Gravitational lensing

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Outline

- What and why?
- Background and notation.
- Current status.
- Simulating weak lensing.
- Systematics and theoretical input.
- Finding clusters with weak lensing.
- Lensing the CMB.

Overview

- *Cosmic shear* is the distortion of the shapes of background galaxies due to the bending of light by the potentials associated with large-scale structure.
- For sources at $z_s \sim 1$ and structure at $0.1 < z < 1$ it is a percent level effect which can only be detected statistically.
- Observationally tractable.
- Contains “interesting” information.
- Theoretically clean.

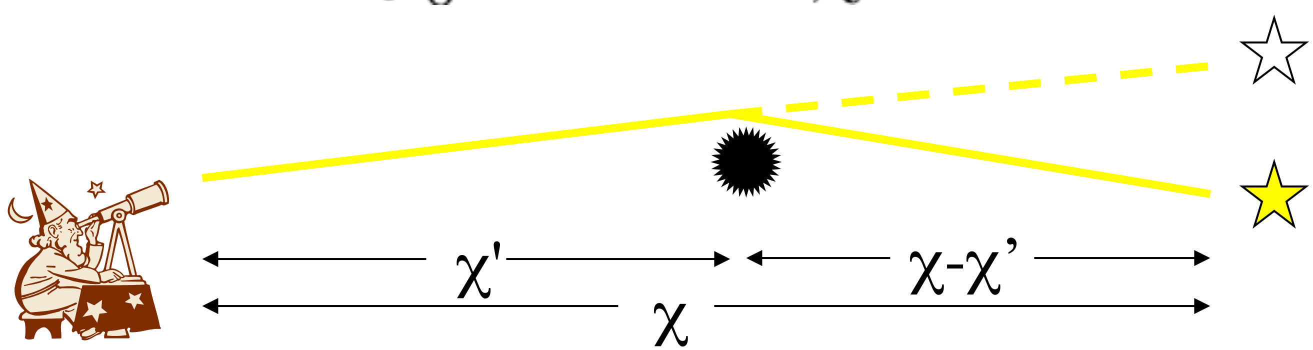
Why Bother?

- Probes large-scale structure
  - Watch structure grow as a function of epoch
  - Observe the formation of objects such as clusters
- Provide estimates of cosmological parameters
  - Measure the mass density (Ω_m)
  - Measure the amplitude of clustering (δ_H, A_s, σ_8)
  - Study Dark Energy (including growth rate!!)
- It is an interesting theoretical problem!

Background

Light from distant sources is deflected by the potentials associated with large-scale structure. Recalling that light deflection goes as the gradient of the potential we can derive the mapping:

$$\vec{\theta}(\chi) = -2\int_0^{\chi} d\chi' \, \frac{\chi - \chi'}{\chi} \nabla_{\perp}\Phi + \vec{\theta}(0)$$



Unfortunately we do not know *a priori* the positions of the galaxies that we observe, so we need to look at distortions in the shape of galaxies, i.e. the Jacobian of this mapping.

Background (contd)

Thus the "distortion matrix", which describes the how a ray bundle is modified by its transit through the universe is

$$\frac{\partial \theta_i(\chi)}{\partial \theta_j(0)} \equiv \delta_{ij} + A_{ij}$$

where A can be written as the gradient of the mapping or

$$A_{ij} = -2 \int d\chi \; g(\chi) \nabla_i \nabla_j \Phi$$

with

$$g(\chi) \equiv \int_{\chi}^{\infty} d\chi_s \; p(\chi_s) \frac{\chi(\chi_s - \chi)}{\chi_s}$$

where $p(\chi_s)$ is the source distribution.

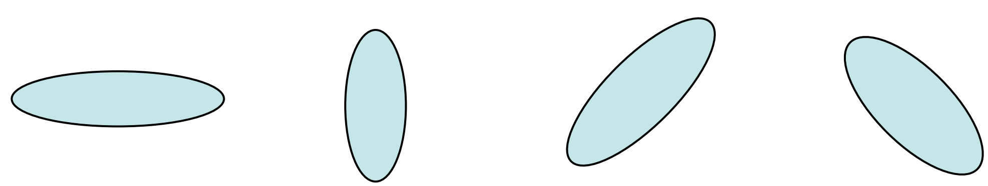
Background (contd)

The distortion matrix ***A*** is conventionally decomposed as

$$(1 + A) = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 - \omega \\ -\gamma_2 + \omega & 1 - \kappa + \gamma_1 \end{pmatrix}$$

where $\kappa \ll 1$ is the convergence and $\gamma \ll 1$ is the shear.
The rotation, ω, only comes from higher order effects and is much smaller than κ or γ.

This maps a circular source to an ellipse:



$\gamma_1 > 0$ $\gamma_1 < 0$ $\gamma_2 > 0$ $\gamma_2 < 0$

The "Born" approximation

The integral defining **A** should be taken along the perturbed photon path, but the deflection is typically small, so to 1st order we can integrate along a straight line (*Born approximation*).

Then **A** is the second derivative of a projected potential:

$$A_{ij} = -2 \int d\chi \; g(\chi) \nabla_i \nabla_j \Phi \rightarrow \nabla_i \nabla_j \phi$$

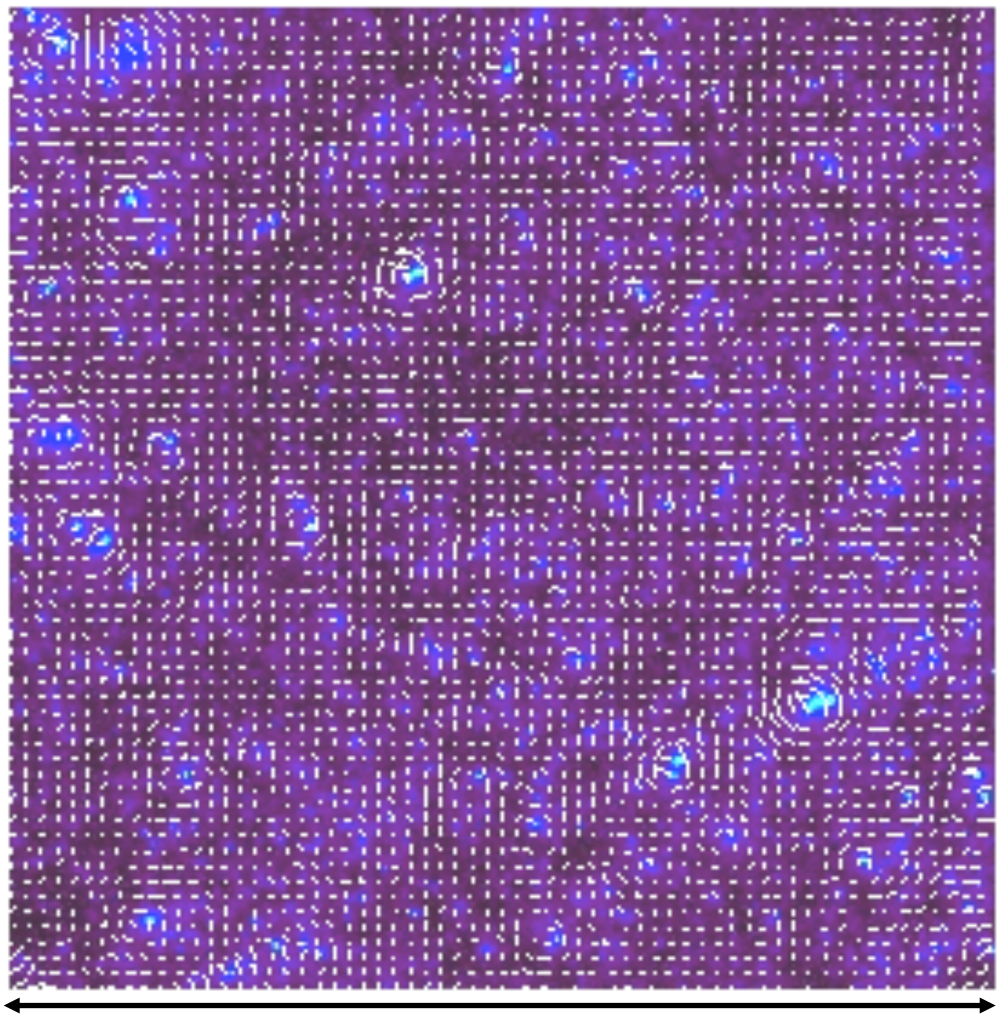
Note κ and γ come from a single potential, ϕ.

If we relate the potential to the density by Poisson's equation, integrate by parts and ignore the surface term

$$\kappa \simeq \frac{3}{2} H_0^2 \Omega_{\rm mat} \int d\chi \; g(\chi) \frac{\delta}{a}$$

In the Born limit, the convergence is (almost) the projected mass.

A simulated shear field



Obvious non-linear structure, with shear tangential about κ peaks of typical size ~1 arcmin.

Filamentary structure erased by projection.

Shear field sampled (regularly) at about the level achievable observationally from deep space based data.

Measuring Shear

The mapping (1+A) changes the shapes of galaxy images. Thus each galaxy provides a (noisy) measure of the shear at its position:

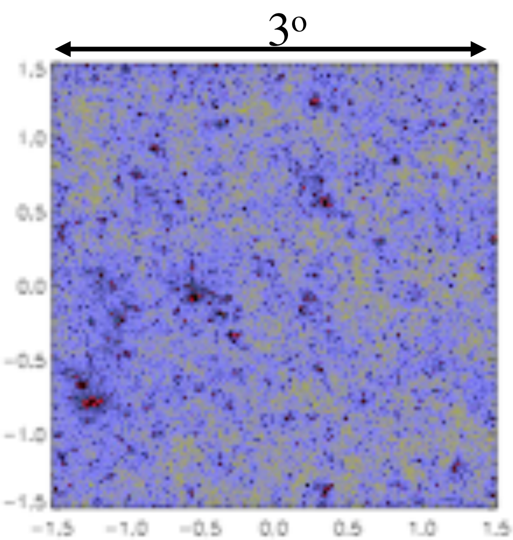
Under the assumption that galaxies are randomly oriented but coherently sheared in some region of the sky, we can simply average the measures of ellipticity to obtain the shear with an error that scales as $e_{rms}/N^{1/2}$ for N galaxies.

$$\gamma_{\mathrm{obs}} = \underbrace{\gamma_{\mathrm{intrinsic}}}_{0 \pm \sqrt{\frac{\gamma_{\mathrm{int}}^2}{N_{\mathrm{gal}}}}} + \gamma_{\mathrm{signal}} \longrightarrow \text{constant}$$

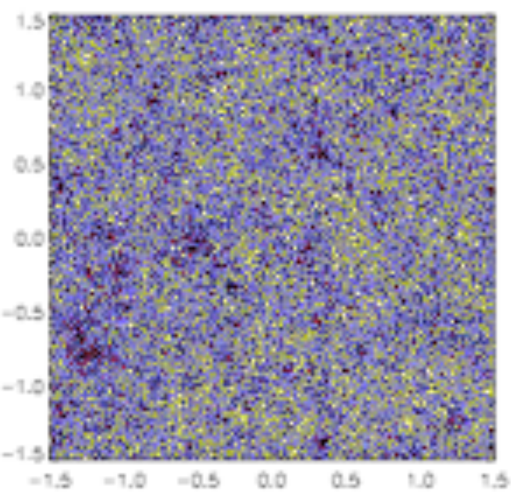
Shot noise

For 10% intrinsic ellipticities and 1% shears we need to average over 100 galaxies to get an estimate of the shear at any position on the sky with S/N~1.

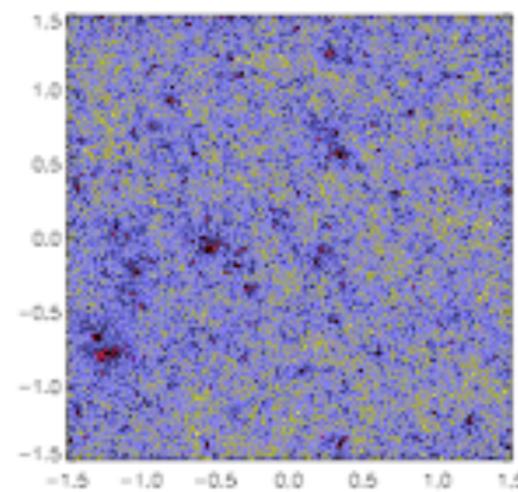
Example: simulated convergence maps with appropriate noise



Input



50 gal/arcmin^2

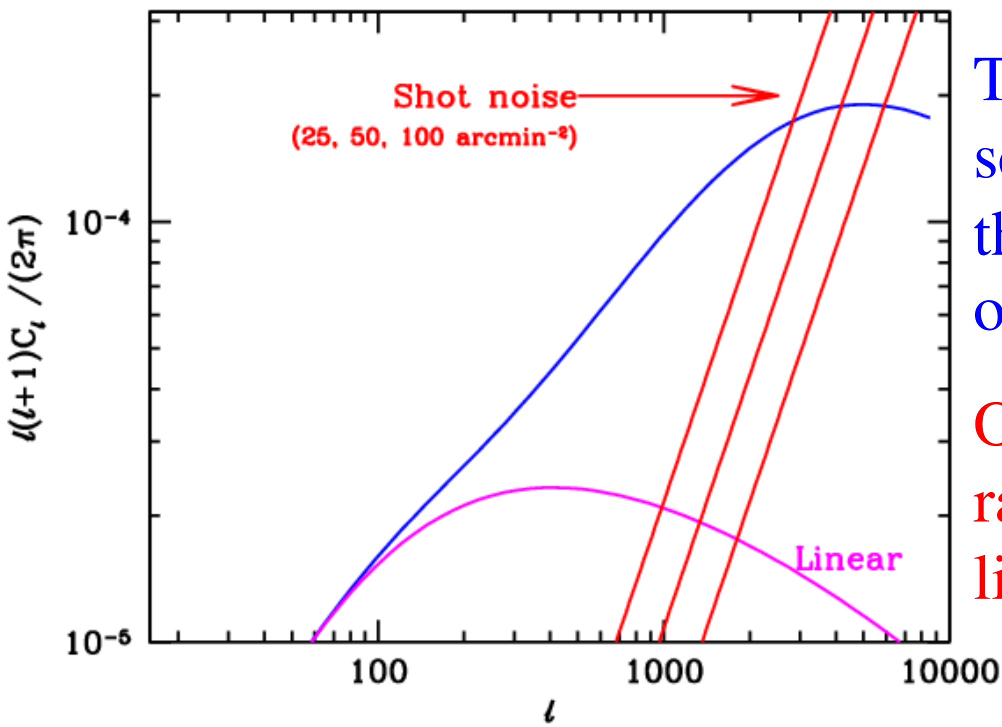


200 gal/arcmin^2

Lensing power spectrum

Within the Born and Limber approximations the shear and convergence power spectra are given by

$$\Delta^2_\kappa(\ell) = \frac{9\pi}{4\ell}\Omega^2_{\rm mat}H_0^4 \int \chi'\,d\chi' \left[\frac{g(\chi')}{a(\chi')}\right]^2 \Delta^2_{\rm m}(k=\frac{\ell}{\chi},a)$$



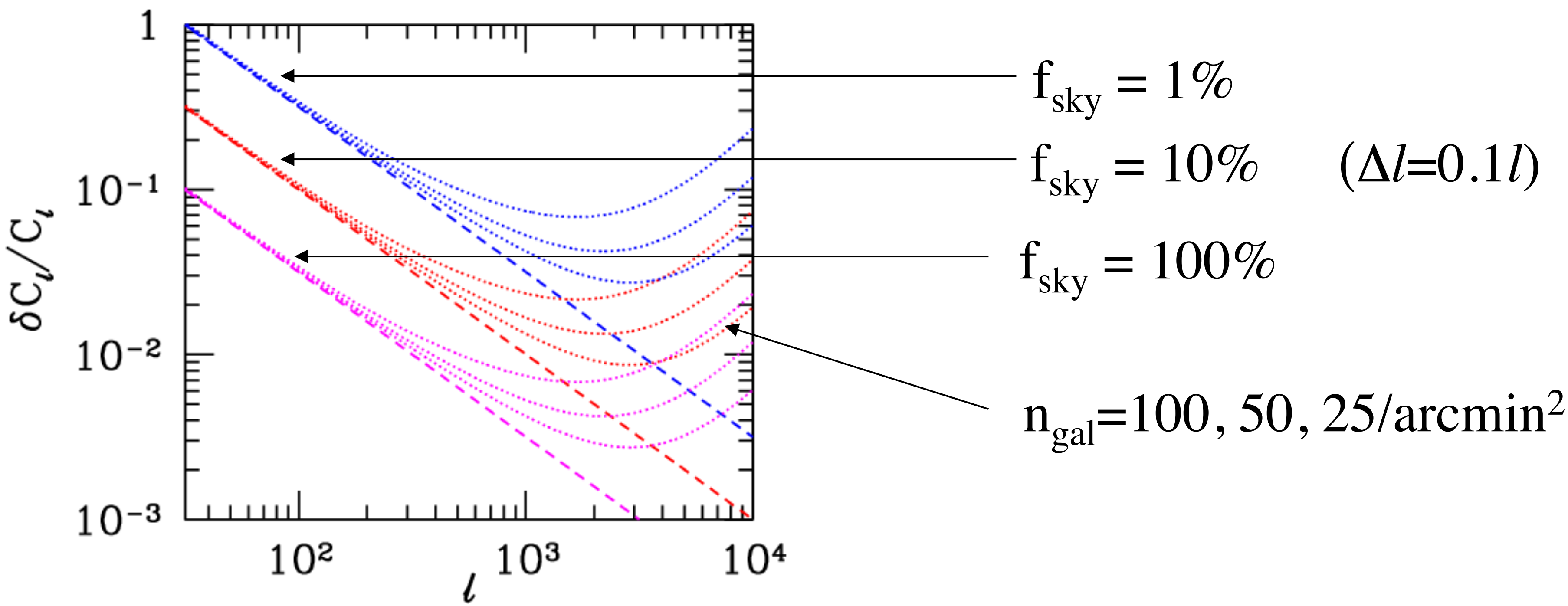
The lensing power spectrum is sensitive to the distance factors, the matter density and the growth of large-scale structure.

Over most of the measurable range it is dominated by non-linear gravitational clustering.

Measuring the power spectrum

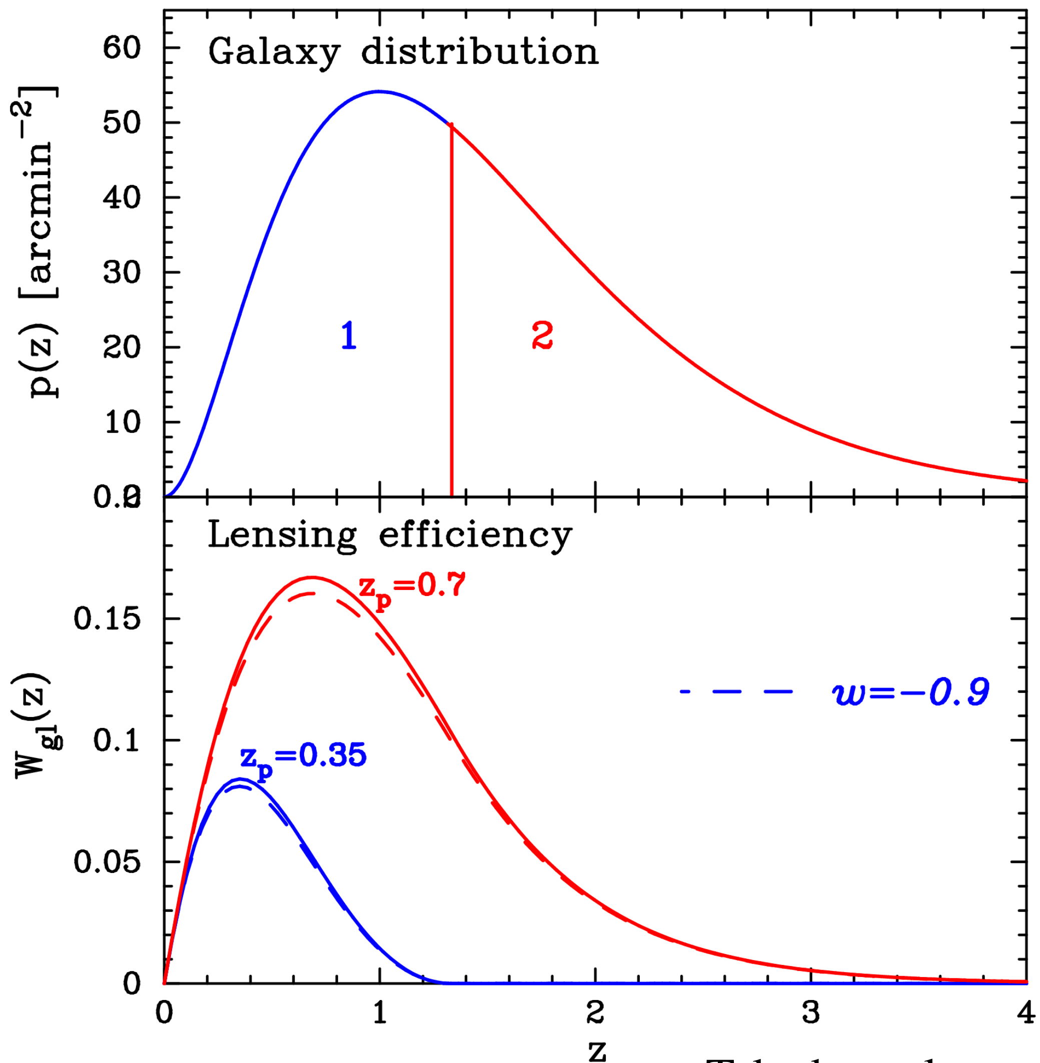
For a Gaussian field measured over f_{sky} of the sky with a finite number of galaxies the error is:

$$\frac{\delta C_\ell}{C_\ell} = \sqrt{\frac{2}{(2\ell+1) f_{\rm sky}}} \left(1 + \frac{\gamma_{\rm rms}^2}{\bar{n}_{\rm gal} C_\ell}\right)$$



Tomography: (2+ε)D surveys

- Tomography refers to the use of information from multiple source redshifts.
- This adds some "depth" information to lensing -- important for evolution studies (Hu 1999).

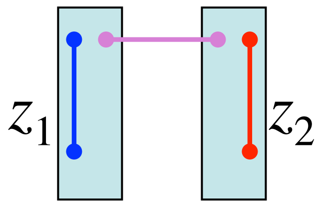


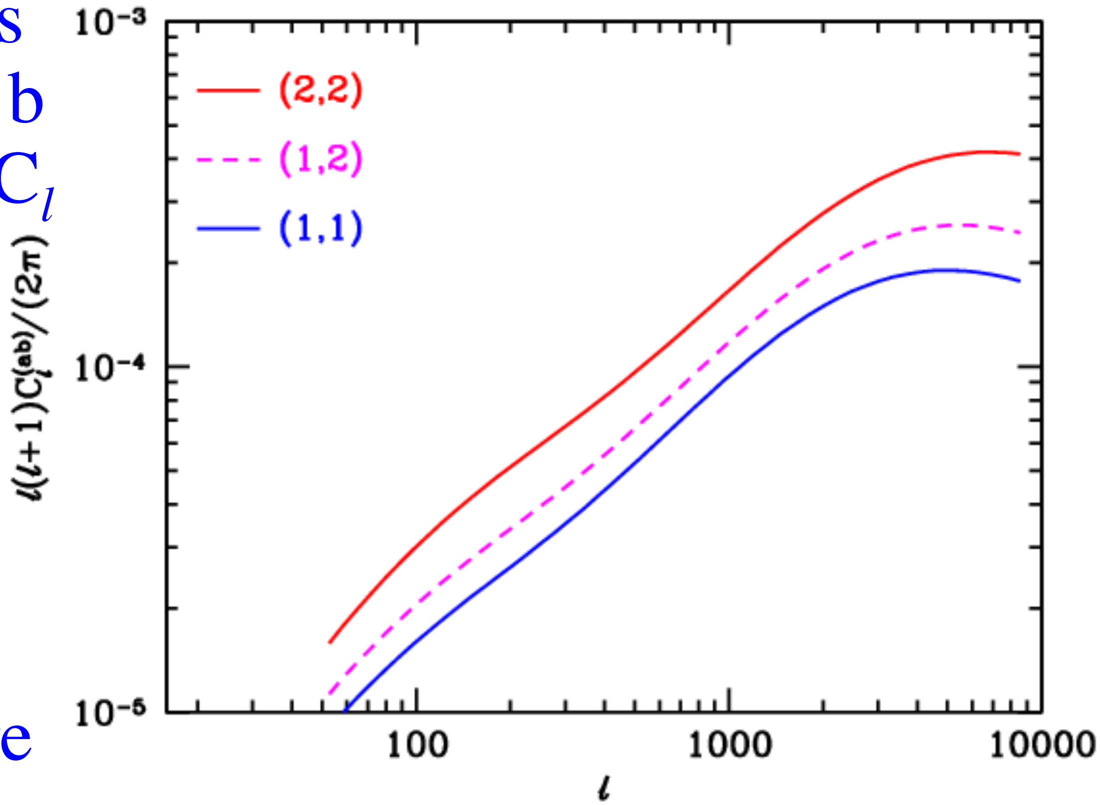
Takada et al.

Tomography (contd)

The generalization is straightforward for any statistic.

If we divide the sources into bins labelled by a, b then we promote C_l to $C_l^{(ab)}$, etc.





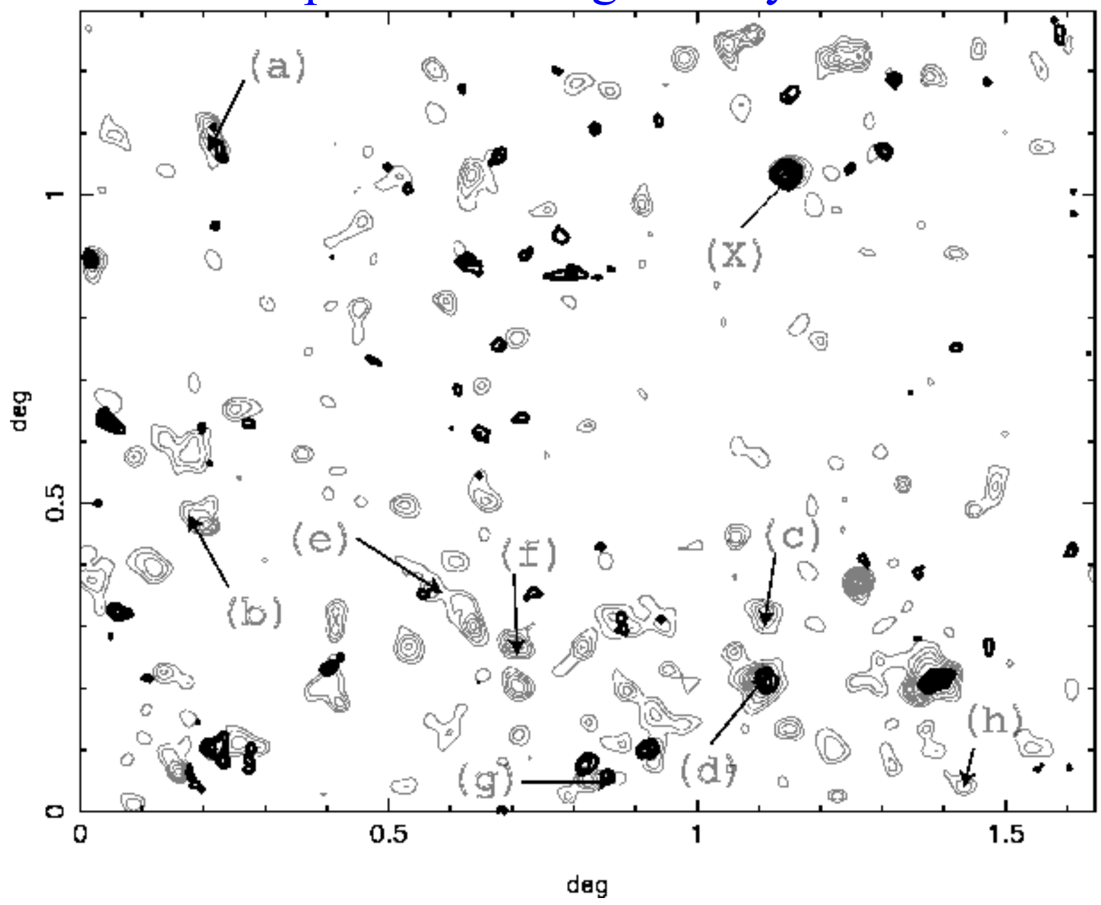
Since $g(\chi)$ is so broad, different source bins are very correlated (r>0.9).

Gains saturate quickly!

Observations

First detections of cosmic shear in Spring 2000

Mass map from 2.1 deg^2 survey with Subaru



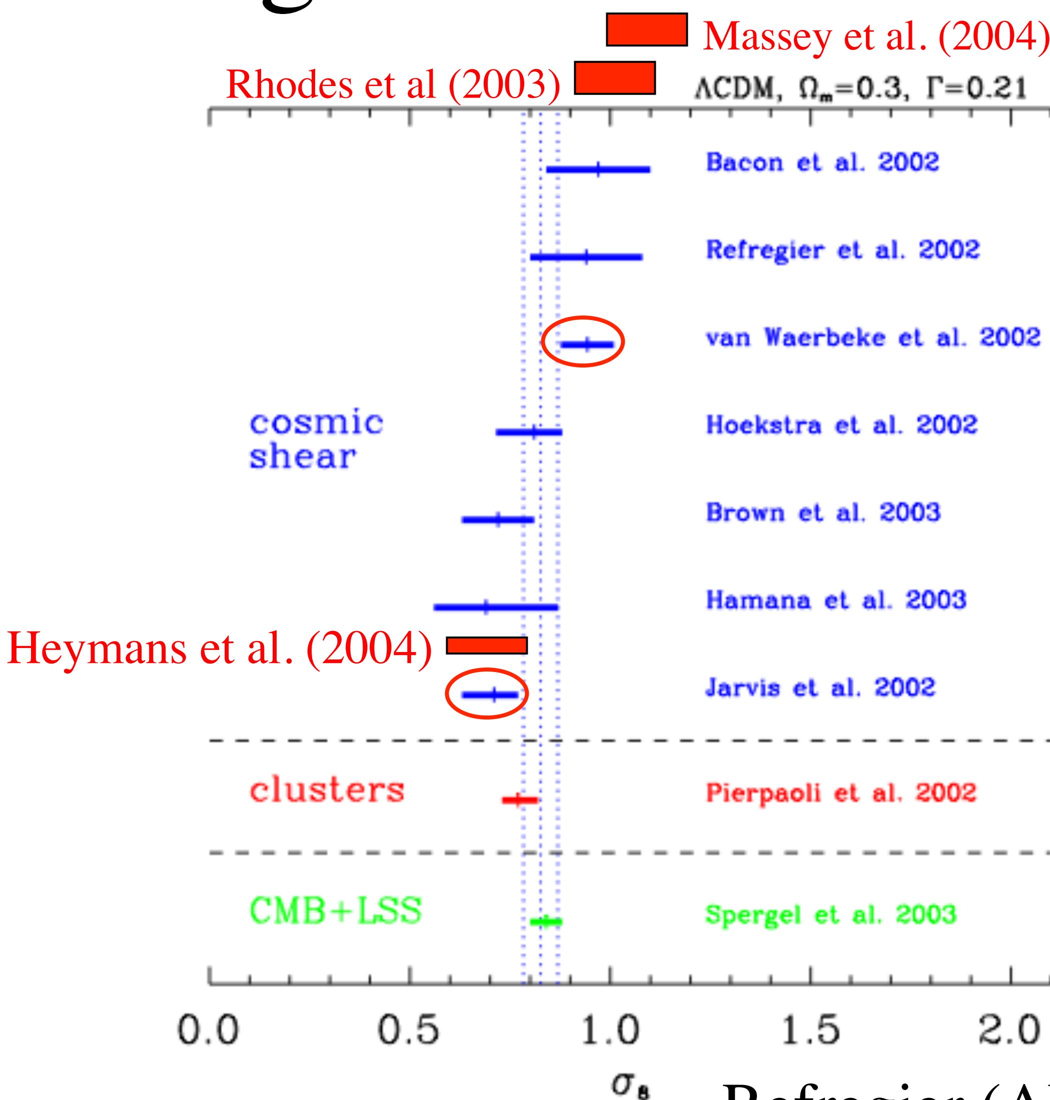
Miyazaki et al. 2002

Observational status through 2003

Typically tens of galaxies per square arcminute

Reference	Year	Telescope	Area (deg^2)	Mag. limit	σ_8
Wittman et al.	2000	CTIO	1.0	$R < 26$	–
van Waerbeke et al.	2000	CFHT	1.7	–	–
Kaiser et al.	2000	CFHT	1.0	$I < 24$	–
Bacon et al.	2000	WHT	0.5	$R < 26$	$1.50^{+0.50}_{-0.50}$
Maoli et al.	2001	VLT	0.7	$I < 25$	$1.03^{+0.03}_{-0.03}$
Rhodes et al.	2001	HST	0.05	$I < 26$	$0.91^{+0.21}_{-0.30}$
van Waerbeke et al.	2001	CFHT	6.5	$I < 25$	$0.88^{+0.02}_{-0.02}$
Hammerle et al.	2002	HST	0.02	–	–
Hoekstra et al.	2002	CFHT	24	$R < 24$	$0.81^{+0.07}_{-0.09}$
van Waerbeke et al.	2002	CFHT	8.5	$I < 25$	$0.98^{+0.06}_{-0.06}$
Refregier et al.	2002	HST	0.4	$I < 24$	$0.94^{+0.14}_{-0.14}$
Bacon et al.	2002	WHT	1.6	$R < 26$	$0.97^{+0.13}_{-0.13}$
Hoekstra et al.	2002	CFHT	53	$R < 24$	$0.86^{+0.04}_{-0.05}$
Jarvis et al.	2002	CTIO	75	$R < 23$	$0.71^{+0.06}_{-0.08}$
Brown et al.	2003	ESO	1.3	$R < 25$	$0.72^{+0.09}_{-0.09}$
Hamana et al.	2003	Subaru	2.1	$R < 26$	$0.69^{+0.18}_{-0.13}$

Agreement isn't bad, but …

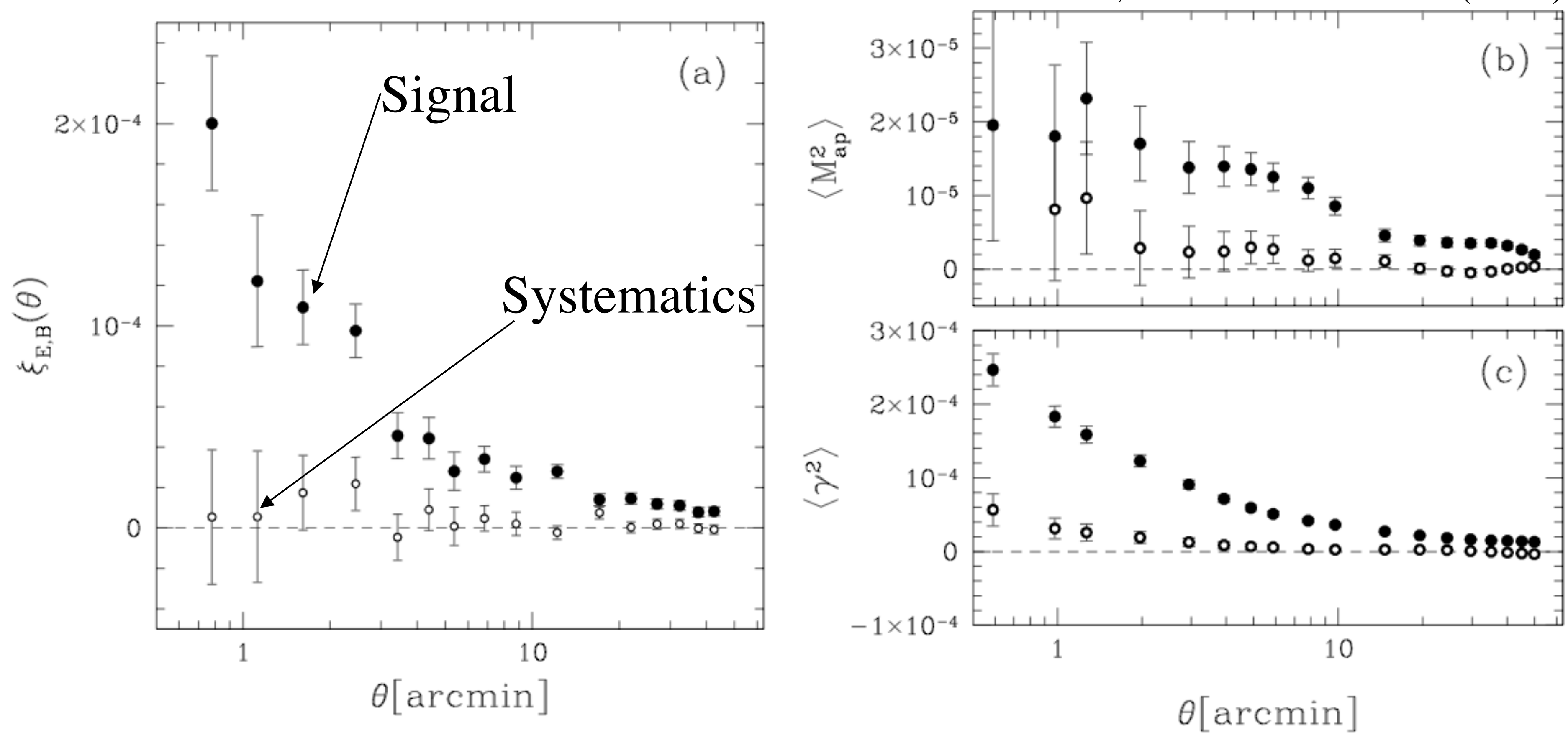


Need the equivalent of 1% precision in σ_8 to be able to measure dark energy *w*.

Refregier (ARAA, 2003)

The 2-point function: state of the art

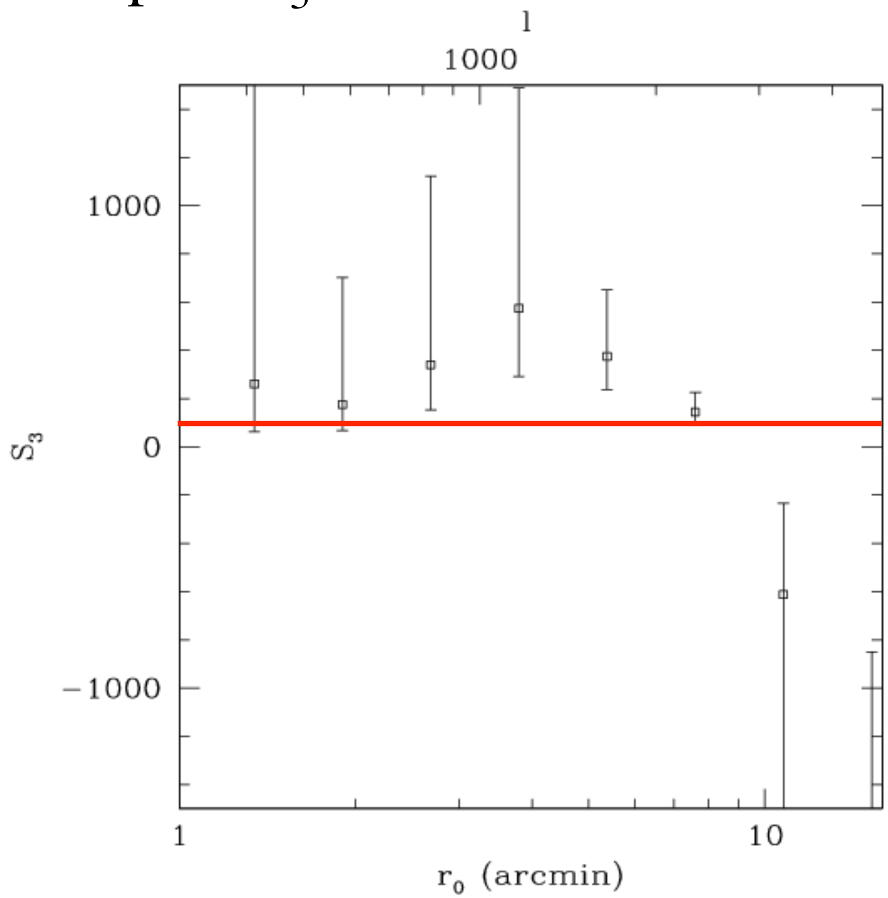
van Waerbeke, Mellier & Hoekstra (2004)



We are beginning to measure the power spectrum. B-modes gone!

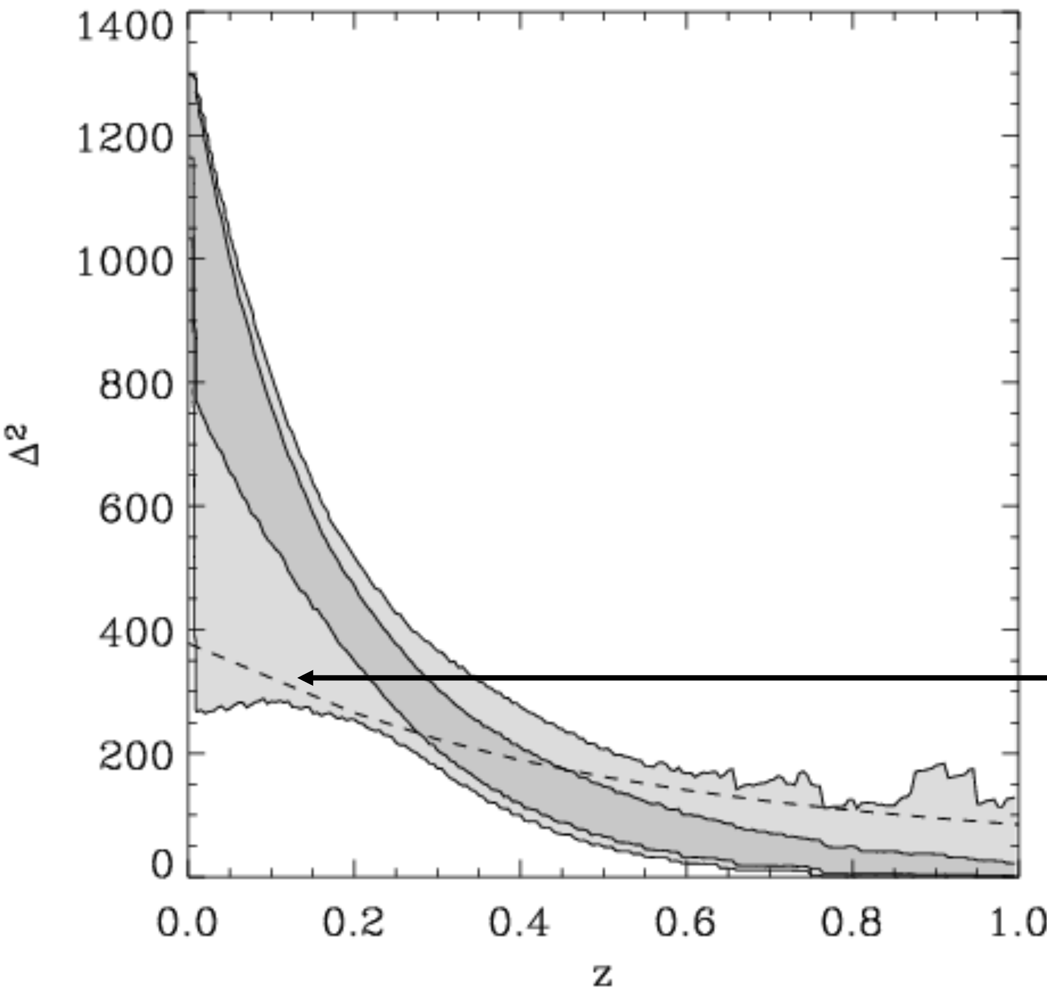
The skewness

By measuring the 2- and 3-point functions of the shear, the VIRMOS-DESCARTES group (Pen et al. 2003) were able to compute $S_3 = \langle\kappa^3\rangle / \langle\kappa^2\rangle^2$ over a range of scales.



The errors include an allowance for the non-zero *B*-mode they found during this earlier analysis.
Re-analysis in progress.

Structure grows!?



Evolution of power, Δ^2 (14/Mpc), from the COMBO-17 survey.

(Bacon *et al*. 2004)

ΛCDM with σ_8=0.7

Future projects

The 2nd generation of surveys will use “good” telescopes and tested observational techniques: expect dramatic improvement.

Survey	Diameter (m)	FOV (deg^2)	Area (deg^2)	Start
DLS	2×4	2×0.3	28	1999
CFHT-LS	3.6	1	172	2003
VST	2.6	1	$x100$	2004
VISTA	4	2	10000	2007
Pan-STARRS	4×1.8	4×4	31000	2008
DES	4	2.1	5000	200X
LSST	8.4	7	30000	201X
SNAP	2	0.7	300-7000	201X

Computing weak lensing

Theory or simulation?

All lensing “theory” is simulation based … whether it uses fits to halo profiles, halo mass functions and N-body power spectra (semi-“analytic”) or direct simulation.

Types and uses of simulations

Lensing lends itself to numerical simulation …

We need numerical simulations to refine and calibrate algorithms and analytic approximations, and potentially serve as templates when the data become available.

Simulations can be used to extract:

- Halo abundances and shapes
- Mass power spectra (and covariance matrices)
- Projected mass maps
- Ray tracing maps
- Mock galaxy catalogues

We have implemented all of these approaches…

Ray tracing: the MLP algorithm

- The gold standard of simulation algorithms is the "multiple lens plane" algorithm, where we trace ray bundles through the evolving mass distribution in an N-body simulation.
- The lensing equations are discretized and the integrals turned into sums:

$$\vec{\theta}_n = -\sum_{p=1}^{n-1} \frac{r(\chi_n - \chi_p)}{r(\chi_n)} \nabla_\perp \psi_p + \vec{\theta}_1$$

$$\mathbf{A}_n = \mathbf{I} - \sum_{p=1}^{n-1} g(\chi_p, \chi_n) \mathbf{U}_p \mathbf{A}_p$$

$$U_{ij} \equiv \frac{\partial^2 \psi_p}{\partial x_i \partial x_j}$$

Tests of the MLP

With Chris Vale we have made extensive tests of the MLPA and its convergence properties:

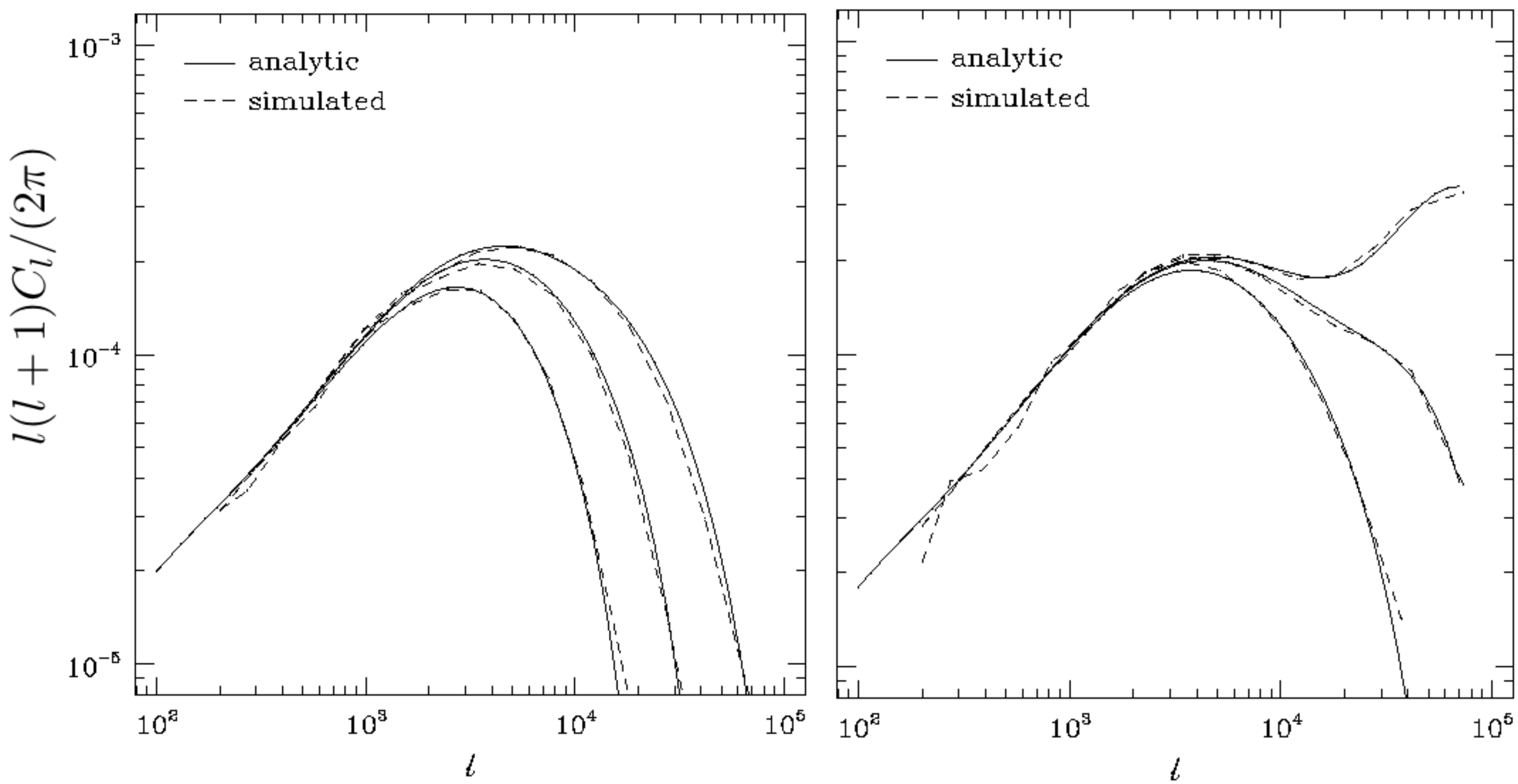
- The effect of border discontinuities
- The "ray-plane perpendicular" approximation
- The first fully 3-d ray tracing protocol
- Time evolution effects
- Number of lens planes necessary
- Numerical resolution issues
- Test common analytic approximations

Vale & White (2003)

Bottom line: MLP is good to at least a few percent in the power spectrum; the limiting computational cost is the generation of N-body simulations.

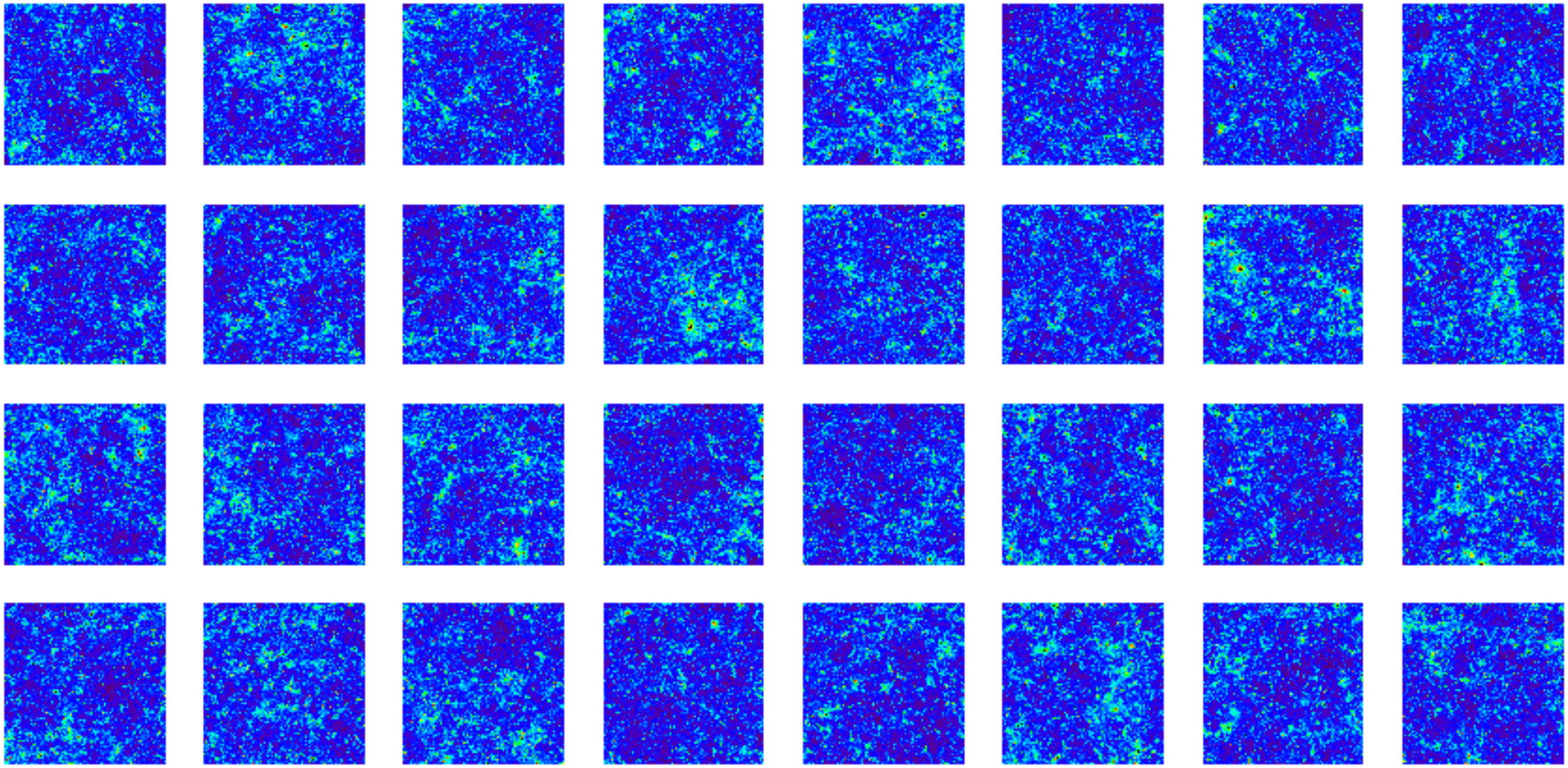
Understanding Our Simulations

(Vale & White 2003)



$\Omega_m = 0.357 \quad \omega = -0.8 \quad h = 0.64 \quad n = 1.00 \quad \sigma_8 = 0.88 \quad \tau = 0.15$

(with Chris Vale)



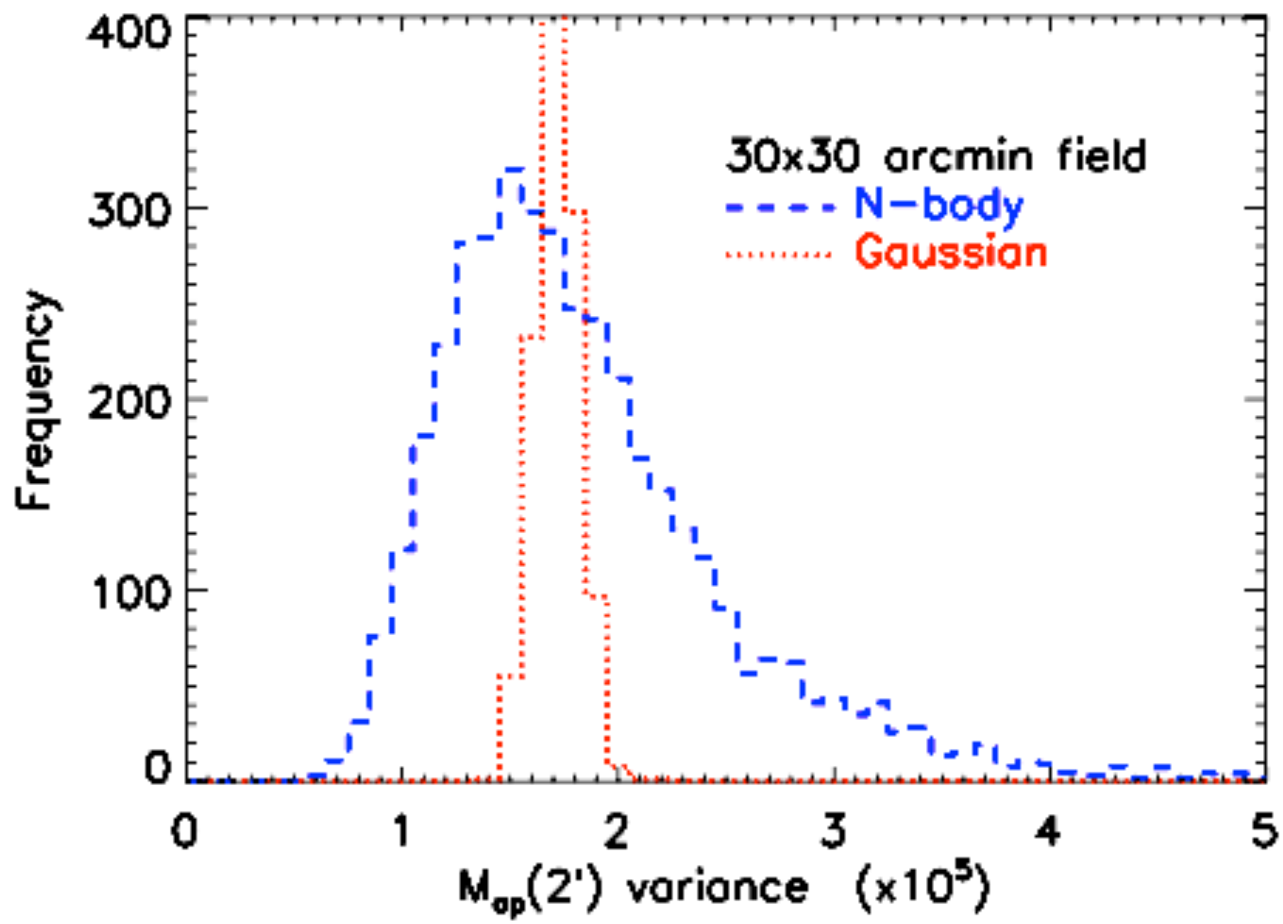
32 convergence maps, 3^o on a side

http://mwhite.berkeley.edu/Lensing/

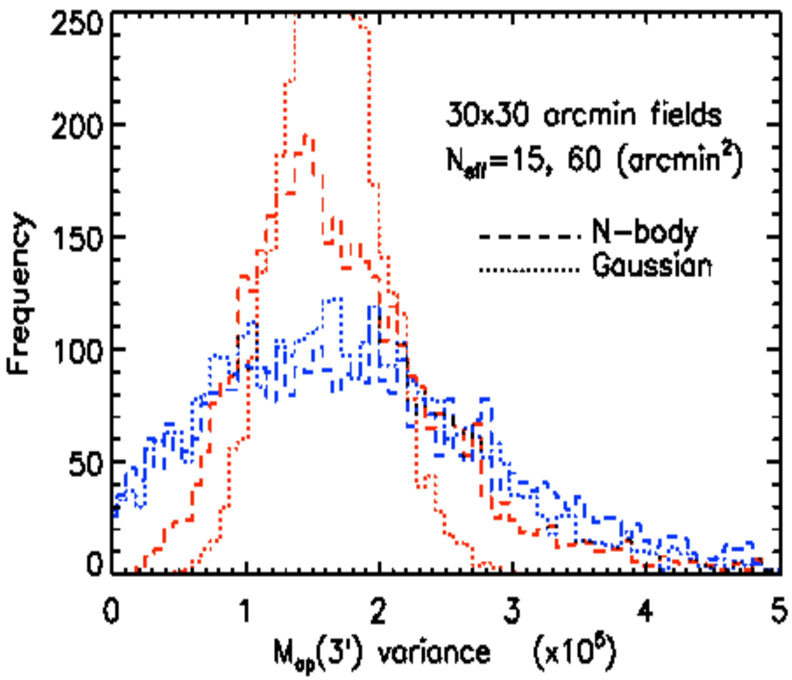
Theory & Analysis

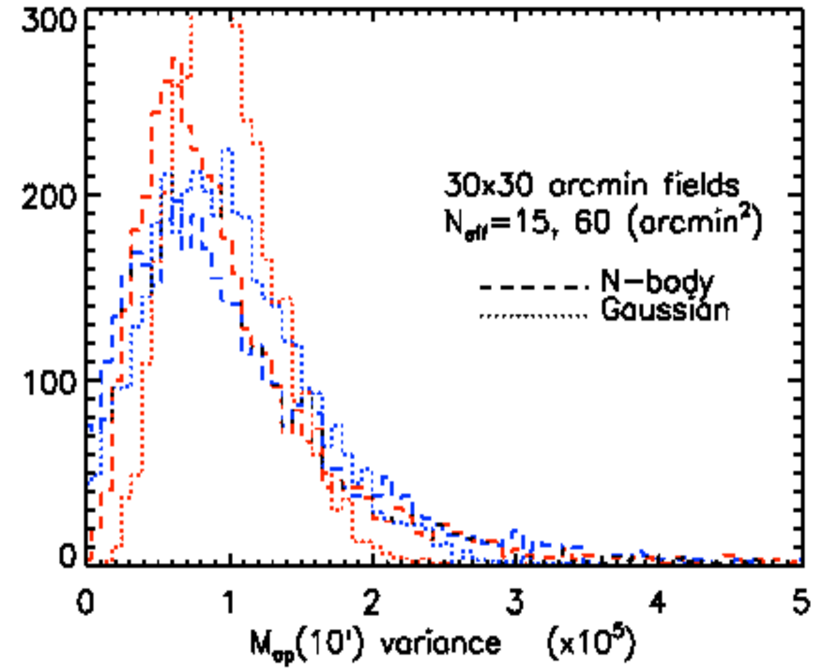
- These maps are very useful for investigating higher order functions.
- The maps make good tests of algorithms.
- The maps can be used to model systematic errors and their removal.
- Available
  - Convergence and shear maps [different $p(z_s)$]
  - Halo catalogs
  - Sheared "galaxy" catalogs
  - Power spectra, …

Non-Gaussianity & Sample variance



The effects of noise





Two abuses of tomography

(1) Nulling tomography and small scale structure

(2) Cross-correlation tomography and intrinsic alignments.

Using better observations to mitigate theory uncertainty …

Nulling tomography

- There are some techniques which are almost theory independent in gravitational lensing. They depend only on distances.
- To probe large-scale structure, to test its growth (&GR) and to get at the full power of lensing requires coupling observations to theory.
- Most "theory" is simulation based.
- Not all calculations are under good control!

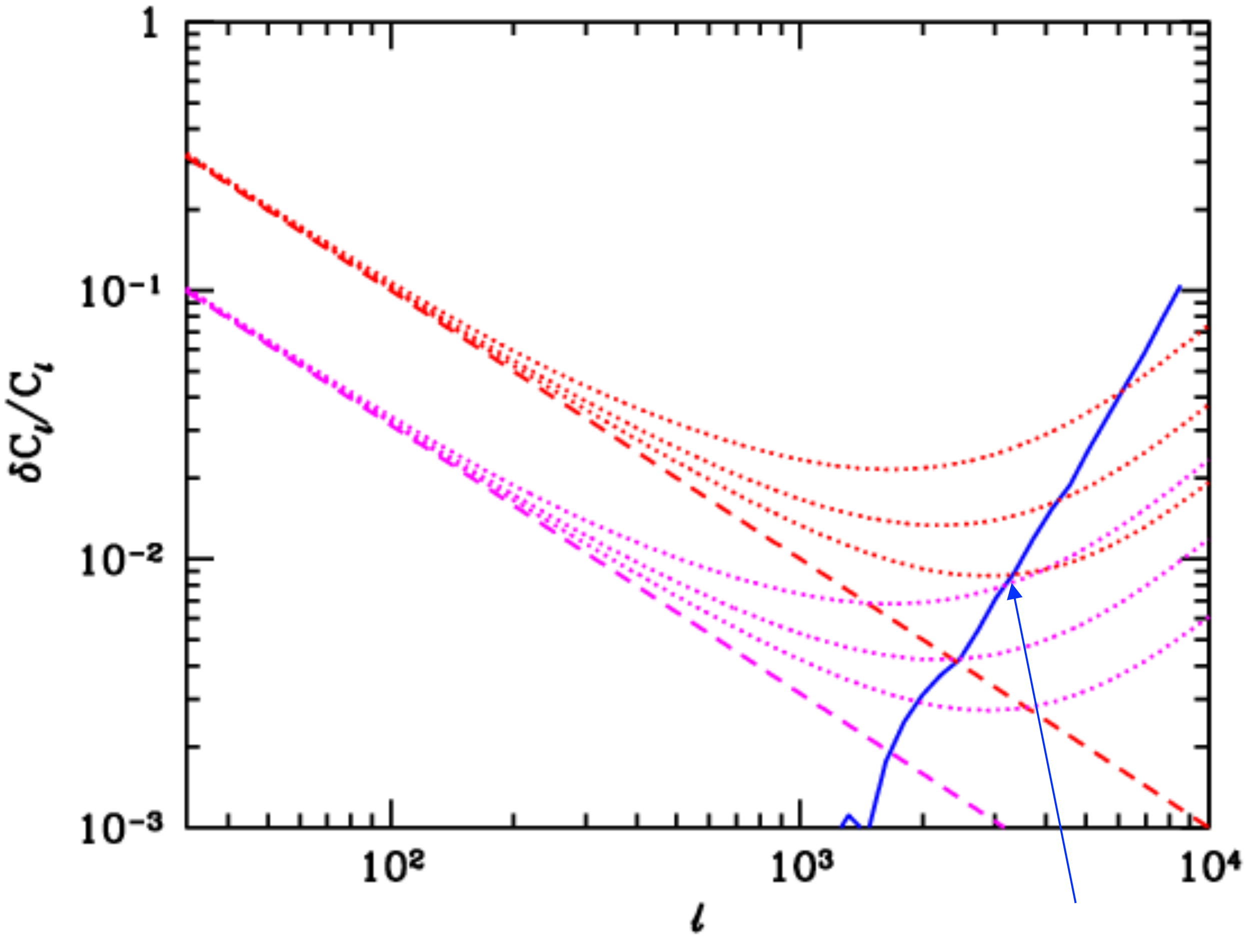
Beyond N-body

Gravitational lensing is "simple" because it involves only gravity, albiet non-linear gravity. However non-gravitational physics does become important on small scales:

- Baryonic cooling produces steep inner cusps in galaxies, leading to strong (extreme) lensing events.
- Contraction of baryons by cooling alters the potential in the surroundings, changing the lensing signal.
- Cooling alters the profiles of sub-halos, affecting lensing.
- Hot gas is "flatter" and "rounder" than DM in cluster centers (Zhan & Knox 2004).

It is difficult to model these effects accurately at present, but we can make toy models to guesstimate the size of the effects.

Baryonic cooling

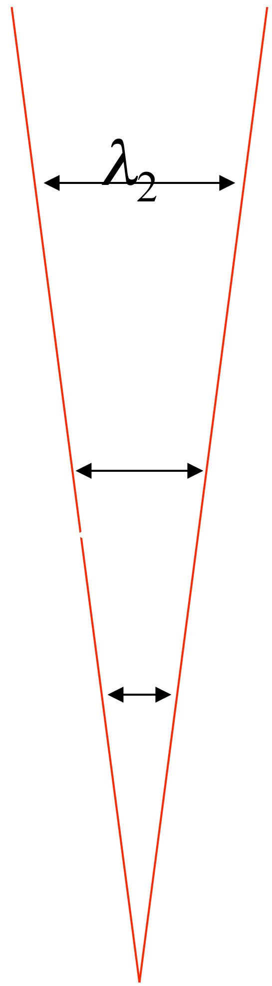


(Cooling)-(no cooling)

Using a simple model of cooling and adiabatic contraction can guess how cooling affects lensing C_l for sources at z~1.

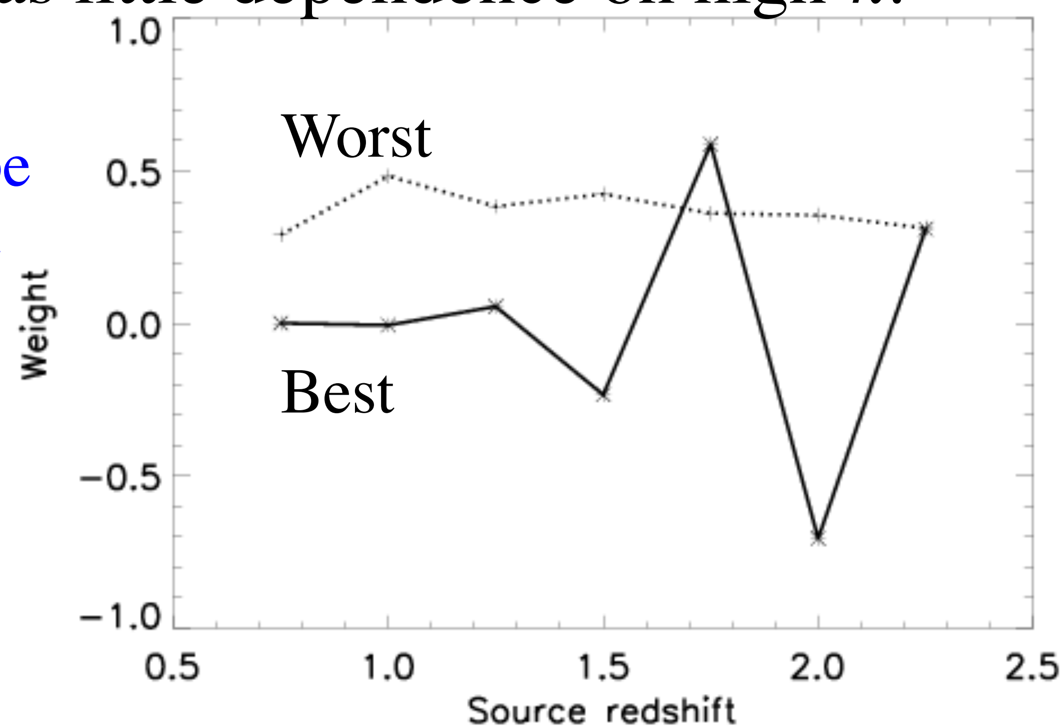
Tomography to the rescue

- Recall however that, within the Limber approximation, a given angular scale depends on small-scale power only through the nearby structure.
- If we use multiple source *z*s and "null out" the contribution to κ from nearby structure, we can reduce our sensitivity to small-scale physics!
- Want to find the weights which reduce sensitivity to high-*k* physics below the measurement noise.



At the 1-point function level

- Nulling tomography can be implemented at any level, 1-point, 2-point, etc.
- The simplest method is for the 1-point function, where we form combinations of the shear so t$\langle\gamma_\alpha^2\rangle$. has little dependence on high-k.



The optimal weights can be computed as a generalized eigenvalue problem

(Huterer & White, in prep.)

Cross-correlation tomography

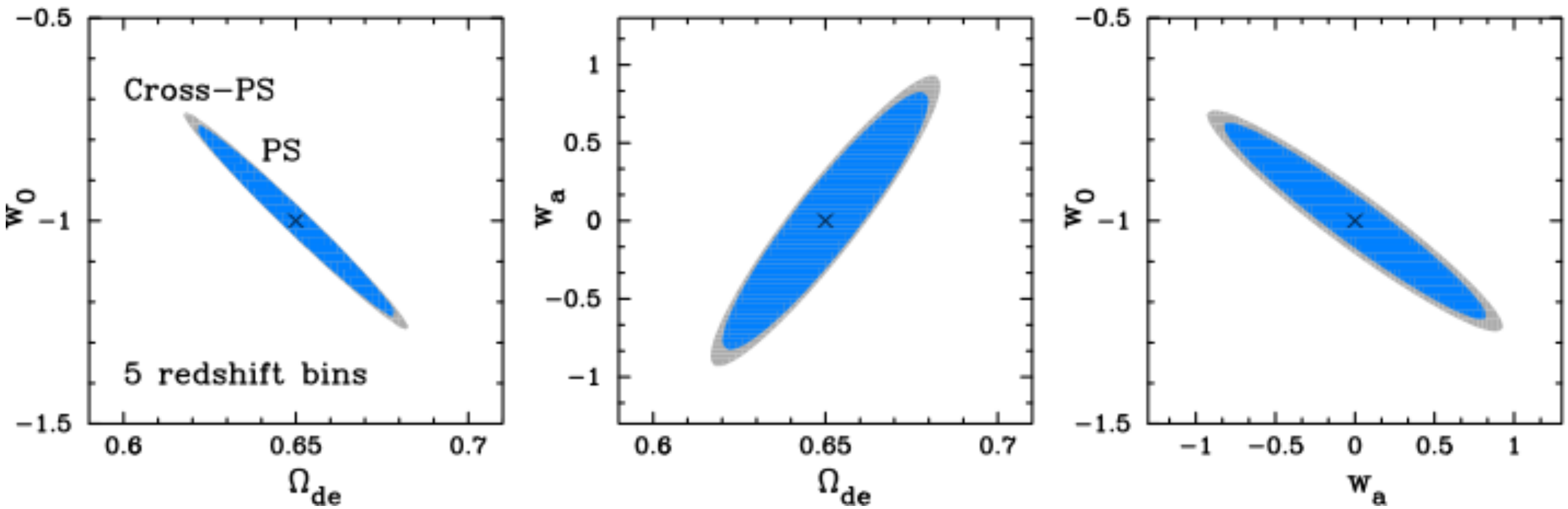
- Even more complex baryonic problems exist!
  - Galaxy formation
- Are galaxies randomly oriented?
- If not, our estimates of the shear made by averaging over neighbouring galaxies are biased!
- Again, we can use tomography to reduce our sensitivity to this uncertainty, or solve galaxy formation and model it out.

Cross-correlation tomography

- While the lensing signal builds up over Gpc, galaxy alignments should fall of rapidly (~10Mpc).
  - Tidal fields generate galaxy spin, and scale similarly to density fields.
- For N_s source *z*-bins the 2-point function becomes an $N_s x N_s$ matrix. The entries are highly correlated.
- If the source bins are thick, only the diagonal entries can have a significant contribution from intrinsic galaxy aligments.
- Just omit the diagonal entries …

CCT results

	$\sigma(\Omega_{\rm de})$		$\sigma(w_0)$		$\sigma(w_a)$		$\sigma(\sigma_8)$	
redshift bin #	PS	Cross-PS	PS	Cross-PS	PS	Cross-PS	PS	Cross-PS
$n_s = 3$	0.024	0.057	0.20	0.51	0.65	1.5	0.025	0.055
$n_s = 4$	0.021	0.026	0.17	0.21	0.58	0.71	0.022	0.027
$n_s = 5$	0.020	0.022	0.16	0.18	0.55	0.62	0.020	0.023
$n_s = 5$ + Bisp	8.2×10^{-3}	8.7×10^{-3}	0.078	0.084	0.27	0.29	8.3×10^{-3}	8.8×10^{-3}
$n_s = 6$	0.019	0.021	0.15	0.17	0.52	0.58	0.020	0.022
$n_s = 10$	0.018	0.019	0.15	0.15	0.49	0.52	0.019	0.020



Takada & White (2003)

Beyond the 2-point function

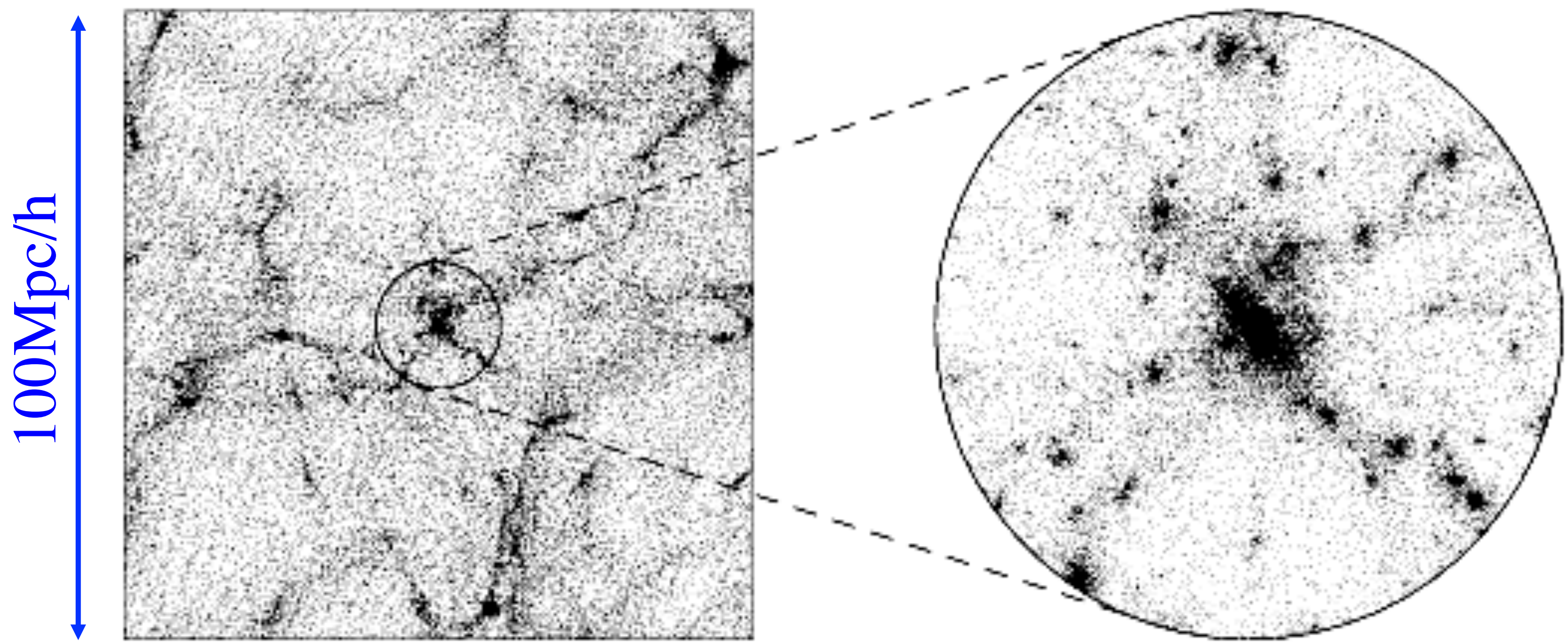
Non-gaussianity as blessing or curse?

Finding clusters with weak lensing

- The obvious extension of non-Gaussian thinking is to look at the extrema of the maps
  - Finding clusters.
- Unfortunately lensing measures the *projected* mass along the line of sight.
- Projection effects can be severe and need to be modeled.

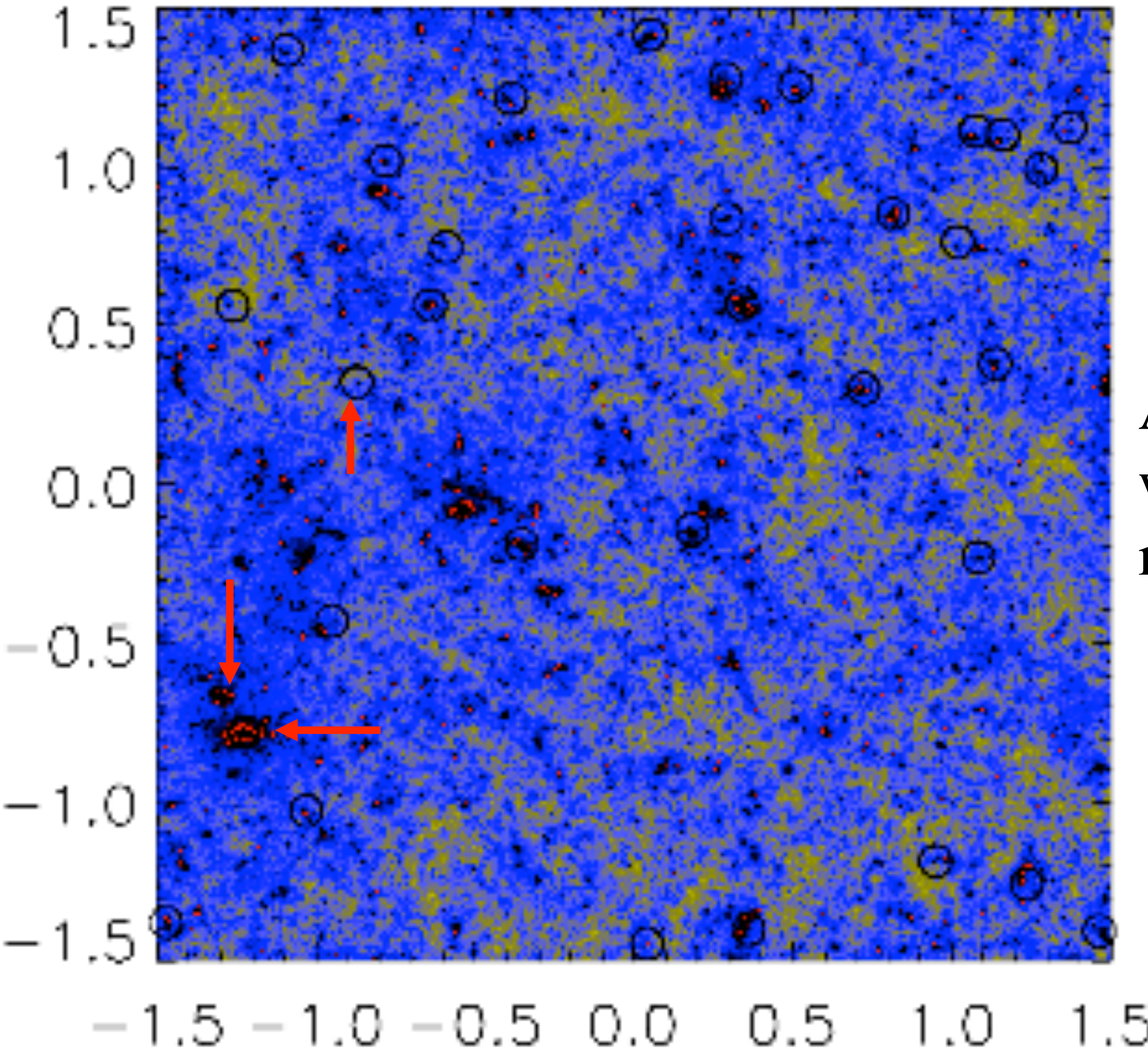
Projection effects lead to scatter in the shear-mass relation

Scatter in the shear-mass relation means lensing does *not* produce a <u>mass</u> selected sample, but a <u>shear</u> selected sample! This has implications for doing cosmology.



Metzler, White & Loken

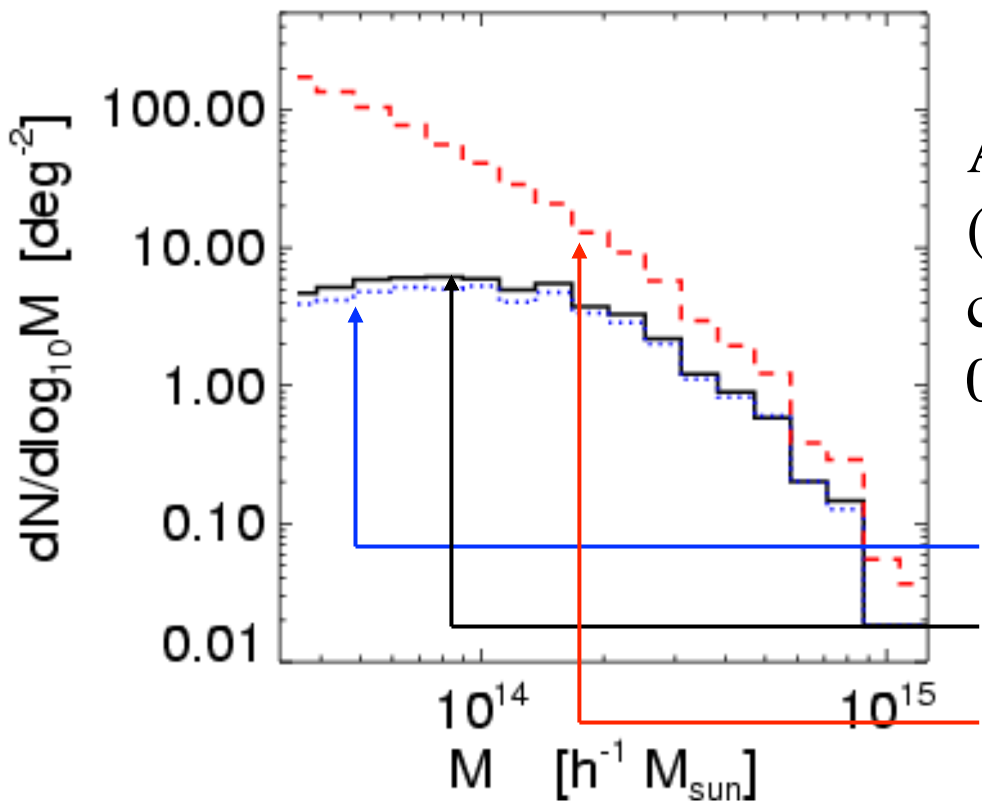
The peak-halo connection



A 3x3 degree κ map with the 32 most massive halos circled!

Projection effects can be severe

Hennawi & Spergel have used a tomographic matched filter algorithm to find clusters and determine their redshifts. The tomographic information helps reduce projection effects, but cannot eliminate them entirely.



Assuming 60% efficiency (3.5σ) and searching for clusters in the range 0.2<z<0.8

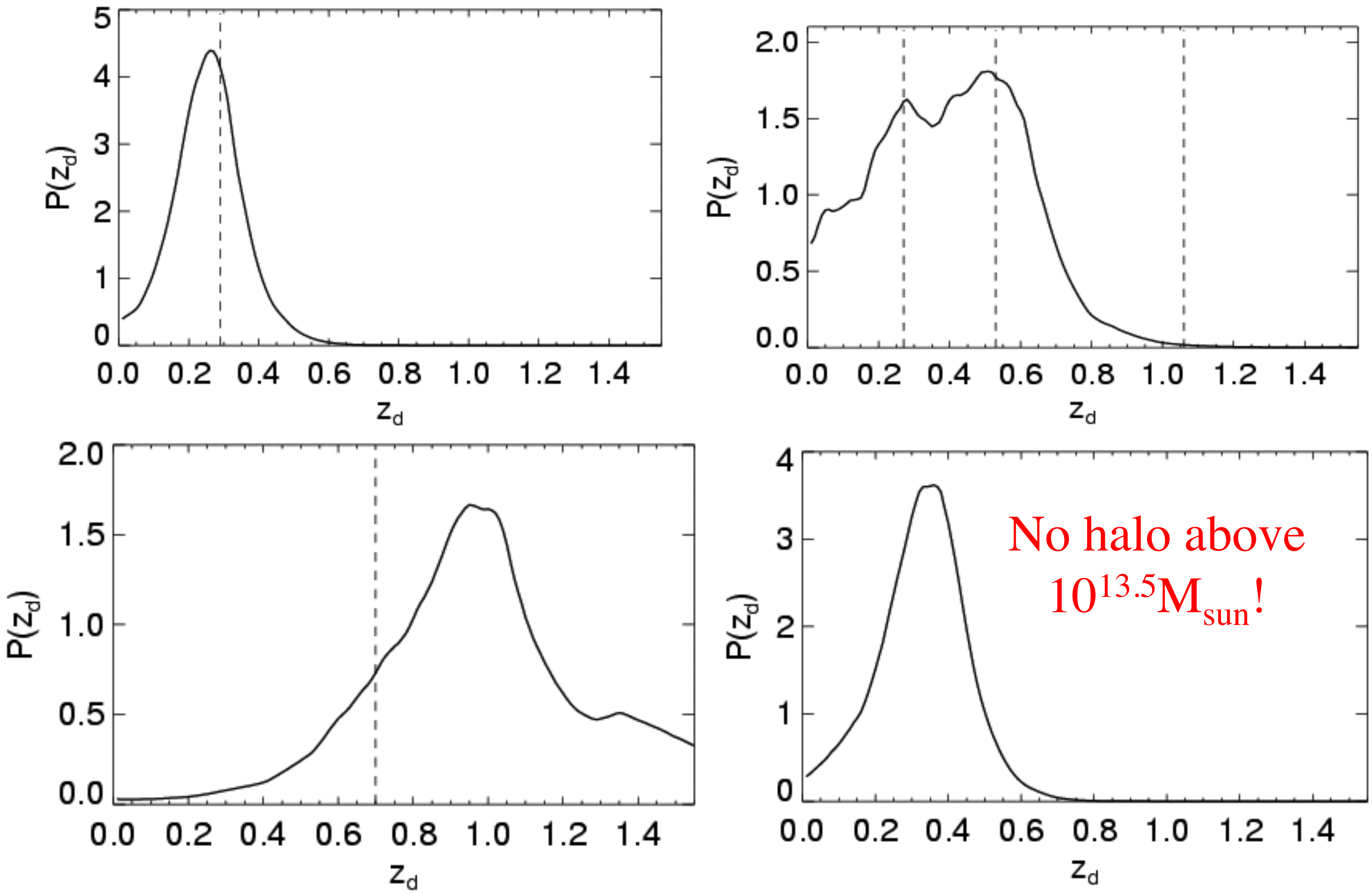
No tomography

Tomography

Truth

Tomographic (MF) redshifts

(Hennawi and Spergel 2004)



Lensing of the CMB

Of course galaxies aren't the only source of (lensed) light in the universe. Any screen will do. The CMB is the furthest screen!

Large-scale structure will lens the CMB anisotropy.

Since we don't know the "shape" of the CMB *a priori* we need to use more statistical information.

Seljak (1996)

Zaldarriaga & Seljak (1999)

Zaldarriaga (2000)

Seljak & Zaldarriaga (2000)

Hu (2001)

Hu & Okamoto (2002)

Okamoto & Hu (2002, 2003)

Cooray & Kesden (2003)

Hirata & Seljak (2003)

Amblard, Vale & MW (2004)

Lensing of the CMB (contd)

Consider the CMB, lensed

$$T(\hat{n}) = \widetilde{T}(\hat{n} - \nabla\Phi) \simeq \widetilde{T}(\hat{n}) - \nabla\Phi \cdot \nabla\widetilde{T}$$

The correlation function will depend on Φ, allowing us to make a quadratic estimator assuming everything is Gaussian and the deflection angle is small.

$$\hat{\kappa} \sim \nabla \cdot \widehat{\nabla\Phi} \sim \nabla \cdot [T_w \nabla T_w]$$ (Hu; Hirata & Seljak)

We should be able to detect this effect with upcoming experiments (e.g. APEX-SZ, SPT, ACT)!

But how well do these estimators work, and how sensitive are they to observational strategy, foregrounds and systematics?

Lensing of CMB by LSS

- Worry about violations of assumptions:
  - Potential field is non-Gaussian.
  - Deflection angle is not infinitesimal.
- Since estimator is looking for small amounts of (lensing induced) non-Gaussianity on top of the Gaussian CMB, worry about the effect of foregrounds,
  - IR sources
  - kSZ and
  - O-V at hi-*z*
  - etc..

Numerical study

Want to simulate some of these issues and investigate whether APEX-SZ, SPT and ACT could see lensing.

- Simulations include
  - Primary CMB map
  - Gaussian and non-Gaussian lensing fields
  - Idealized detector noise
  - Kinetic SZ signal (optional)
- Primary configuration
  - 30x30 degrees
  - 0.8' FWHM resolution
  - 2μK/arcmin (white) noise or more
- Make lensed maps, apply quadratic estimator, apply corrections …

Assessing the results

It is non-trivial to assess the numerical results. We use visual inspection and two power spectra as our metrics:

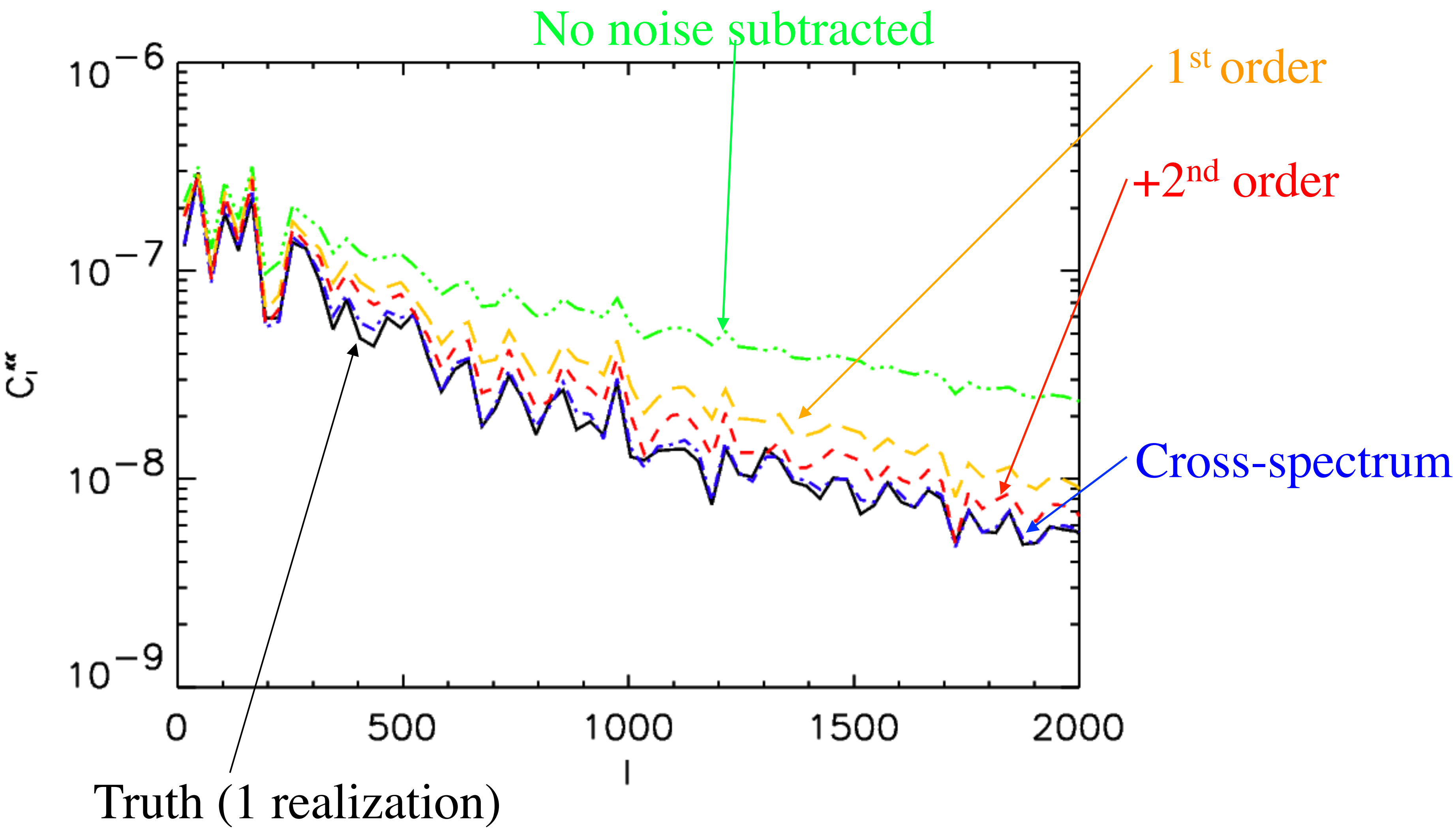
- Cross-spectrum $\langle \kappa^{\rm true} \kappa^{\rm est} \rangle$
  - Not measurable
  - Looks for bias in the method or a mis-estimated normalization for κ
- Auto-spectrum $\langle \kappa^{\rm est} \kappa^{\rm est} \rangle - \mathcal{N}$
  - Measurable
  - Can be both multiplicatively and additively biased due to misestimates of the noise terms.

Biases

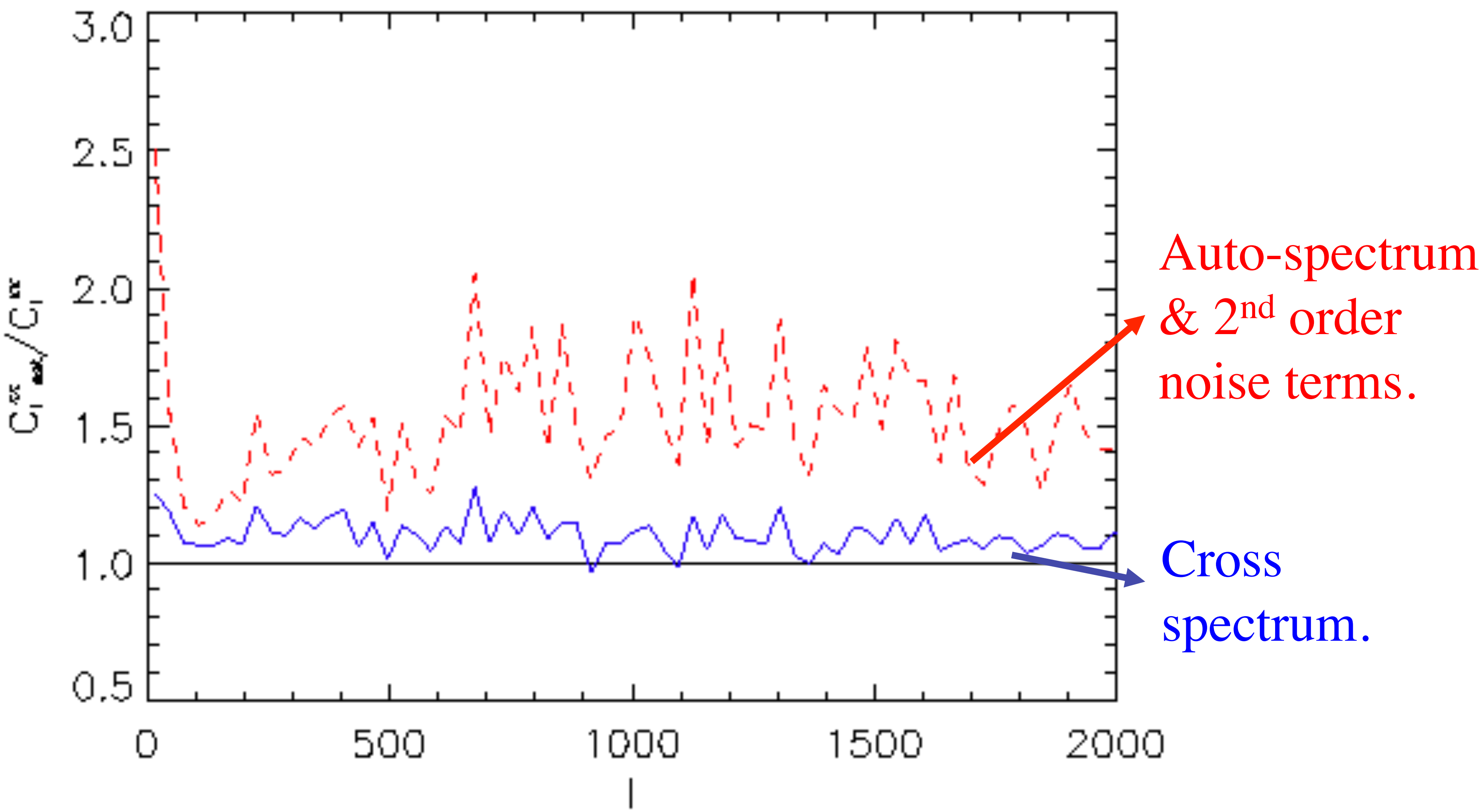
- Even in the absence of foregrounds we find that the quadratic estimator is both multiplicatively and additively biased.
- The bias depends on the level of signal.
- The additive bias comes from
  - Higher order terms in the noise
  - Non-Gaussianity in the lensing field.
- The multiplicative bias is due to non-linearity

Assumption	C_l Error
Quad Est	70%
2nd order	25%
NG	20-30%

Additive bias for Gaussian maps



Adding non-Gaussianity



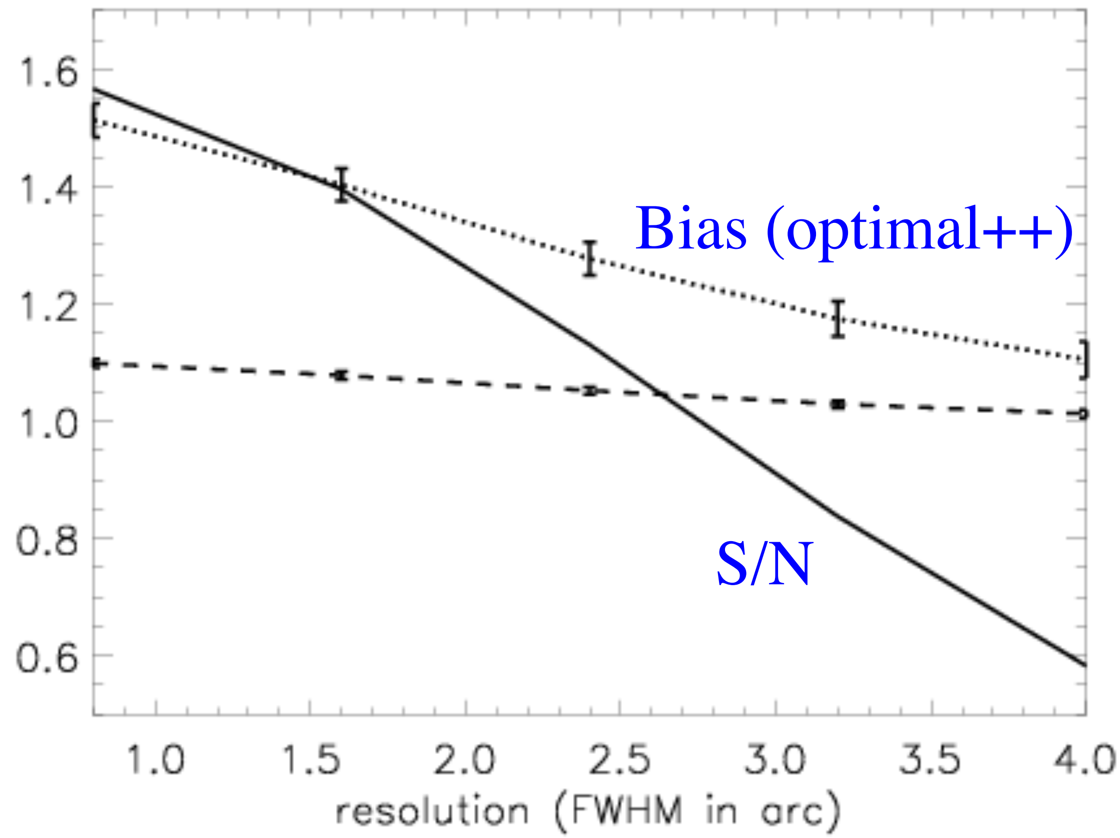
Now have non-vanishing 3-point function, etc., so Gaussian estimate for normalization is insufficient.

Effects of resolution

Working at higher resolution increases the S/N.

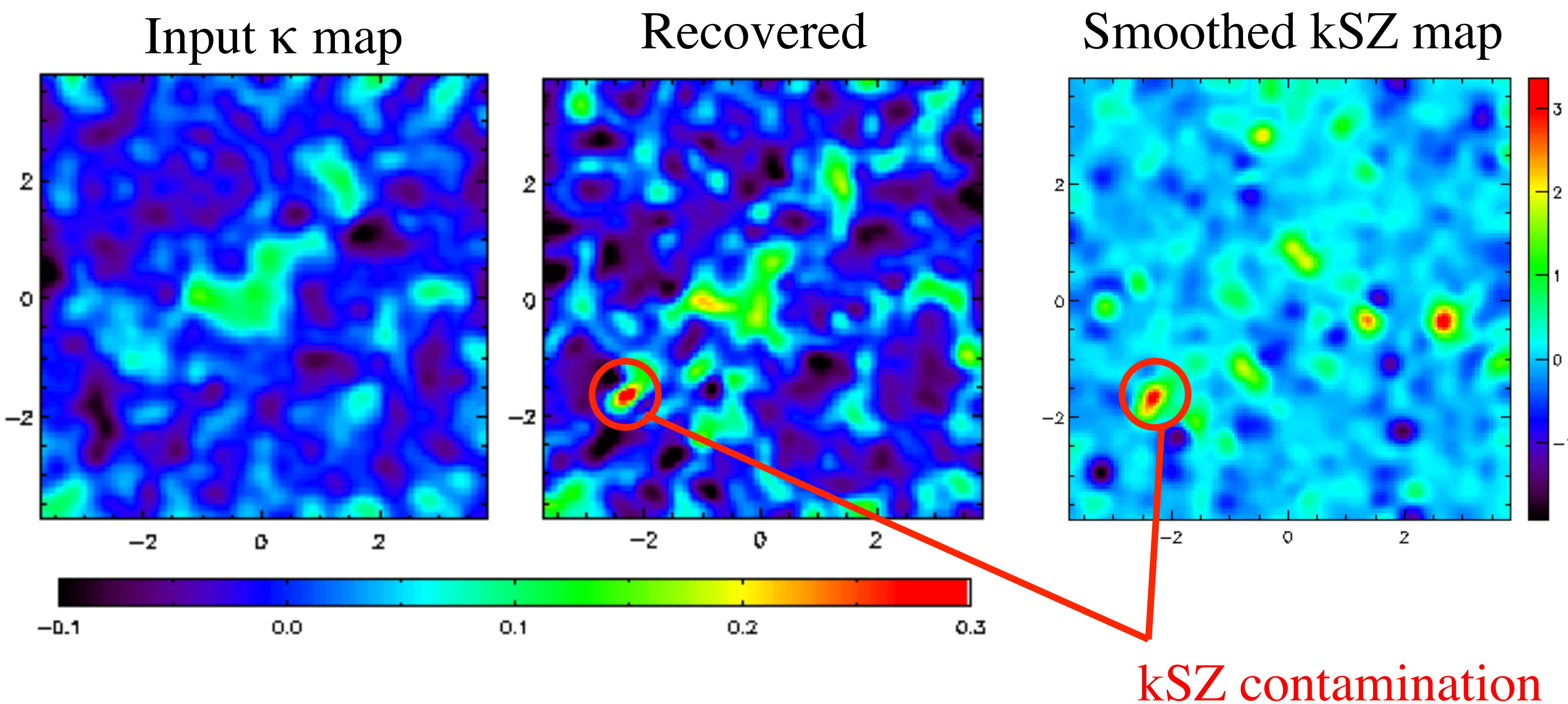
Higher resolution brings more signal, but additional theory uncertainty:

- LSS is non-Gaussian
- Deflection angle not small



Similar effects for polarization (Amblard & White, in prep)

Adding foregrounds: kSZ

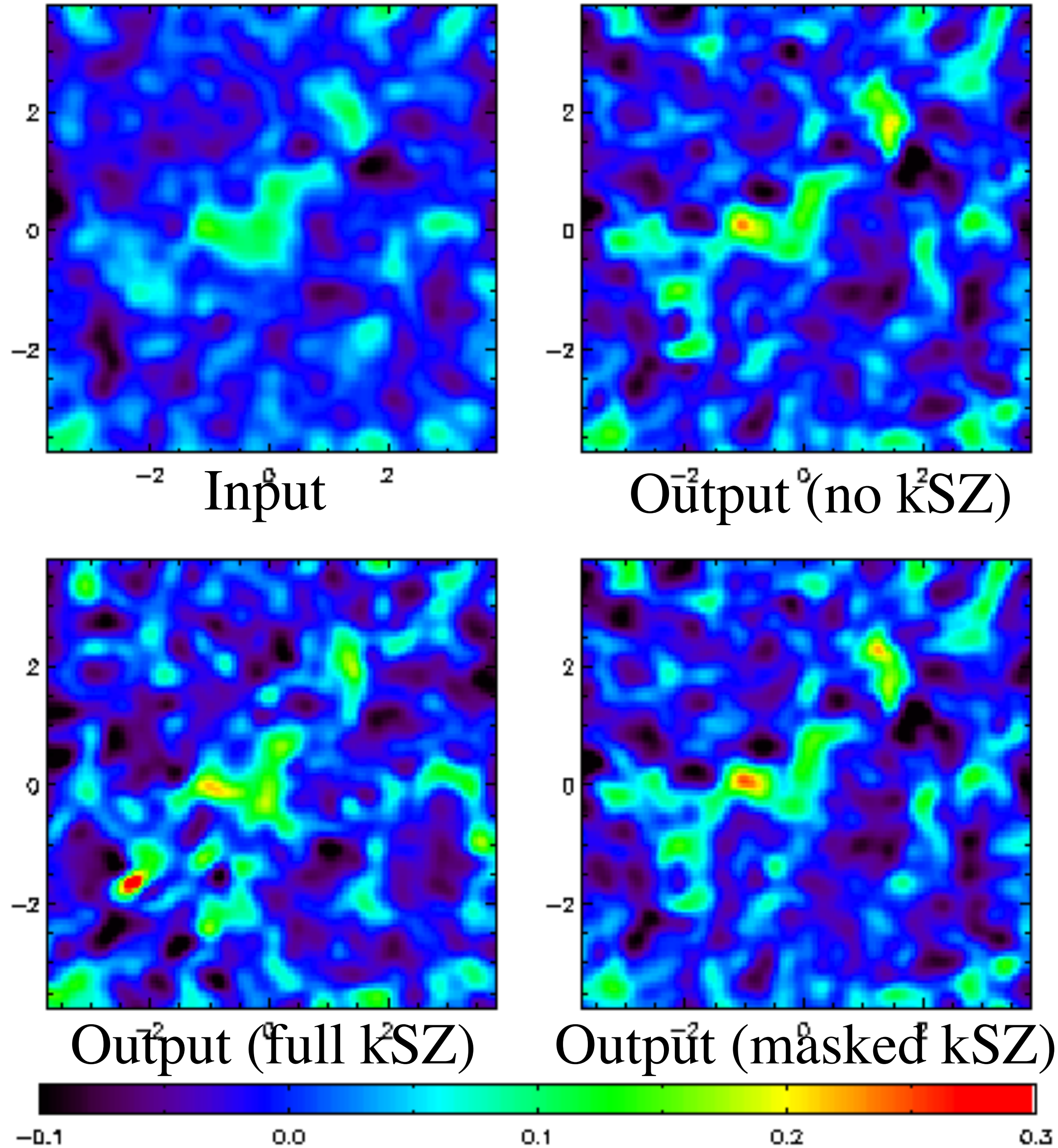


Maps have been smoothed from 0.8’ to 20’ to enhance the S/N.

Masking "bright" clusters

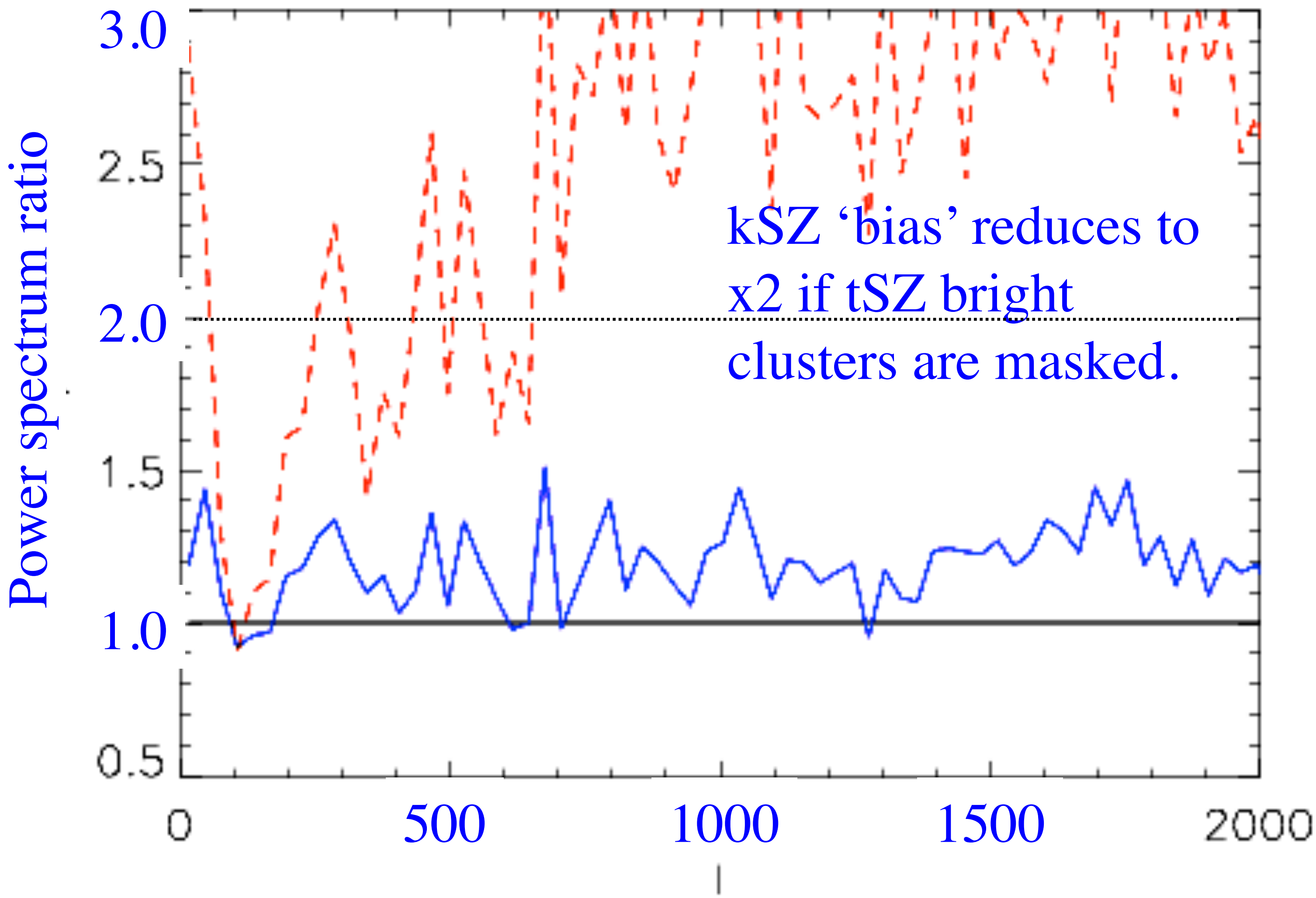
Can use thermal SZ maps to find bright clusters and "mask" them. The recovered map looks very similar to the "no kSZ" map.

Masking has only a modest effect on the lensing power spectrum. The most massive clusters do not dominate the lensing reconstruction.

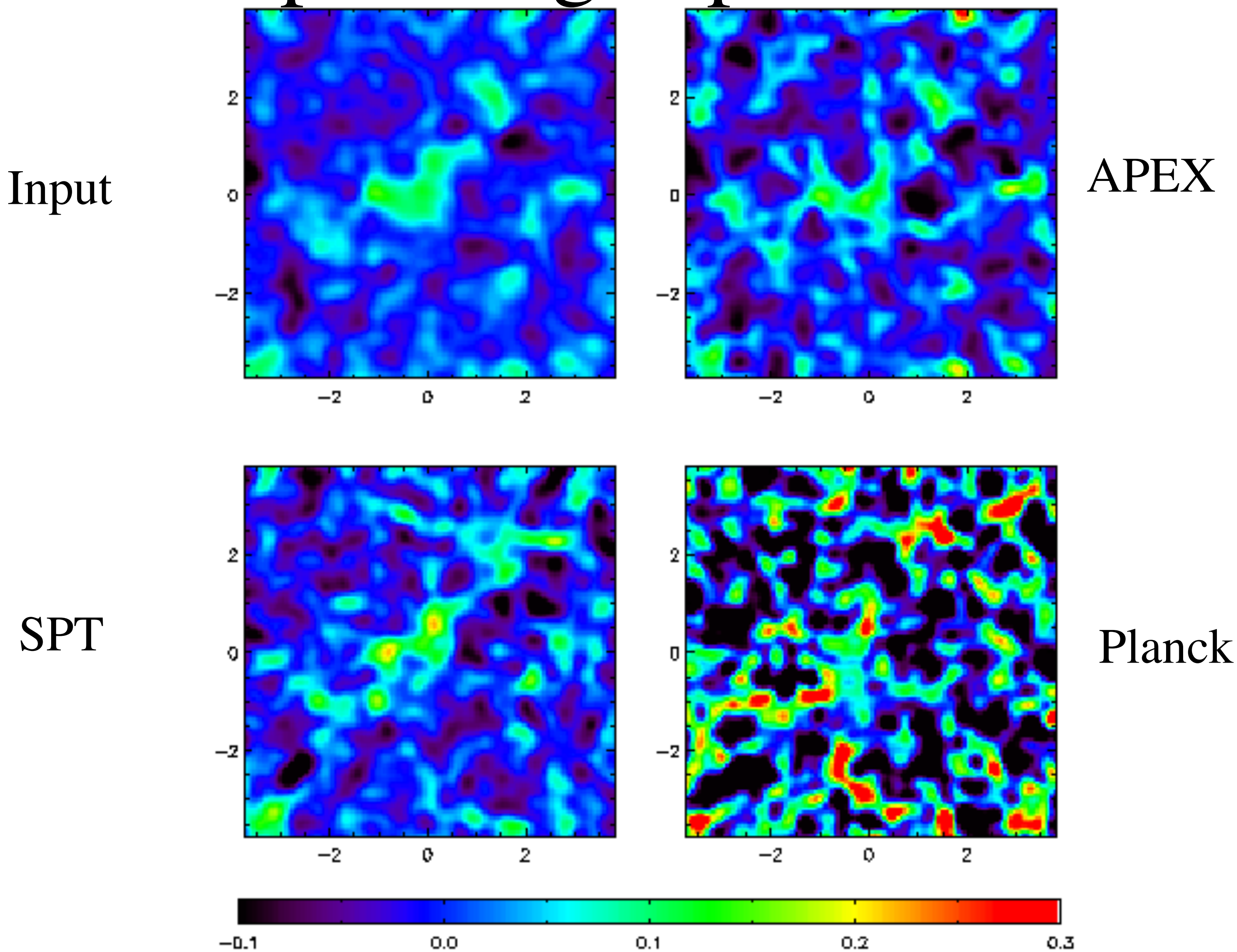


kSZ and power spectrum

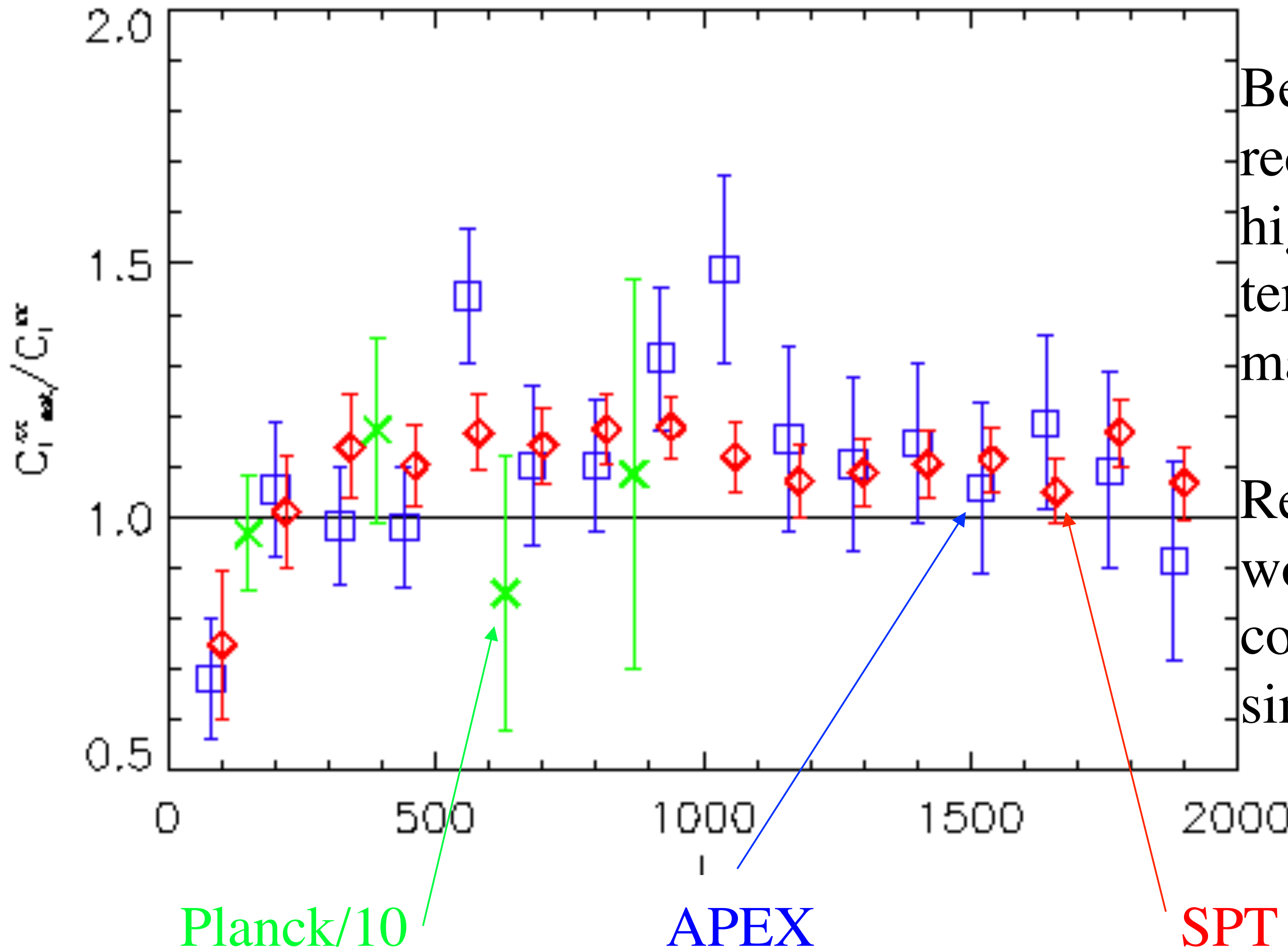
Foregrounds?!



Upcoming experiments



Power spectra



Best reconstruction: higher order terms and kSZ masked.

Remaining bias would need to be corrected by simulations.

Conclusions

- Gravitational lensing has come of age!
- We can accurately simulate lensing fields on scales of arcminutes to degrees.
- Multiple source redshifts offer advantages in both science reach and systematic effect mitigation.
- *Non-Gaussianity offers rich opportunities and difficult challenges.*
- We may soon detect gravitational lensing of the cosmic microwave background.

The "theory" team?

- Many of the questions we are asking in cosmology are becoming more subtle, and the design of experiments and analysis of data more complex/demanding.
  - Mock galaxy catalogs.
  - Simulated gravitational lensing maps.
  - Simulated, multi-wavelength skies (e.g. X-ray, SZ, …)
  - Halo profiles, mass functions, power spectra, …
- Many of the methods described above require sophisticated theoretical machinery.
- Who will do these "experiment specific" calculations?
  - How will we fund them?
  - How will we reward/motivate them?
  - What can we learn from particle physics?

The End