A 1-1, volume and local structure preserving remapping of periodic cubic boxes

Jordan Carlson & Martin White  
UC Berkeley, LBNL  
(paper, in prep)
Running simulations in non-cubical volumes has numerical issues.
Sometimes the desired geometry is not cubical.
By viewing the periodic cube as a hyper-torus one can devise “wrappings” of light rays which generate light-cones or cube remappings which allow non-cubical geometries.
- But this can become complex.

We have a new way of thinking about generating non-cubical geometries from cubical simulations which is
- Fast (so can do on-the-fly calculations, e.g. lightcones)
  - Computer graphics, “collision detection”.
- Volume preserving (one longer and two shorter sides).
- One-to-One: every particle appears once and only once.
- Structure preserving
  - Local neighboring structures are mapped to neighboring places.
Two views (2D): shifting and shearing

\[ 1 \times 1 \rightarrow \sqrt{2} \times \frac{1}{\sqrt{2}} \]
Example: a slice through a simulation

(Can mask “boundaries” if desired)
The method generalizes easily to 3D and gives a fast way of evaluating the remapping.

Final possible configurations are specified by integers $m$ and $n$ s.t.

$$L_x = \sqrt{1 + m^2 + n^2}, \quad L_y = \frac{\sqrt{1 + n^2}}{\sqrt{1 + m^2 + n^2}}, \quad L_z = \frac{1}{\sqrt{1 + n^2}}$$

For example, for a cube $500h^{-1}$Mpc on a side (e.g. Millennium)

<table>
<thead>
<tr>
<th>$(m,n)$</th>
<th>$L_x$</th>
<th>$L_y$</th>
<th>$L_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,2)</td>
<td>1120</td>
<td>500</td>
<td>220</td>
</tr>
<tr>
<td>(1,1)</td>
<td>870</td>
<td>410</td>
<td>350</td>
</tr>
<tr>
<td>(3,2)</td>
<td>1870</td>
<td>300</td>
<td>220</td>
</tr>
</tbody>
</table>
Consider a “survey” 100 sq. deg. to $z=1$.

- $z=1$ is $\chi = 2400 \, h^{-1} \text{Mpc}$, so 10 deg is $\approx 400h^{-1}\text{Mpc}$ (comoving) on a side.
- Total volume is $2400 \times 400 \times 400 \,(h^{-1}\text{Mpc})^3 \approx 4 \times 10^8 \,(h^{-1}\text{Mpc})^3 \approx (700 \, h^{-1}\text{Mpc})^3$
- If we run a $1h^{-1}\text{Gpc}$ box we can embed this easily as (e.g.)

<table>
<thead>
<tr>
<th>$(m,n)$</th>
<th>$L_x$</th>
<th>$L_y$</th>
<th>$L_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,2)</td>
<td>2450</td>
<td>910</td>
<td>450</td>
</tr>
<tr>
<td>(2,1)</td>
<td>2450</td>
<td>580</td>
<td>710</td>
</tr>
<tr>
<td>(2,2)</td>
<td>3000</td>
<td>750</td>
<td>450</td>
</tr>
<tr>
<td>(3,1)</td>
<td>3320</td>
<td>430</td>
<td>710</td>
</tr>
<tr>
<td>(3,2)</td>
<td>3740</td>
<td>600</td>
<td>450</td>
</tr>
</tbody>
</table>

Can do 2 regions side by side (volume ratio is 2.6)
The End