Probing the high-redshift Universe
(Cosmic Brunch)

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Outline

- Some reasons to go to high redshift.
- Upcoming observational opportunities/motivations.
- Lightening review of CMB lensing.
- Dropout selection of high $z$ galaxies.
- Measuring the growth of large-scale structure.
- Bias, bias, bias, ...
- Future directions.
Searching under the lamppost?

- Increasing precision in cosmology is essentially a mode counting exercise – reduce sample variance by increasing the number of samples.
  - Inflation ($f_{NL}$, features, flatness, ...)
  - Neutrino masses (and other light relics)
  - Test General Relativity to the largest scales.

- In general non-linearity is not your friend, especially if you are after ‘primordial’ physics.

- Ideally maximize S/N of ‘new physics’ or ‘new insights’
  - I don’t have a crystal ball which tells me where/if new physics will show up, so I can’t increase ‘S’. How do I lower N?

- All else being equal, I’d like a problem with well controlled and interesting theory!

Going to high redshift gives me more volume, more modes and less non-linear structure ...
One example: growth rate

Between $z \approx 10^3$ and today, fluctuations grow by $\sim 10^3$.

GR predicts growth very precisely.

Marginalizing over unknown parameters, growth is predicted to 1.1% per bin (dominated by $m_\nu$ uncertainty).

Is GR right?
Light neutrinos

Since neutrinos free stream on small scales, \( \delta_m \propto a^{1-3f_\nu/5} \) where \( f_\nu \) is the \( \nu \) mass fraction.

Yu+19
We are about to “turn on” several new facilities representing billions of dollars and hundreds of person-years of investment ...
Optical surveys

Major new imaging and spectroscopic facilities ...

- Dark Energy Survey (DES)
- DECam Legacy Survey (DECaLS)
- Dark Energy Spectroscopic Instrument (DESI)
- Subaru Hyper Suprime-Cam (HSC)
- Prime Focus Spectrograph (PFS)
- Large Synoptic Survey Telescope (LSST)
- Euclid
- Wide-Field Infrared Survey Telescope (WFIRST)

These facilities can map large areas of sky to unprecedented depths!
## Optical Surveys

<table>
<thead>
<tr>
<th>Survey</th>
<th>$u$</th>
<th>$g$</th>
<th>$r$</th>
<th>$i$</th>
<th>$z$</th>
<th>$y$ ($Y$)</th>
<th>Area [deg$^2$]</th>
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<tr>
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<td>–</td>
<td>25.4</td>
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<td>–</td>
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<td>14K</td>
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<td>26.4</td>
<td>25.2</td>
<td>24.5</td>
<td>14K</td>
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CMB Surveys

A similar revolution is happening at longer wavelengths ...

<table>
<thead>
<tr>
<th>Survey</th>
<th>Map RMS [µK-arcmin]</th>
<th>Resolution [ℓ]</th>
<th>Area [deg²]</th>
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<tbody>
<tr>
<td>Planck</td>
<td>30.0</td>
<td>7.0</td>
<td>21K</td>
</tr>
<tr>
<td>Simons Observatory</td>
<td>6.0</td>
<td>1.0</td>
<td>27K</td>
</tr>
<tr>
<td>CMB-S4</td>
<td>1.0</td>
<td>1.4</td>
<td>17K</td>
</tr>
<tr>
<td>LiteBIRD</td>
<td>2.5</td>
<td>30.0</td>
<td>30K</td>
</tr>
</tbody>
</table>

A natural “by-product” of next generation CMB experiments to constrain primordial gravitational waves is high fidelity CMB lensing maps.
The anisotropies we see in the CMB are the seeds of large-scale structure in the Universe. General Relativity makes precise predictions for the growth of this large-scale structure once the constituents are known. The gravitational potentials associated with this structure lens the CMB photons on their way to us ... ... imprinting a characteristic pattern which can be used to probe the structure itself. This provides an important consistency check and sensitivity to the low redshift Universe.
Characteristic scales

The lensing-induced deflections of CMB photons

- are $\mathcal{O}(2' - 3')$ in size
- are coherent over $2^\circ - 3^\circ$
- arise from structures over a wide redshift range ...
- ... but are most sensitive to $z \sim 2 - 3$.

The CMB is 14 Gpc away.
δΦ nearly scale invariant on large scales, damped below horizon size at equality ($\sim 300$ Mpc).
There are $\sim 14000/300 \sim 50$ lenses along the line of sight, each with $\delta \Phi \sim 3 \times 10^{-5}$ or deflection $\alpha \sim 10^{-4}$ so $\alpha_{\text{tot}} \sim 50^{1/2} \times 10^{-4} \sim 2'$. 
Half-way to the surface of last scattering 300 Mpc subtends $300/7000 \sim 2^\circ$. 
Measuring lensing from the CMB

- CMB fluctuations have a characteristic scale.
- Lensing “reconstruction” finds $\kappa$ by measuring a local stretching of the power spectrum.
- Magnified regions shift power to larger scales (smaller $\ell$).
- Demagnified regions shift power to smaller scales (higher $\ell$).
Planck lensing map

Planck Collaboration (2018)
Planck was definitely not the first experiment to
▶ to measure lensing,
▶ ... by large scale structure,
▶ ... of the CMB

However, it was the first experiment to measure CMB lensing by large scale structure over a significant fraction of the sky and with enough signal to noise that it provided a sharp test of the theory and could drive fits.

In some sense Planck was a “coming of age” for CMB lensing, and a taste of things to come – much of the science from future CMB surveys will come from lensing ...

... in combination with ...
Dropout or Lyman-Break Galaxy (LBG) selection

Dropout color-color selection targets the steep break in an otherwise shallow $F_\nu$ spectrum bluewards of the 912Å Lyman limit due to absorption by the neutral hydrogen rich stellar atmospheres and interstellar photoelectric absorption. Lyman-series blanketing along the line-of-sight further suppresses flux short-ward of 1216Å for $z > 2$ sources.
Composite spectra

\[ \lambda \cdot F_{\lambda} \left[ 10^{-17} \text{ergs/s/cm}^2/\text{Å} \right] \]

- +52.6 Å, \( z = 2.5 \)
- +14.3 Å, \( z = 3.5 \)
- +11.0 Å, \( z = 4.5 \)
- −14.9 Å, \( z = 5.5 \)
Dropout or Lyman-Break Galaxy (LBG) selection

- Dropout selection requires only 3 filters, so is observationally efficient.
  - Easier to model selection than a photo-z based case.
- These objects have been extensively studied (for decades!) over the range $2 < z < 7$.
- Selects massive, actively star-forming galaxies – and a similar population over a wide redshift range.
- Rest-frame UV spectra dominated by O5 and B star emission with $M > 10 \ M_\odot$ and $T > 2.5 \times 10^4 \ K$.
- LBGs lie on the main sequence of star formation and UV luminosity is approximately proportional to stellar mass.
- Galaxies of interest have $M_* \sim 10^{10-11} \ M_\odot$, 


Large numbers of galaxies

\[ N(< m) / \text{deg}^2 \]

\( 2 < z < 3 \) QSO
BX-dropouts
\( u \)-dropouts
\( g \)-dropouts
\( r \)-dropouts

\[ m \]

\[ 23 \ 24 \ 25 \ 26 \ 27 \]
Tomographic lensing

The combination of galaxies with known redshifts and CMB lensing with its long lever arm can be particularly powerful ...
The opportunity

A new generation of deep imaging surveys and CMB experiments offers the possibility of using cross-correlations to

- constrain the early Universe
- investigate light particles
- test General Relativity
- probe the galaxy-halo connection
- measure the growth of large-scale structure

The combination can be more than the sum of its parts!
In particular we can use the optical survey to isolate the $\kappa$ contribution from narrow $z$ slices, increase $S/N$ and downweight systematics.
Signal to noise: now

unWISE galaxies crossed with Planck lensing ...

Krolewski+19
Signal to noise: the future

We could achieve $S/N \gtrsim 10^2$ at $z \sim 3$ and at 4, larger than or comparable to $S/N$ we can achieve in one bin at low $z$ at present.
Fiducial samples

<table>
<thead>
<tr>
<th>Sample</th>
<th>Band</th>
<th>$m_{\text{lim}}$</th>
<th>$\log_{10} \bar{n}$</th>
<th>$n_{\theta}$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BX</td>
<td>$R$</td>
<td>25.5</td>
<td>-2.06</td>
<td>26300</td>
<td>-</td>
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<tr>
<td>$u$-dropouts</td>
<td>$i$</td>
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<td>-3.15</td>
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<td>$g$-dropouts</td>
<td>$i$</td>
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<td>-2.45</td>
<td>5250</td>
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<tr>
<td>$r$-dropouts</td>
<td>$z$</td>
<td>25.8</td>
<td>-3.00</td>
<td>1300</td>
<td>5.4</td>
</tr>
</tbody>
</table>
Signal-to-noise: BX galaxies ($z \sim 2$)

- $C_{gg}$, $C_{kg}$ and $C_{kk}$.
- Grey lines: noise levels (per $\ell$) for AdvACT, SO and S4.
- Horizontal line: shot noise.
- Band at $L_{nl}$.
**Signal-to-noise: \( u \)-dropouts \( (z \sim 3) \)**

- **\( C_{gg} \), \( C_{\kappa g} \)** and \( C_{\kappa\kappa} \).
- Grey lines: noise levels (per \( \ell \)) for AdvACT, SO and S4.
- Horizontal line: shot noise
- Band at \( L_{nl} \).
Signal-to-noise: $g$-dropout ($z \sim 4$)

- $C_{gg}$, $C_{kg}$ and $C_{κκ}$.
- Grey lines: noise levels (per $ℓ$) for AdvACT, SO and S4.
- Horizontal line: shot noise
- Band at $L_{nl}$. 
Signal-to-noise: \textit{r}-dropouts ($z \sim 5$)

- $C_{gg}^\ell$, $C_{\kappa g}^\ell$ and $C_{\kappa \kappa}^\ell$.
- Grey lines: noise levels (per $\ell$) for AdvACT, SO and S4.
- Horizontal line: shot noise
- Band at $L_{nl}$. 

\begin{align*}
C_{gg} & = \ldots \\
C_{\kappa g} & = \ldots \\
C_{\kappa \kappa} & = \ldots \\
N_{gg} & = \ldots \\
N_{\kappa \kappa} & = \ldots 
\end{align*}
Example: Measuring $P_{mm}(k, z)$

- A proper accounting of the growth of large scale structure through time is one of the main goals of observational cosmology – key quantity is $P_{mm}(k, z)$.
- Schematically we can measure $P_{mm}(k, z)$ by picking galaxies at $z$ and
  \[
P_{mm}(k) \sim \frac{[bP_{mm}(k)]^2}{b^2 P_{mm}(k)} \sim \frac{[P_{mh}(k)]^2}{P_{hh}(k)} \sim \left[ \frac{C_{\ell=k\chi}^{\kappa g}}{C_{\ell=k\chi}^{gg}} \right]^2
  \]
  
- Operationally we perform a joint fit to the combined data set.
  - With only the auto-spectrum there is a strong degeneracy between the amplitude ($\sigma_8$) and the bias parameters ($b$).
  - However the matter-halo cross-spectrum has a different dependence on these parameters and this allows us to break the degeneracy and measure $\sigma_8$ (and $b$).
- Need a model for the auto- and cross-spectra of biased tracers.
As always …

Improvements in data require concurrent improvements in the theoretical modeling in order to reap the promised science.

What is the right framework for analyzing such data?

We need a model which can predict the auto- and cross-spectra of biased tracers at large and intermediate scales.

▶ Even though we are at high $z$ and “large” scales it turns out that linear perturbation theory isn’t good enough.

▶ Need to include non-linear corrections – and as soon as you do that you need to worry about scale-dependent bias, stochasticity and a whole host of other evils.
Highly biased objects

- From existing surveys we can estimate the bias.
- Even at faint magnitudes, these are highly biased objects.
Scale-dependent bias

\[ w(\theta) \times 2 \]

\[ z \approx 3 \text{ HOD} \]
\[ z \approx 4 \text{ HOD} \]
\[ u\text{-drops (H09)} \]
\[ g\text{-drops (O17)} \]

\[ b_{gg}(z \approx 3) \]
\[ b_{gm}(z \approx 3) \]
\[ b_{gg}(z \approx 4) \]
\[ b_{gm}(z \approx 4) \]
Scale-dependent bias

$L = k \cdot \chi_*$

$P(k)\ [h^{-3}\text{Mpc}^3]$ for $z = 3.0$

$P_{gg}$
$P_{gm}$
$bP_{gm}$
$b^2 P_{mm}$

$L = k \cdot \chi_*$

$P(k)\ [h^{-3}\text{Mpc}^3]$ for $z = 4.0$

$P_{gm}$
$P_{mm}$
$P_{\text{lin}}$
“Standard” model

- The most widely used model to date is based on the HaloFit fitting function for $P_{mm}(k)$ (auto-magically computed by CAMB and CLASS).
- Most analyses assume scale-independent bias (but this is barely sufficient even “now”).
- One extension, motivated by peaks theory, is to use $b(k) = b_{10}^E + b_{11}^E k^2$.
- We will find we need to augment this with a phenomenological linear ($k$) term

\[
\begin{align*}
P_{mh}(k) &= \left[ b_{10}^E + b_{1\frac{1}{2}}^E k + b_{11}^E k^2 \right] P_{HF}(k) \\
P_{hh}(k) &= \left[ b_{10}^E + b_{1\frac{1}{2}}^E k + b_{11}^E k^2 \right]^2 P_{HF}(k)
\end{align*}
\]

Note the (necessary) assumption that $b_{hh} = b_{mh}$!
Our approach

- Much of the information available from combining galaxy and CMB surveys lies at high $z$ and low $k$.
- This is the regime where PT excels!
- Less sensitive to baryonic effects, galaxy formation physics, etc.

Extend the highly successful linear perturbation theory analysis of primary CMB anisotropies which has proven so impactful!

[Formalism in PT similar to CMB lensing formalism]
Perturbation theory

- As surveys get larger and more powerful more of the modes we measure well are “quasi-linear” \(\Rightarrow\) analytic models.

- Over the last several decades, cosmological perturbation theory has developed steadily.
  - New ideas from particle physics and condensed matter.
  - Advances in modeling bias.
  - Generalizations beyond \(\Lambda\)CDM.

- At Berkeley we have been developing analytic models based on Lagrangian perturbation theory.

- Our original goal was baryon acoustic oscillations (BAO) and redshift-space distortions (RSD). But I will argue these tools (and others like them) are “perfect” for the coming world of survey cross-correlations...
CLEFT model

(Large scales, high z, it sounds like a job for ...)

The Lagrangian PT framework we have been developing for many years naturally handles auto- and cross-correlations in real and redshift space for Fourier or configuration space statistics. For example:

\[ P_{mg}(k) = \left(1 - \frac{\alpha k^2}{2}\right) P_Z + P_{1-\text{loop}} + \frac{b_1}{2} P_{b1} + \frac{b_2}{2} P_{b2} + \cdots \]

where \( P_Z \) and \( P_{1-\text{loop}} \) are the Zeldovich and 1-loop matter terms, the \( b_i \) are Lagrangian bias parameters for the biased tracer, and \( \alpha \) is a free parameter which accounts for \( k^2 \) bias and small-scale physics not modeled by PT.
Comparison with N-body

Let’s look at the ingredients going into the prediction of $C_{\ell}^{XY}$, for three cases:

- Linear theory, constant bias.
- HaloFit, constant bias (for now!).
- PT, $b_1 - b_2$. 
Comparison with N-body

\[ k P(k) \left[ h^{-2} \text{Mpc}^2 \right] \]

\( z = 1.0 \)

\( z = 2.0 \)

\( z = 3.0 \)

\( h - h \)  
Lin  
HF  
PT

\( m - m \)  
Lin  
HF  
PT

\( h - m \)  
Lin  
HF  
PT

Ratio  
\( k \left[ h \text{Mpc}^{-1} \right] \)  
0.2  
0.4  
0.6  
0.8
Model fit

\[ z = 3 \]

\[ \frac{L}{L_{\text{max}}} = 3 \]

PT

HF
Model fit

The likelihoods hide a lot of information about how the fit is performing. If we look at the best fit models:
Part of the issue with \texttt{HaloFit} is with the fit to $P_{mm}$, much of it is with the $b(k)$ assumption.

- Fitting functions for $P_{mm}$ are good to $\mathcal{O}(5 - 15\%)$, but the error bars will be smaller than this.
- Once $b$ is large it is not a constant and $b_{hh} \neq b_{mh}$.

At high $z$, modeling bias is at least as important as modeling non-linear structure formation.

In the EFT language: $k_{NL}$ shifts to higher $k$ at higher $z$, but the scale associated with halo formation (the Lagrangian radius) remains constant for fixed halo mass.

In general there is a “sweet spot”, where $b$ is not too scale dependent but non-linearity is not too pronounced.

How $b_{ij}(k)$ depends upon complex tracer selection is unknown.
Future directions

- There are good reasons to work in configuration space, not Fourier space ... (with compensated filters?)
- Go to 2-loop, so we can work to lower \( z \) and higher \( \ell \).
- Add \( m_\nu > 0 \) or MG, \( \nu_{bc} \), ...
- More explicit modeling of lensing.
- Inclusion of baryonic effects using EFT techniques.
- Look at non-Gaussianity from inflation (low \( \ell \)).
- Combining 3D surveys with 2D surveys. More modes to a fixed \( \ell \), but more difficult to model.
- Clean low \( z \). Can model \( C_{\ell}^{\kappa\kappa} (> z_{\text{min}}) \) and the decorrelations using PT.
- Simultaneously fitting \( dN/dz \) and \( \sigma_8 \) using clustering redshifts.
- Multi-tracer techniques (Schmitfull & Seljak 2017).
Conclusions

- We are on the cusp of a dramatic increase in the quality and quantity of both CMB and optical data. The combination can be more than the sum of its parts.
- As always, better data requires “better” modeling.
  - With primary anisotropies, linear theory is 99% of the story.
  - At lower redshift this is no longer the case.
- We need to model both non-linear matter clustering and bias.
- The fields of LSS and CMB have grown apart, but now are recoupling.
- The combination of high redshift and “large” scales makes this an attractive problem for analytic/perturbative attack.
- Generalizes to other high-z probes, in real- and redshift-space (e.g. LIM).
The End
Ancillary material
Signal to noise
Exposure time

- Exposure time needed to get a redshift of a high-EW dropout with PFS.
- Using the highest EW quartile of Shapley et al.
- Lines show 1- and 2-hour exposure limiting magnitudes.
The landscape

A natural “by-product” of next generation CMB experiments to constrain primordial gravitational waves is high fidelity CMB lensing maps.

- CMB lensing is sensitive to the matter field and to the space-space metric perturbation, over a broad redshift range.
- CMB lensing has radically different systematics than cosmic shear (and measures $\kappa$ and $\gamma$).
- CMB redshift is very well known (but can’t change it)!
- CMB lensing surveys tend to have large $f_{\text{sky}}$, but relatively poor resolution.
- The lensing kernel peaks at $z \sim 2 - 3$ and has power to $z \gg 1$, where galaxy lensing becomes increasingly difficult.
- The CMB is behind “everything” ... but projection is a big issue.
The noise in our measurements goes as

\[
\text{Var} \ [C^\kappa g_\ell] = \frac{1}{(2\ell + 1)f_{\text{sky}}} \left\{ (C^\kappa \kappa_\ell + N^{\kappa \kappa}_\ell) (C^{gg}_\ell + N^{gg}_\ell) + (C^{\kappa g}_\ell)^2 \right\}
\]

where \( f_{\text{sky}} \) is the sky fraction, \( C^{ii}_\ell \) represent the signal and \( N^{ii}_\ell \) the noise in the auto-spectra. Similarly

\[
\text{Var} \ [C^{gg}_\ell] = \frac{2}{(2\ell + 1)f_{\text{sky}}} (C^{gg}_\ell + N^{gg}_\ell)^2
\]

At low \( \ell \) we are sample variance limited, and at high \( \ell \) we are noise limited. For future experiments the transition will be \( \ell \sim 10^3 \).
Noise model II

For the galaxies the noise is simply shot-noise: $N_{\ell}^{gg} = 1/\bar{n}$

For the lensing we approximate the noise as

$$N_{L}^{\kappa\kappa} = \left[ \frac{\ell(\ell + 1)}{2} \right]^2 \left[ \int \frac{d^2 \ell}{(2\pi)^2} \sum_{(XY)} K^{XY}(\vec{\ell}, \vec{L}) \right]^{-1}$$

with e.g.

$$K^{EB}(\ell, L) = \frac{[ (\vec{L} - \vec{\ell}) \cdot \vec{L} C_{\ell-L}^{B} + \vec{\ell} \cdot \vec{L} C_{\ell}^{E} ]^2}{C_{\ell}^{tot,E} C_{\ell-L}^{tot,B}} \sin^2(2\phi_\ell)$$

and similar expressions for $TT$, $TE$ and $EE$.

(Ignore foregrounds and iterative methods.)
Effective redshift

- It is often the case that we wish to interpret the $C_\ell$, which involve integrals across cosmic time, as measurements of the clustering strength at a single, “effective”, epoch or redshift.

- Define

$$z_{XY}^{\text{eff}} = \frac{\int d\chi \left[ W^X(\chi) W^Y(\chi)/\chi^2 \right] z}{\int d\chi \left[ W^X(\chi) W^Y(\chi)/\chi^2 \right]}$$

such that the linear term in the expansion of $P(k, z)$ about $z_{XY}^{\text{eff}}$ cancels in the computation of $C_{XY}^\ell$.

- The $C_\ell$ computed with $P(k, z_{\text{eff}})$ fixed are within 1.5% of the full result for $\Delta z \leq 0.5$ and $\ell > 10$ for $1 < z < 3$. 
Fitting function accuracy

$\frac{\Delta^2_{\text{emu}}}{\Delta^2_{\text{Takahashi}}}$

$k$

$0.75$

$0.8$

$0.85$

$0.9$

$0.95$

$1$

$1.05$

$1.1$

$1.15$

$1.2$

$1.25$

$2 \text{emu} / 2 \text{Takahashi}$

$M038, w_a = 0$

$M039, w_a = 0$

$M040, w_a = 0$

$M041, w_a = 0$

$M042, w_a = 0$

$M043, w_a = 0$

$M044, w_a = 0$

$M000$

Lawrence et al. (2017)
Fitting function accuracy

\[ \frac{\Delta^2_{\text{sim}}}{\Delta^2_{\text{Mead}}} \]

\( k \)

\(|\Delta^2_{\text{sim}}| / \Delta^2_{\text{Mead}}\)

\( k \)

\( 10^{-3} \) to \( 10^{0} \)

Lawrence et al. (2017)
Perturbation theory

- CMB anisotropies are “everyone’s favorite”, linear, cosmological perturbation theory calculation ...
- Arguably, CMB anisotropies form the gold standard for cosmological inference and cosmological knowledge.
- A well controlled, analytic calculation which can be compared straightforwardly to observations.
- As we move to lower redshifts we need to start worrying about structure going non-linear and about the relation between the matter field and what we see (bias).
Lowest order I

\[ P_{\text{tree}} = 4\pi \int q^2 \, dq \, e^{-(1/2)k^2(X_L+Y_L)} \left\{ \right. \\
\left. \begin{array}{c}
1 + b_1^2 \left( \xi_L - k^2 U_L^2 \right) - b_2 \left( k^2 U_L^2 \right) + \frac{b_2^2}{2} \xi_L^2 \bigg| j_0(kq) \\
+ \sum_{n=1}^{\infty} \left[ 1 - 2b_1 \frac{q \, U_L}{Y_L} + b_1^2 \left( \xi_L + \left[ \frac{2n}{Y_L} - k^2 \right] U_L^2 \right) \\
+ b_2 \left( \frac{2n}{Y_L} - k^2 \right) U_L^2 \right] \\
- 2b_1 b_2 \frac{q \, U_L \, \xi_L}{Y_L} + \frac{b_2^2}{2} \xi_L^2 \bigg| \left( \frac{k \, Y_L}{q} \right)^n j_n(kq) \bigg| \right. \right\} \\

For cross-correlations: \( b_1 \rightarrow \frac{1}{2} (b_1^A + b_1^B), \ b_2 \rightarrow b_1^A b_1^B, \) etc.
Lowest order II

Where

\[ \xi_L(q) = \frac{1}{2\pi^2} \int_{0}^{\infty} dk \ P_L(k) \left[ k^2 j_0(kq) \right] \]

\[ X_L(q) = \frac{1}{2\pi^2} \int_{0}^{\infty} dk \ P_L(k) \left[ \frac{2}{3} - 2 \frac{j_1(kq)}{kq} \right] \]

\[ Y_L(q) = \frac{1}{2\pi^2} \int_{0}^{\infty} dk \ P_L(k) \left[ -2j_0(kq) + 6 \frac{j_1(kq)}{kq} \right] \]

\[ U_L(q) = \frac{1}{2\pi^2} \int_{0}^{\infty} dk \ P_L(k) \left[ -kj_1(kq) \right] \]

The integrals over \( q \) can be done efficiently using fast Fourier transforms or other methods.

The full expressions contain “1-loop” terms which are integrals of \( P_L^2 \).
Scale-dependent bias

In detail P-S isn’t right, but ...

Note the bias is scale-dependent, and the scale dependence is different for the auto- and cross-spectra.
Model fit: galaxies

\( b \) means something different in each theory.
Knowing $dN/dz$

We can use the Fisher forecasting formalism to investigate where the signal is coming from, degeneracies, and biases.

Can work at relatively low $\ell$, but need to know $dN/dz$ well.
Model fit

- Consider a future experiment, motivated by LSST and CMB-S4 but it could be a number of things.
- Imagine cross-correlating the CMB lensing map with the (gold sample) galaxies in a slice $\Delta z = 0.5$ at $z = 1, 2$ and $3$.
  - $i_{\text{lim}} = 25.3$.
  - $\theta_b = 1.5', \Delta T = 1 \mu\text{K-arcmin}.$
- Compare two ‘models’:
  - HALOFIT with $b(k) = b_{10}^E + b_{1\frac{1}{2}}^E k + b_{11}^E k^2$.
  - Perturbation theory with $b_1, b_2$ (and $\alpha_i$).
- Concentrate on just measuring an amplitude of matter clustering, $\sigma_8$.
- Jointly fit $C_{\ell}^{\kappa g}$ and $C_{\ell}^{gg}$ ...
Hidden Valley

A set of \( > 10^{12} \) particle N-body simulations directed at IM science ...
Hidden Valley

![Graphs showing the behavior of $k_\perp$ for $z = 2.0$ and $z = 6.0$. The plots illustrate the change in $k_\perp$ with varying $k_\parallel$ and $k_\perp$ values at different redshifts.]
**Hidden Valley**

![Graphs showing model predictions and data points for $\Omega_{HI}$ and $b_{DLA}(z)$](#)
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\[ b(k) \quad k \,[h\,\text{Mpc}^{-1}] \]

\[ b(r) \quad r \,[h^{-1}\,\text{Mpc}] \]

Different colors and line styles represent different redshifts:
- Blue: \( z=2.0 \)
- Green: \( z=4.0 \)
- Orange: \( z=6.0 \)

Legend:
- \( b_a \)
- \( b_x \)
Hidden Valley

- Graphs show the behavior of $d\log(P(k))/d\log(\theta)$ as a function of $k$ (h/Mpc) for different redshifts ($z = 2.0$, $z = 4.0$, $z = 6.0$) at different times ($t = 0$, $t = 2$).
- The graphs display different models: $\sigma_8$, $b_1$, and combinations thereof.

- The plots illustrate the evolution of power spectra at different epochs.
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$P_l(k) [h^{-3} \text{Mpc}^3]$

$z = 2.0$

$P_N/P_Z$

$\ell = 0$

$z = 6.0$

$P_N/P_Z$

$\ell = 2$
Hidden Valley

\[ \frac{P(k)}{P_{nw}(k)} + \text{offset} \]

- \( z = 2.0 \)
- \( z = 4.0 \)
- \( z = 6.0 \)

\[ r^2 \xi(r) + \text{offset} \]

\( r \) [Mpc/h]

\( k \) [\( h \text{ Mpc}^{-1} \)]