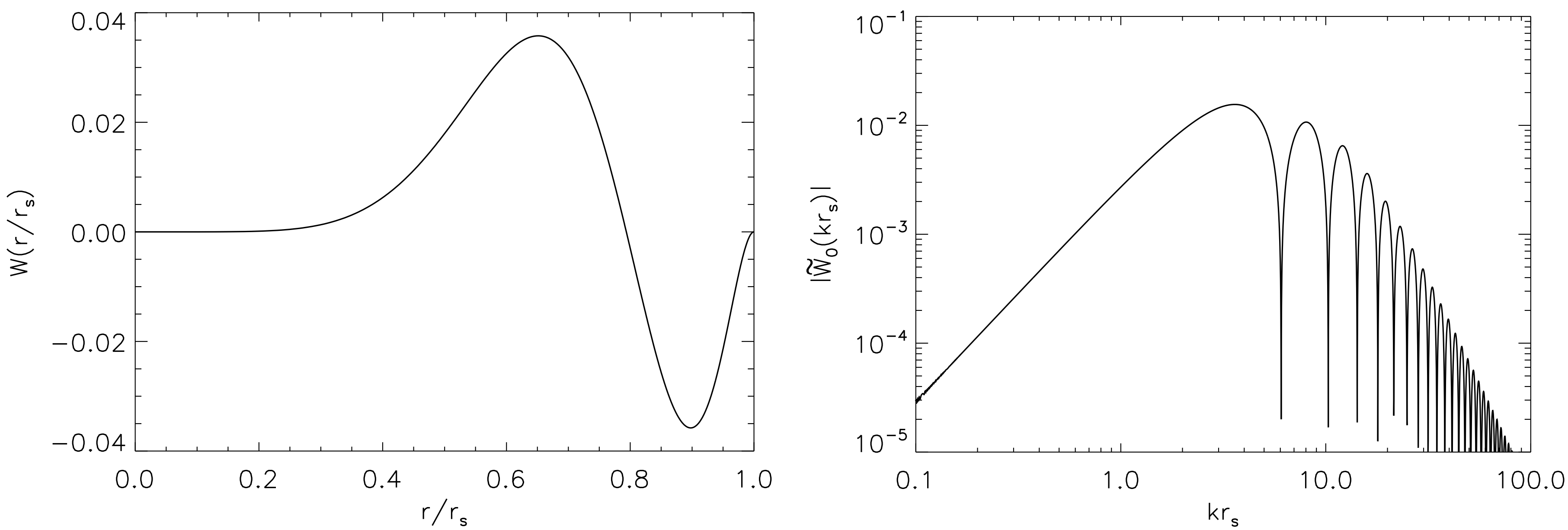
Xu++2010

Compensated filters

- If we make the filter compensated ($\int r^2 dr W(r) = 0$)
  - $W_l(k)$ is reduced for k~0.
  - Reduce sensitivity to poorly constrained low *k* power
  - Make ω_l depend multiplicatively on mean density (no integral constraint).
  - Decorrelates estimates of ω_l from different sub-volumes of the survey.

An example

$$W(x) = (2x)^2(1-x)^2\left(\frac{1}{2}-x\right)\frac{1}{r_s^3} \qquad ; \qquad x = (r/r_s)^3$$



$$\widetilde{W}_\ell \sim k^2 \text{ as } k \to 0 \quad ; \quad \widetilde{W}_\ell(kr_s) \to \cos(kr_s)/(kr_s)^4 \text{ as } k \to \infty$$

Pair counts

- Computation of ω_l is actually as easy (or easier!) than computing $\xi(r)$.

$$\omega_\ell(r_s) = i^\ell \int d^3r W_\ell(r) L_\ell(\mu) \frac{DD(r,\mu)}{RR(r,\mu)}$$

$\longleftarrow$ No -1!

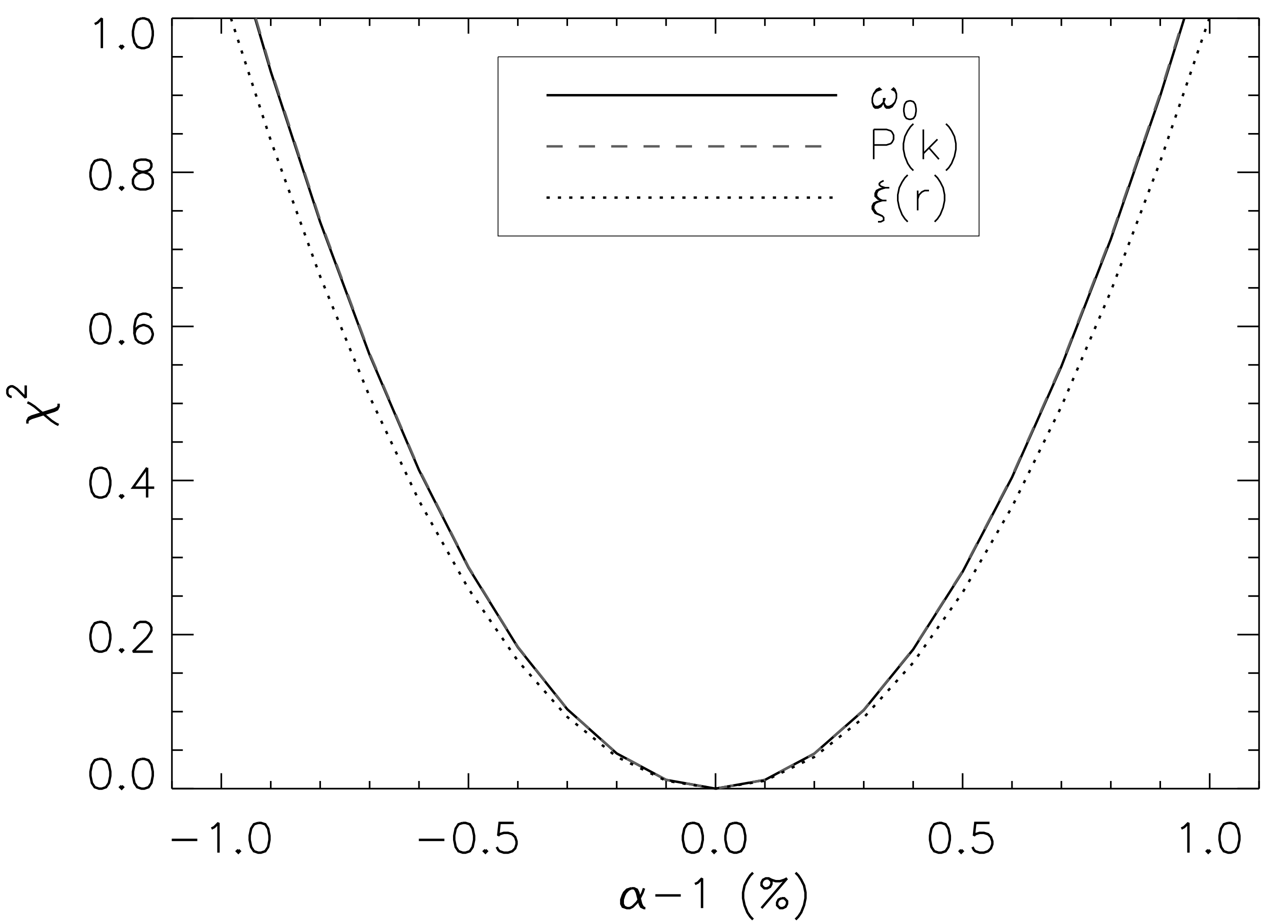
$$RR(r,\mu) = n_R N_R V \Phi(r,\mu) dr d\mu$$

$\longleftarrow$ Defines Φ, (smooth)

$$\omega_\ell(r_s) = i^\ell \sum_{i \in DD} \frac{W_\ell(r_i) L(\mu_i)}{n_D N_D V \hat{\Phi}(r_i,\mu_i)}$$

$\longleftarrow$ Sum over data pairs!

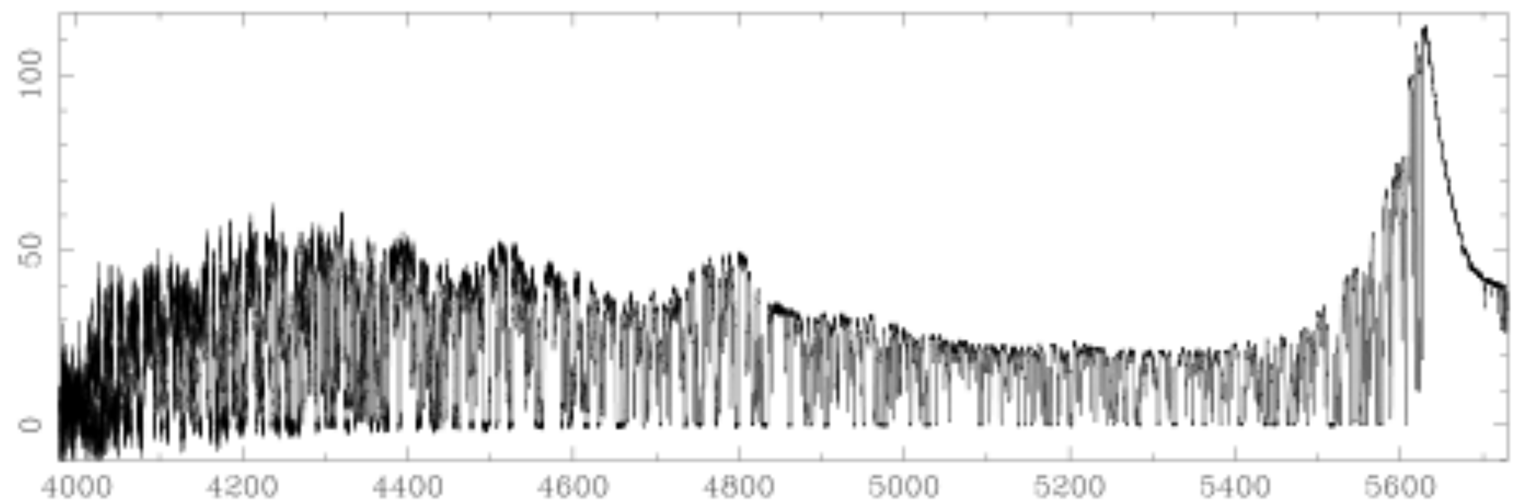
Loses no information



Fitting:
0<k<1.2h/Mpc
50<r_s<200Mpc/h
50<r<200Mpc/h
Gives the same constraints on the acoustic scale.

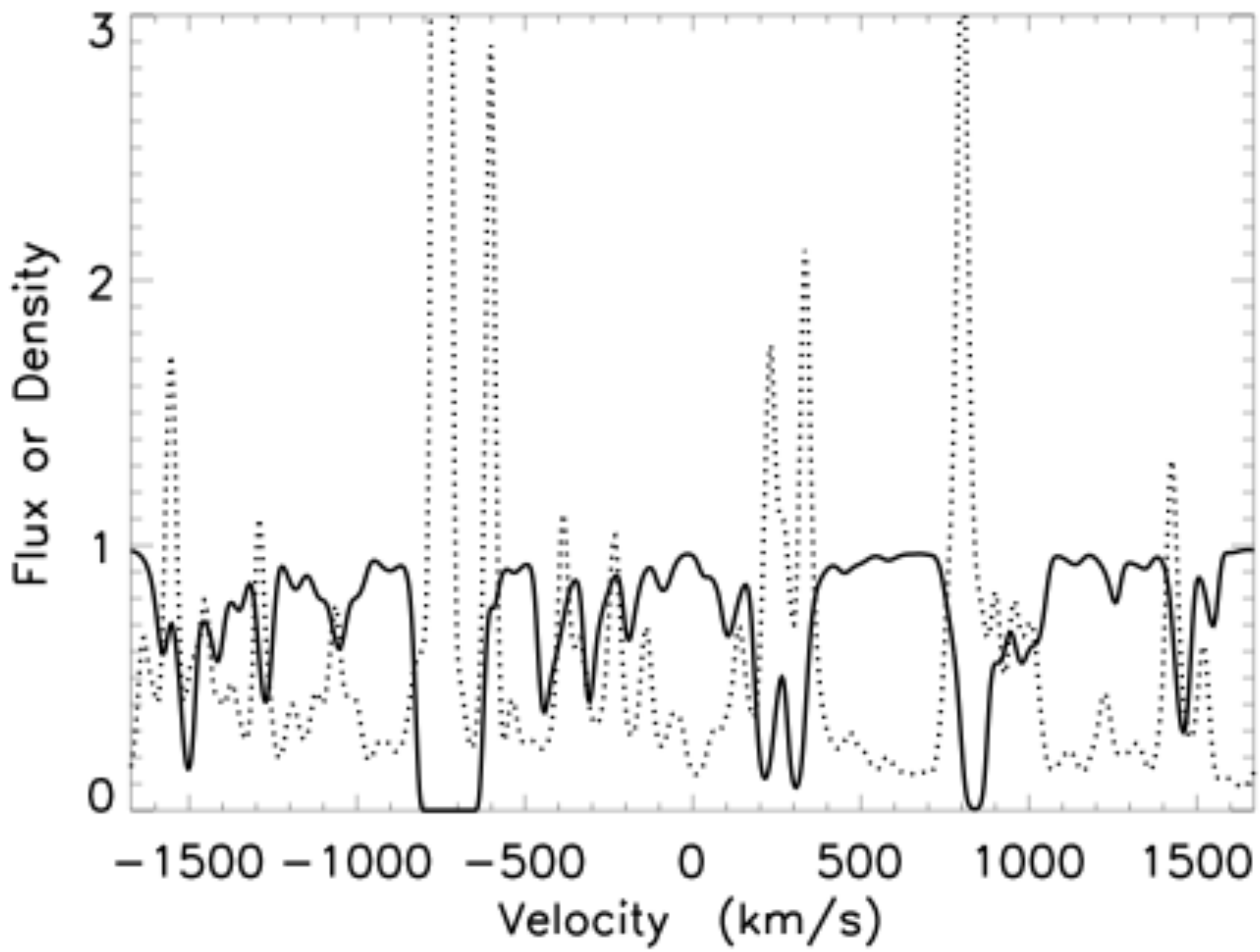
BAO at high z
The IGM and LyαF

(Meiksin 2009; Rev Mod Phys. 81, 1405)



QSO 1422+23

Spectrum ‘=’ density



The basic observations

- Observations of the Ly-α forest go back to the 70s and early 80s when the basic properties were established.
- Low resolution spectra provide mean flux or distributions of equivalent widths.
- High resolution spectra provide column densities (N_{HI}) and doppler parameters (*b*).

$N_{HI} < 10^{12}$ cm^{-2}	Not currently observable
$10^{12} < N_{HI} < 10^{17}$ cm^{-2}	Ly-α forest
$10^{17} < N_{HI} < 10^{20}$ cm^{-2}	Lyman limit systems
$10^{20} < N_{HI}$	Damped Ly-α systems

Power laws everywhere

- Equivalent width distribution
  - $d^2N/dWdz \sim e^{-W/W^*} (1+z)^\gamma$
  - $W_* \sim 0.27$A and $1.5<\gamma<3$
- Column density distribution
  - $dN/dN \sim N^{-1.5}$ $12<\log N<22$!!!
    - (Some evidence for "break", e.g. Prochaska++10)
  - Slight steepening above logN=14
- *b* distribution
  - Gaussian of mean ~ 30km/s, width 10km/s
  - *b* decreases to higher *z*
- Absorbers are weakly clustered

Cosmic web

- IGM is the main baryonic reservoir for $z>2$
  - Galaxies are “flotsam”
- Hierarchy of structure
  - Sheets for $N_{HI}<10^{14}$ cm^{-2}
  - Filaments for $N_{HI}\sim10^{15}$ cm^{-2}
  - Clouds for $N_{HI}>10^{16}$ cm^{-2}
- Smaller lines come from cold but low density material -- Hubble expansion dominates the broadening!
- Basic properties of the forest depend very weakly on cosmology or indeed hydrodynamics!

Interpretation

- But the entire framework for *interpreting* these observations has changed dramatically in "recent" years.
- No longer discuss (spherical) halos, shock, pressure or gravity confined clouds, minihalos etc.
- Now we discuss continuous density fields - the flux is a 1D, non-linear map of the density field (in redshift space).
- Much of the structure of the IGM can be understood as a consequence of the spatial coherence and properties of the "cosmic web".
- Beware misleading language and toy model concepts!

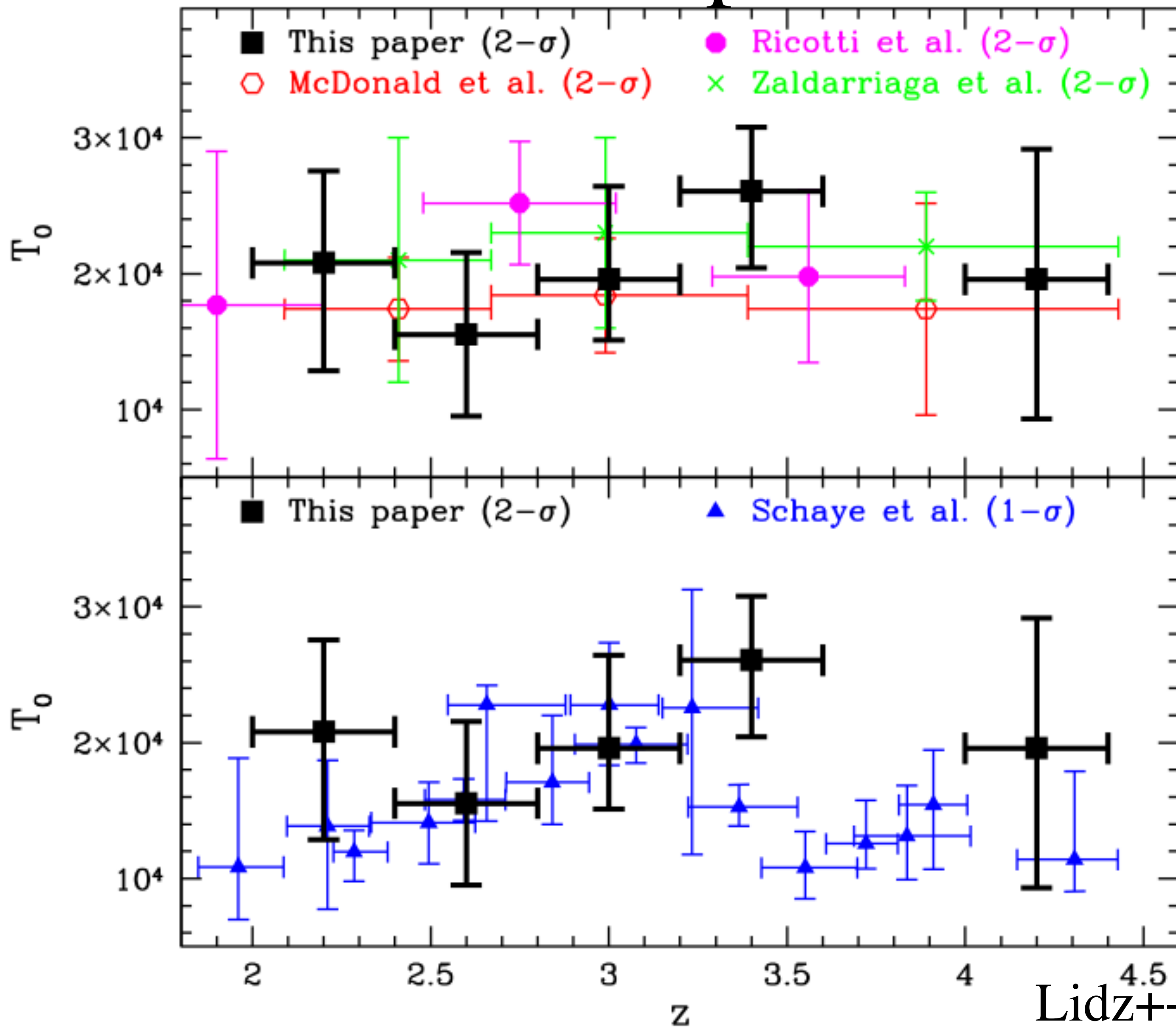
Orientation: distances & redshifts

z	**λ_α**	**$\Delta\chi$**	**$d\lambda/d\chi$**	**$dv/d\chi$**
2.0	3657	575	1.11	91
2.5	4255	546	1.37	97
3.0	4863	518	1.66	102

FGPA

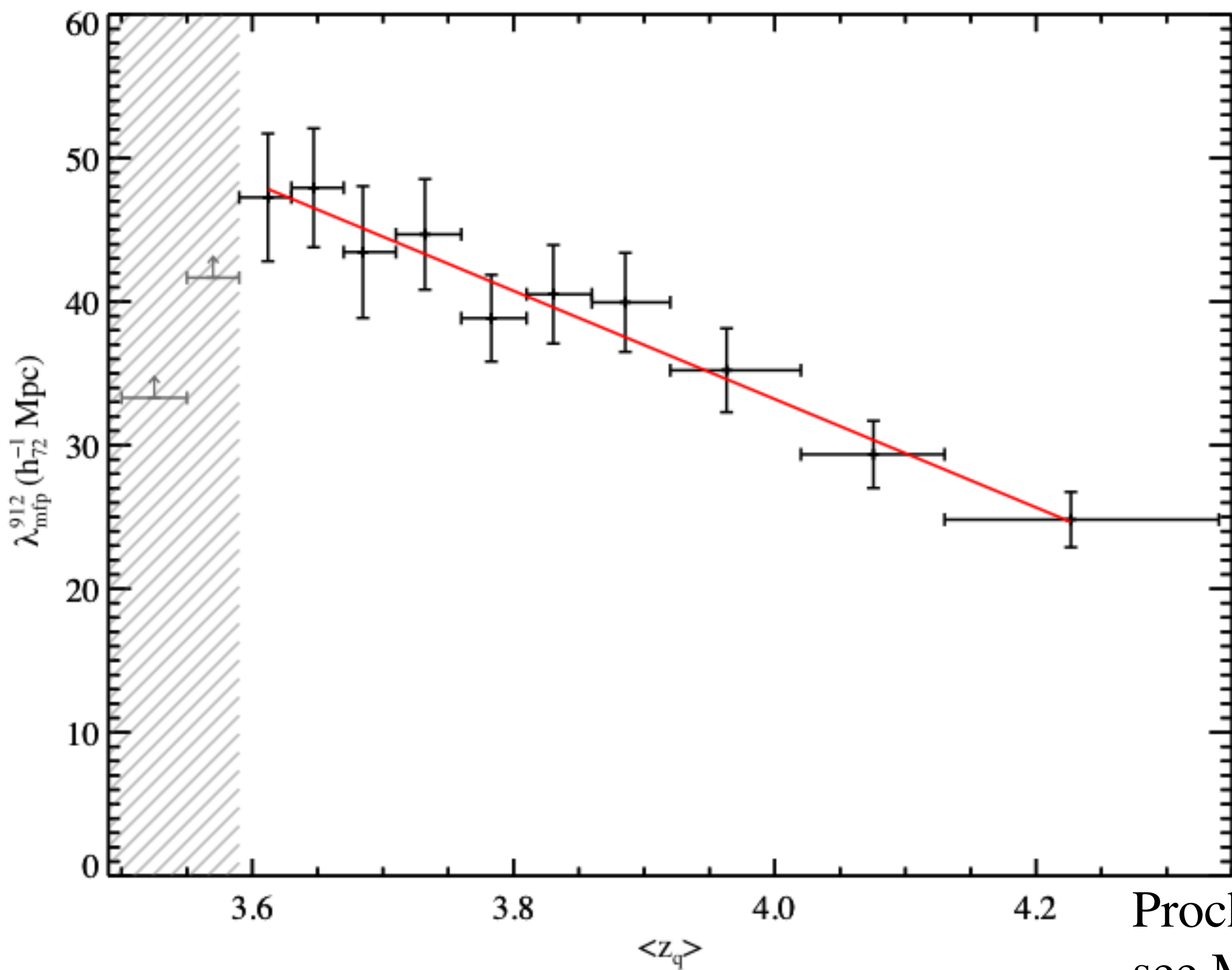
- Physics of the forest is straightforward.
  - Gas making up the IGM is in photo-ionization (but not thermal) equilibrium with a (uniform?) ionization field which results in a tight ρ-T relation for the absorbing material
    - $T = T_0 (\rho/\rho_0)^{\gamma-1}$
    - Expect $\gamma \sim 1$ at reionization to ~1.5 at late time and T_0~2. 10^4K
  - The HI density is proportional to a power of the baryon density.
    - For z<5, $x_e \sim 1$ so $n_e \sim n_p \sim n_b$ thus $n_{HI} \sim \alpha(T)\, n_b^2/\Gamma \sim n_b^p$

IGM temperature



Lidz++10

Ionizing background



How the mean free path of a photon at 912A varies with redshift.

Distances here are proper, not comoving.

Prochaska++10
see Meiksin &White 04 for results at low *z*.

FGPA

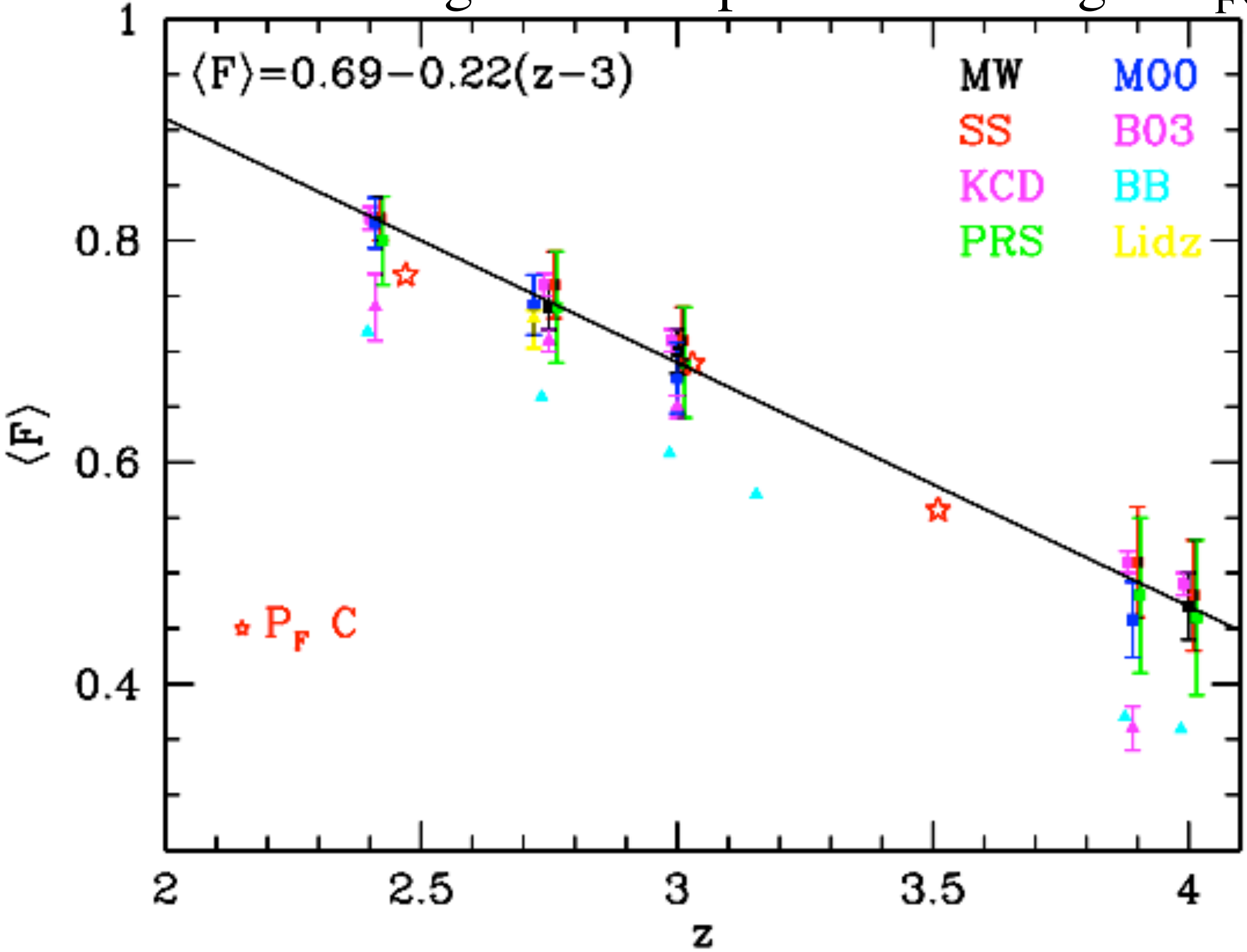
- Physics of the forest is straightforward.
  - Since pressure forces are sub-dominant on “large” scales, the gas traces the dark matter (0.1-10 Mpc/h).
  - The structure in the QSO spectrum thus traces, in a calculable way, the fluctuations in the matter density along the line-of-sight to the QSO. The Ly-α forest arises from overdensities ~ 1.

$$\tau(u) \propto \int dx \left[\frac{\rho(x)}{\bar{\rho}}\right]^2 T(x)^{-0.7} \frac{e^{-(u-u_0)^2/b^2}}{b} \quad \text{with} \quad b = \sqrt{2k_B T/m_{\mathrm{H}}}$$

  - Observed flux is $e^{-\tau}$ (times quasar continuum, plus noise, etc.)
  - The pre-factor is in principle calculable (depends e.g. on Γ) but is usually fixed by an external data point, typically <F>, or fit to the data.

Mean flux

Can be used to fix τ normalization in the FGPA, otherwise degenerate in parameter fitting to $P_F(k)$



A compilation of data from the literature.

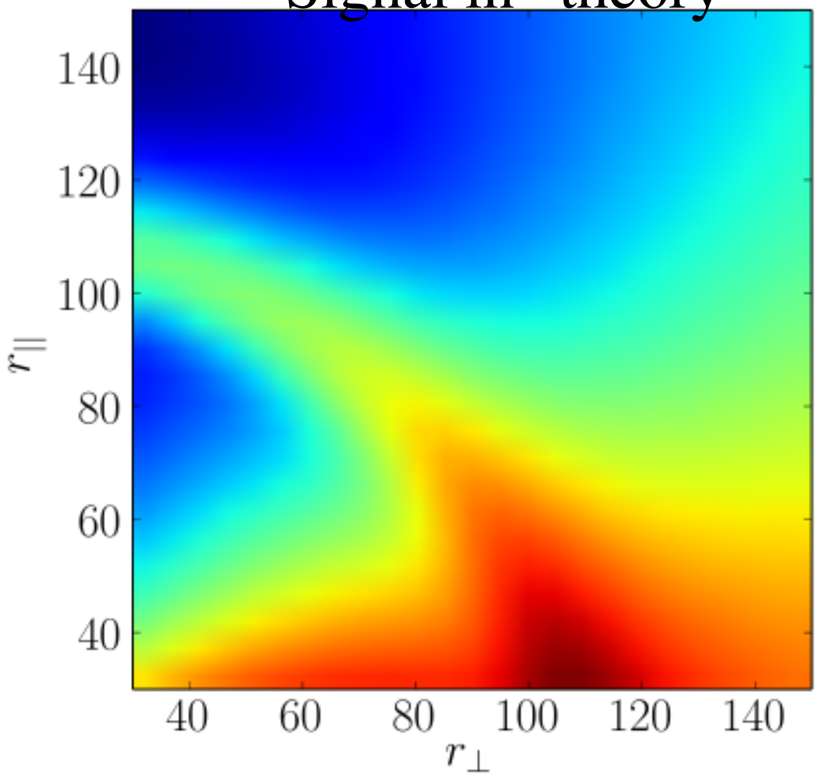
On large scales

- Now on large-scales we have that the flux is some (complicated) function of the density.
  - Flux traces mass, with a bias.
- Expect to see a BAO signal in the flux.
- Differences with the galaxies
  - Projection/finite sampling.
  - Signal is $e^{-\tau}$, so downweights high-δ.
  - Need to be slightly careful about redshift space distortions (τ conserved, not *n*).

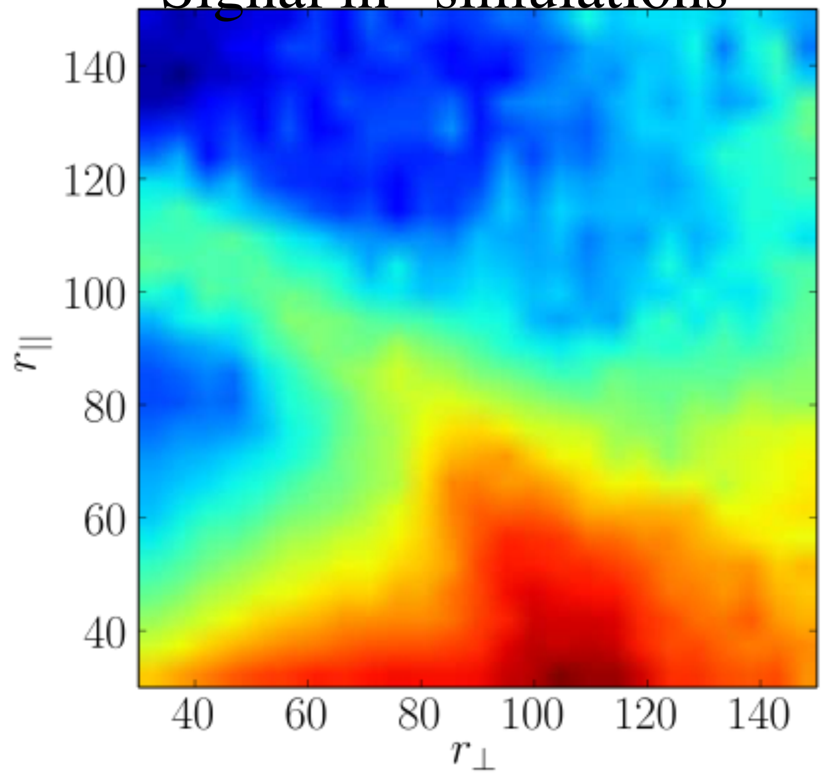
BAO at high z

Slosar, Ho, White & Louis (2009)

Signal in “theory”



Signal in “simulations”



BAO feature survives in the LyA flux correlation function, because on large scales flux traces density. Relatively insensitive to astrophysical effects.

(see also Norman++09, White++10)

Lower dimensional fields

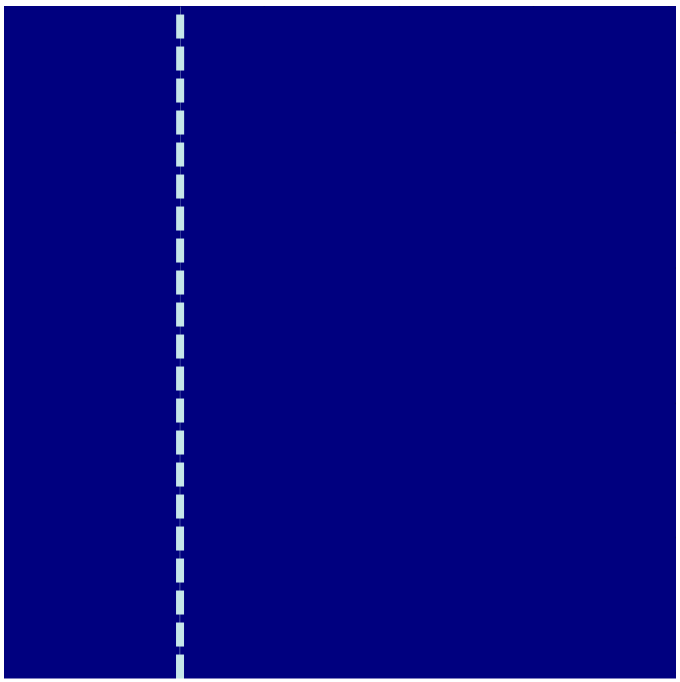
- Imagine $\delta(\boldsymbol{x})$ is a 3D stochastic field.
- Let $W(\boldsymbol{x})$ be a window function we multiply the field by in configuration space
  - $\delta_W(\boldsymbol{x}) = \delta(\boldsymbol{x})W(\boldsymbol{x})$.
- In Fourier space
  - $\delta_W(\boldsymbol{k}) = [\delta * W](\boldsymbol{k})$.
  - $P_W(k)=[P*W^2](k)$.
- For a 1D field along z: $W(\boldsymbol{x})=\delta_D(x)\delta_D(y)\boldsymbol{1}(z)$
  - $W(\boldsymbol{k})=\boldsymbol{1}(k_x)\boldsymbol{1}(k_y)\delta_D(k_z)$

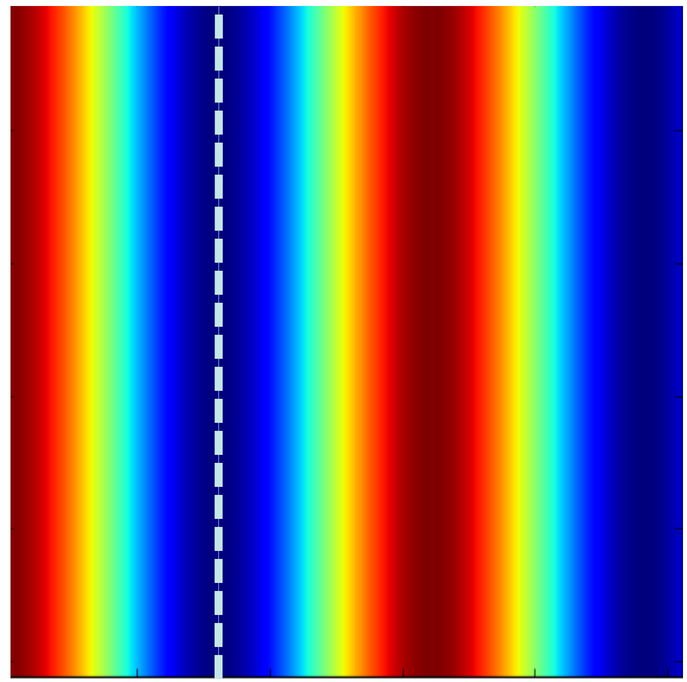
$$\Delta^2_{1D}(k) = \frac{kP(k)}{\pi} = k\int_k^\infty \frac{d^3\mathbf{k}'}{(2\pi)^3}\,\frac{P(k')}{k'}$$

Power at k_{1D} comes from $k_{3D} \geq k_{1D}$.

Aliasing

$$\Delta^2_{1D}(k) = \frac{kP(k)}{\pi} = k\int_k^\infty \frac{d^3\mathbf{k}'}{(2\pi)^3}\,\frac{P(k')}{k'}$$





Can't tell the difference between a constant field (k_x=k_y=k_z=0) and one varying transverse to the line-of-sight (k_x>0 or k_y>0)

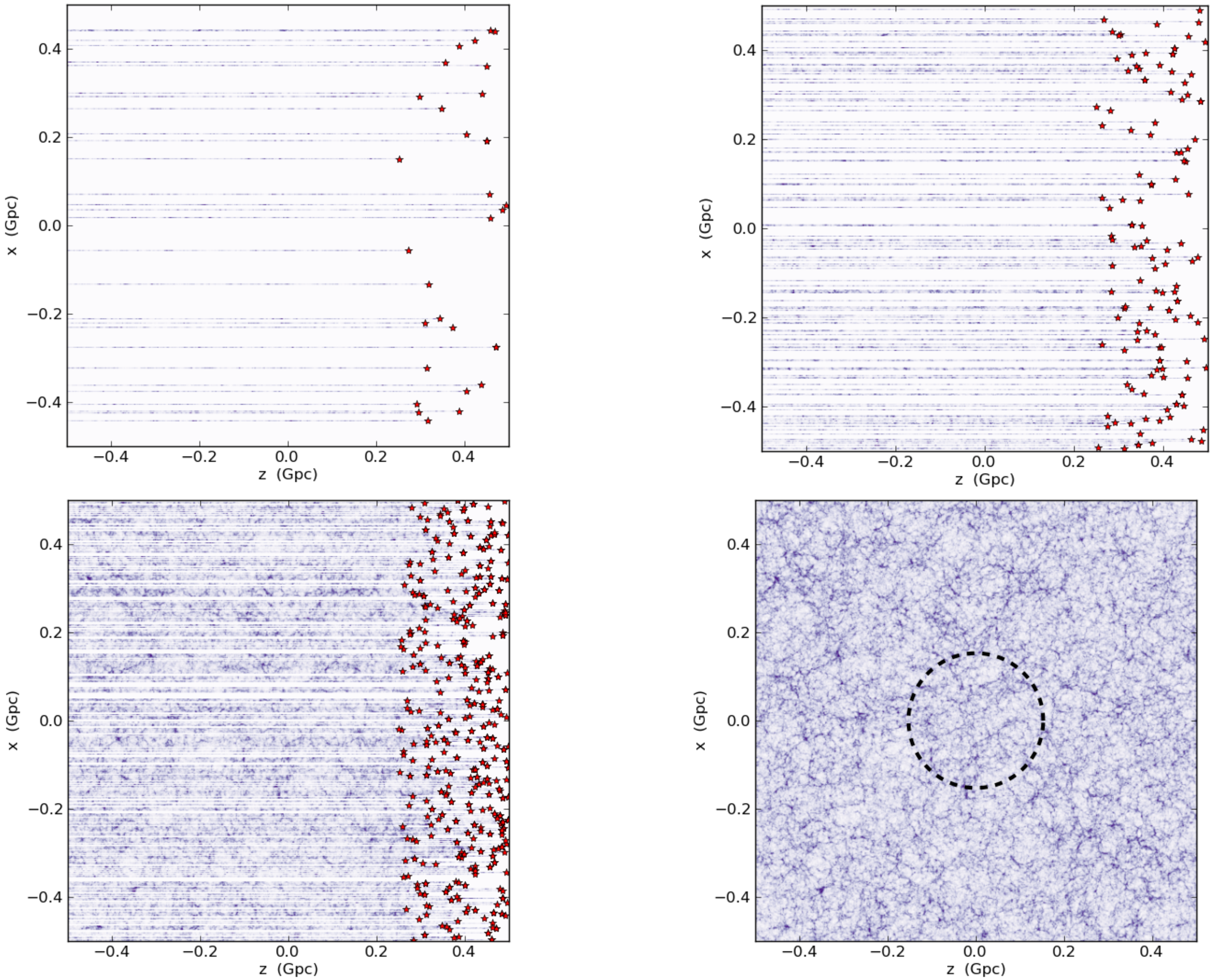
Skewer density

- Looking along a finite number of sightlines leads to power aliasing.
  - Washes out acoustic signal.
  - Increases variance.
- As the number of sightlines increases this aliasing is tamed – eventually reach sample variance.
- Variance arising from aliasing equals sample variance at a critical 2D number density of sightlines:

$$\bar{n}_{\rm crit} = \frac{\Delta^2_{1D}(k)}{k\,P(k)/\pi} \approx 0.01 h^2\,{\rm Mpc}^{-2} \qquad , \qquad \Delta^2_{1D} = k \int_k^\infty \frac{d^3k}{(2\pi)^3}\,\frac{P(k)}{k}$$

- corresponding to about 50 quasars/sq. deg.

Skewer density



Conclusions

- Baryon oscillations are a firm prediction of CDM models.
  - Baryon-photon fluid: tight-coupling, Silk damping, driving, …
- Method is "simple" geometry, with few systematics.
- The acoustic signature has been detected in the SDSS!
- With enough samples of the density field, we can measure $d_A(z)$ and $H^{-1}(z)$ to the percent level and thus constrain DE.
  - Was Einstein right?
- Require "only" a large redshift survey - we have >20 years of experience with redshift surveys.
- Exciting possibility of doing high z portion with QSO absorption lines, rather than galaxies.
- It may be possible to "undo" non-linearity.
- Understanding structure formation well enough to understand the many subtle effects we will measure with future surveys is an interesting theoretical challenge!

Thank you!

- I would like to thank
  - the organizers of this school for the invitation to speak and their hospitality.
  - the participants for their attention and questions.
  - the staff for their support.

The End