Lagrange, Zeldovich and large-scale structure

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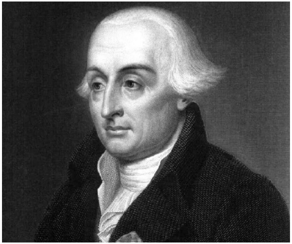
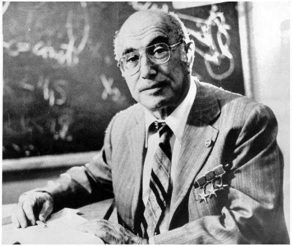




Outline

Large-scale structure is one of our premier laboratories for fundamental physics, cosmology and astrophysics.

- Growth of structure: RSD
- The streaming model and Lagrangian PT.
- The failure of PT and EFT.
- Including bias.
  - EFT & the connection to the ‘peaks formalism’.
  - Euler vs. Lagrange
- Conclusions

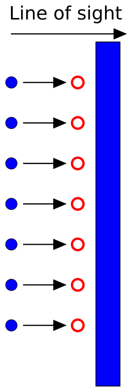
Growth of structure

- For a fixed expansion history/contents, GR makes a unique prediction for the growth of structure (and the velocity field).
  - This prediction is at the percent level – allowing percent level tests of paradigm (in principle!).
  - Growth is a competition between expansion and gravity.
  - In an expanding Universe collapse is slower than the usual Jeans instability – power-law not exponential – so we retain some memory of ICs.
- Growth of structure could help distinguish DE/MG models.
  - Also helps break some DE degeneracies ...
- Do we understand how large-scale structure forms?

Redshift-space distortions: RSD

Growth of structure measured using redshift-space distortions

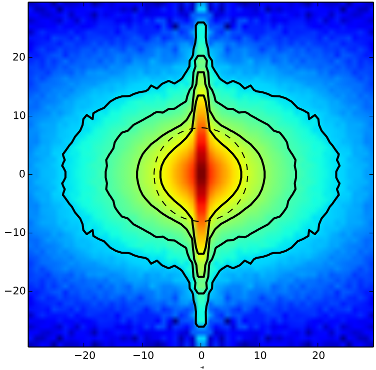
- $z_{\rm obs} = Hr + v_{\rm pec}^{\rm los}$.
- $v_{\rm pec}$ sourced by gravity, which is sourced by densities!
- Since $\delta = \rho/\bar{\rho} - 1$ grows by inflow of material, shifting by $v_{\rm pec}$ is like "looking into the future", but only in the line-of-sight direction!
- Comparison of clustering along and across the line-of-sight is a measure of growth rate.



We constrain $d\delta/d\ln a \sim (d\ln\delta/d\ln a)\delta \sim f\sigma_8$ by measuring the anisotropy in the clustering.

The 2D correlation function

In cosmology we frequently work in Fourier space. Today I will take a mostly configuration space approach.



Plotting the counts of pairs of objects, "above random", in bins of separation across and along the line-of-sight gives the 2D correlation function.

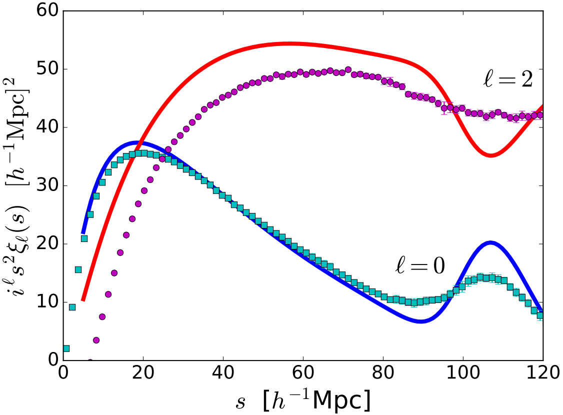
It is very smooth in angle – usually integrate over angle to get the multipole moments: ξ_ℓ.

It is the lowest order moments which are of interest.

Modeling RSD

Kaiser taught us how to model RSD on large scales, showing that within linear theory $\xi_0^{(s)} \propto \xi^{(r)}$, $\xi_2^{(s)}$ and $\xi_4^{(s)}$ are integrals of $\xi^{(r)}$.

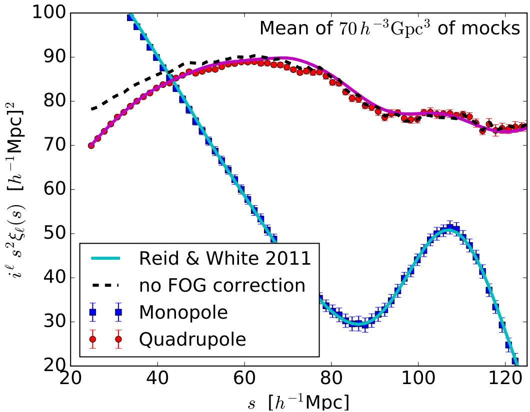
Unfortunately linear theory is not very accurate!



Comparison of N-body and linear theory at $z = 0.5$

Fingers of God

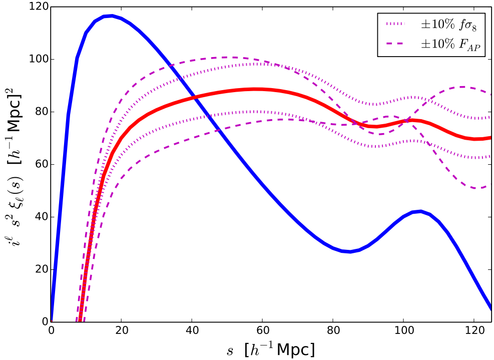
Perturbative non-linearity is not the only thing we need to worry about. Virial motions within collapsed objects also contribute to our signal – suppressing ξ_2 at small scales.



FoG are already a 10% effect by $s \sim 25\,h^{-1}\mathrm{Mpc}$ [$k \sim 0.15$]

Growth-geometry degeneracy

Anisotropies induced by changes in the growth rate can be mistaken for anisotropies induced by having the wrong model to convert θ and z to (R, Z).



This partial degeneracy can be broken with a long enough lever arm. But this means we want to fit over a wide range of scales ...

What we want in our theory ...

- Needs to work over a wide range of scales.
- Fingers of God need to be included.
- Go beyond linear perturbation theory
  - For the monopole, ξ_0, near the BAO peak.
  - For the quadrupole, ξ_2, on essentially all scales.
- Need to be able to handle biased tracers in a flexible and natural manner.
- For RSD part of the difficulty is that we are dealing with two forms of "non-smallness".
  - The density and velocity field are non-linear.
  - The mapping from real- to redshift-space is "non-small"
  - These two forms of correction interact (and can partially cancel) and depend on parameters differently.
  - 'Simple' PT doesn't work ...

Perturbation theory

- This problem is in principle amenable to direct simulation.
  - Though the combination of volume, mass and force resolution and numerical accuracy is actually extremely demanding – especially for next gen. surveys.
  - PT guides what range of k, M_h, etc. scales are necessary and what statistics need to be best converged.
  - N-body can be used to test PT for 'fiducial' models.
- However PT can be used to search a large parameter space efficiently, and find what kinds of effects are most important.
  - And can be much more flexible/inclusive, especially for biasing schemes, ...
- Hopefully we gain insight, not just numbers!
- Our goal is to do highly precise computations at large scales, in preparation for next gen. surveys, *not* to push to very small scales.

Streaming model: I

Displacements due to the velocity field are not "small", and we want to treat them non-perturbatively (as far as we can). One approach is to use a "streaming model".

- Recall that a shift in configuration space corresponds to a phase in Fourier space.
- To compute the redshift-space power spectrum we thus need to consider an object like

$$1 + \mathcal{M} = \left\langle [1 + \delta_1] \, [1 + \delta_2] \, e^{\cdots (u_{1\parallel} - u_{2\parallel})} \right\rangle$$

- Expand $\ln [1 + \mathcal{M}]$ in powers of $\Delta u^i = u_1^i - u_2^i$ (the cumulant expansion for $\langle e^x \rangle$ in terms of $\langle x^n \rangle$).

Streaming model: II

Keeping only cumulants up to second order:

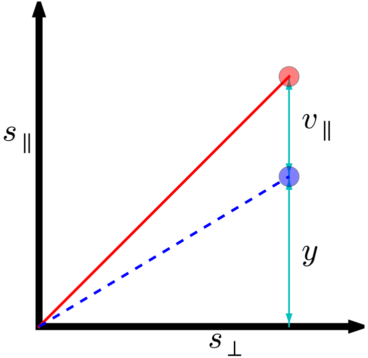
$$
\begin{aligned}
C &= \ln[1+\xi] \\
C^i &= \frac{\langle (1+\delta)(1+\delta')\Delta u^i \rangle}{1+\xi} \equiv v_{12}^i \\
C^{ij} &= \frac{\langle (1+\delta)(1+\delta')\Delta u^i \Delta u^j \rangle}{1+\xi} - v_{12}^i v_{12}^j \equiv \sigma_{12}^{ij}
\end{aligned}
$$

and doing the Fourier transform gives the Gaussian streaming model:

$$
1+\xi^s(s_\perp, s_\parallel) = \int \frac{dy}{\sqrt{2\pi}\;\sigma_{12}} [1+\xi] \exp\left\{ -\frac{[s_\parallel - y - \mu v_{12}]^2}{2\sigma_{12}^2} \right\}
$$

Streaming model: III

- This expression has a very simple interpretation in terms of the conservation of pairs.
- Pairs at $s_{\|}$ come from pairs at "true" separation y which have $v_{\|}$ such that $s_{\|} = y + v_{\|}$.



$$1 + \xi^s = \int dy \, [1 + \xi] \, \mathcal{P}\left(v_{\|} = s_{\|} - y | \mathbf{r}\right)$$

With the "Gaussian streaming model" having $\mathcal{P}$ be a Gaussian.

Streaming model: IV

- The Gaussian streaming model works very well for describing the clustering of halos in simulations.
  - Higher order cumulants small correction on 10s Mpc scales.
- To complete the model we need predictions for ξ, v_{12} and σ_{12}^2.
- We turn to (Lagrangian) peturbation theory
  - Introduced by Zeldovich in the 1970's and developed in the late 80's and early 90's.
  - Lagrangian PT is experiencing a resurgence.
  - Easily handles RSD and bias (Matsubara).
  - Has been a focus of my group for the last ~ 3 years.

Lagrangian perturbation theory I

- Consider fluid elements (or DM particles) which start at $\mathbf{q}$ and move to $\mathbf{x} = \mathbf{q} + \mathbf{\Psi}(\mathbf{q})$, with $\ddot{\mathbf{\Psi}} + 2H\dot{\mathbf{\Psi}} = -\nabla\Phi(\mathbf{x})$
- Expand $\mathbf{\Psi}$ as a power series in linear δ:

$$\Psi_i^{(n)}(\mathbf{k}) = \int \prod_{i=1}^{n} \frac{d^3 k_i}{(2\pi)^3} (2\pi)^3 \delta^{(D)} \left(\mathbf{k} - \sum \mathbf{k}_i\right) L_i^{(n)} \overbrace{\delta(\mathbf{k}_1) \cdots \delta(\mathbf{k}_n)}^{n \text{ terms}}$$

  with $L_i^{(n)}(\mathbf{k}; \mathbf{k}_1, \cdots, \mathbf{k}_n)$ a mode-coupling kernel (just ratios of dot products of $\mathbf{k}_i$).
- The density field is given from $\mathbf{\Psi}(\mathbf{q})$ as

$$\begin{aligned} 1 + \delta_{LPT}(x) &= \int dq\ \delta^{(D)} \left[x - q - \Psi(q)\right] \\ \delta_{LPT}(k) &= \int dq\ e^{-ikq} \left[e^{-ik\Psi(q)} - 1\right] \end{aligned}$$

Lagrangian perturbation theory II

- The $L_i^{(n)}$ can be derived, to arbitrary order, from the equations of motion.
- Products of $\delta(k)$s can be computed as expectation values of exponentials, with each term a product of Gaussian fields.
- Example: The Zeldovich approximation ($1^{\rm st}$ order LPT):
  - Consider computing $P_Z(k) = \langle|\delta_Z|^2\rangle$, with each $\delta_Z(k)$ an integral of $\exp[ik\Psi(q)]$ and $\Psi = (ik/k^2)\ \delta$ a Gaussian.
  - For a zero mean Gaussian, x, the "cumulant expansion":

    $$\left\langle e^x \right\rangle = e^{-\langle x^2 \rangle/2} \qquad \text{(complete the square)}$$

    so in the Zeldovich approximation we have

    $$P_Z(k) = \int dq\ e^{-ikq} \left[e^{-k^2\sigma^2(q)/2} - 1 \right]$$

    where σ^2 is the displacement 2-point function, $\left\langle \Psi^2 \right\rangle$.
- Higher order proceeds analogously ...

Limitations of PT

But is PT right? Well no, not really, ...

- Looking at PT for "perfect" sheets moving in 1D proves very instructive!
  - Zeldovich approximation is "exact" (up to sheet crossing).
- The LPT and SPT solutions are identical to all orders, even though they describe different systems after shell crossing.
  - A perfectly cold, pressureless fluid vs. a collection of non-interacting particles/sheets.
- Both perturbation theories converge smoothly to a well-defined solution.
- **That solution is wrong!** (c.f. N-body)
- Obviously any resummation scheme based purely in perturbation theory cannot cure this problem.

Effective field theory

Traditional perturbation theory treats *all* scales as if they were perturbative and the matter field as a perfect fluid.

- Use "effective field theory" to parameterize our ignorance.
- EFT has a long history in other areas of physics (but beware there are significant differences with e.g. particle physics!).
- Basic idea is to write the equations in terms of long-wavelength fields with no small-scale fields explicitly involved ("integrated out").
- The effects of these small-scale fields then show up as additional terms in the equations of motion.
  - Trivial example: smoothing ρv isn't the same as multiplying smooth ρ by smooth v. Put difference on rhs of eom.
  - $\dot{\tilde{\rho}} + \nabla \cdot [\tilde{\rho}\tilde{v}] = Q \qquad (= -\nabla \cdot [\widetilde{\rho v} - \tilde{\rho}\tilde{v}])$

Procedure

- All we know about these terms is that they must obey the symmetries of the theory.
- We need to make some approximations to make progress.
- Assume we can expand the "extra terms" in derivatives (powers of k) and powers of δ with unknown coefficients.
- Then integrate these source terms against the known Green's function for the perturbative solution.
- It's easy to show that the additional terms asymptotically cancel any cutoff dependence in the theory.
  - However for reasonable k and (cutoff) Λ the final answers depend on Λ if we keep only lowest order.
  - Typically we take the limit $\Lambda \to \infty$.
  - This is the limit we were trying to avoid, but hope that EFTLSS is less sensitive to this limit than SPT.

Lagrangian EFT

- EFT for Lagrangian PT follows the same logic as Eulerian PT.
- What "extra" terms contribute to $\boldsymbol{\Psi}$?
- You can treat this as a theory of extended objects, or you can simply ask

$$\boxed{\text{What term, constructed from a derivative of terms linear in } \delta \text{, transforms under rotations as a vector?}}$$

- The answer, to lowest order, is trivially $\nabla\delta$, so:

$$\boldsymbol{\Psi} = \boldsymbol{\Psi}^{(1)} + \boldsymbol{\Psi}^{(2)} + \boldsymbol{\Psi}^{(3)} + \cdots + \alpha\nabla\delta + \nabla J$$

  with α a free parameter and J a 'stochastic' term, uncorrelated with δ.
- Now can proceed as before, ...

Bias, peaks and EFT

- To make contact with galaxies, QSOs, 21 cm, Lyα, etc. we need to include bias.
- In many ways, having a good model for bias is more important than dealing carefully with the non-linearities – even on large scales.
  - Even 'small' terms become important for next gen. surveys!
  - EFT: the "cut off" for bias can be at lower k than for non-linearities so at fixed $k \ll k_{\rm cut}$ need higher powers in $k/k_{\rm cut}$ for same accuracy.
  - Peaks: the Lagrangian radius (R) of a massive halo can be larger than the non-linear scale, so at fixed $k \ll R^{-1}$ need to include higher powers in kR for the same accuracy.
- We can start to model things like assembly bias.
- This bias can be generalized and 'renormalized'.

Flexible bias model important even on large scales

- Symmetry arguments (EFT) are even more powerful for bias that non-linear gravity, since we really don't understand the small-scale physics of bias.
- However, flexibility comes with a *huge* cost in number of parameters, and associated degeneracy issues.
- Physical models (e.g. galaxies form from peaks in the initial density field) can provide priors.
- Lagrangian formalism has advantages over Eulerian.
    - Naturally includes effects due to bulk motion.
    - Easier to connect to N-body sims.
    - If BBKS were perfect, would have no tidal tensor bias and k^2 terms would arise only due to peak constraint.

The GSM + LEFT

Now we compute the 3 ingredients of the GSM within Lagrangian effective field theory, including biased tracers. Everything reduces to Gaussian integrals of integrals of $P_L(k)$, e.g.

$$1 + \xi(r) = \int d^3q \ M_0(\vec{r}, \vec{q}) \ .$$

with

$$\begin{aligned} M_0 \ &= \ \frac{1}{(2\pi)^{3/2}|A_{\rm lin}|^{1/2}} e^{-(1/2)(q_i - r_i)(A^{-1}_{\rm lin})_{ij}(q_j - r_j)} \\ &\times \ \Bigg\{ 1 - \frac{1}{2} G_{ij} A^{1-\rm loop}_{ij} + b_1^2 \xi_L + \frac{1}{2} b_2^2 \xi_L^2 - 2 b_1 U_i g_i + \frac{1}{6} W_{ijk} \Gamma_{ijk} \\ &\quad - [b_2 + b_1^2] U_i^{(1)} U_j^{(1)} G_{ij} - b_1^2 U_i^{11} g_i - b_2 U_i^{20} g_i \\ &\quad - 2 b_1 b_2 \xi_L U_i^{(1)} g_i - b_1 A^{10}_{ij} G_{ij} - \frac{1}{2} \alpha_\xi {\rm tr}\, G + b_{\nabla^2} \mathcal{B} + b_2 b_{s^2} \chi^{12} \\ &\quad - b_{s^2} \left(G_{ij} \Upsilon_{ij} + 2 g_i V_i^{10} \right) + b_{s^2}^2 \zeta - 2 b_1 b_{s^2} g_i V_i^{12} + \cdots \Bigg\} \ . \end{aligned}$$

The GSM + LEFT

Each of the terms can be expressed as simple integrals over $P_L(k)$, e.g.

$$V_i^{10} = \left\langle s^2(\mathbf{q}_1)\Psi_i^{(2)}(\mathbf{q}_2)\right\rangle_c = -\frac{2\,\hat{q}_i}{7}\int \frac{k\,dk}{2\pi^2} Q_{s^2}(k)\, j_1(kq)$$

with

$$Q_{s^2}(k) = \frac{k^3}{4\pi^2}\int dr\ P_L(kr)\int dx\ P_L(k\sqrt{y})Q_{s^2}(r,x)$$

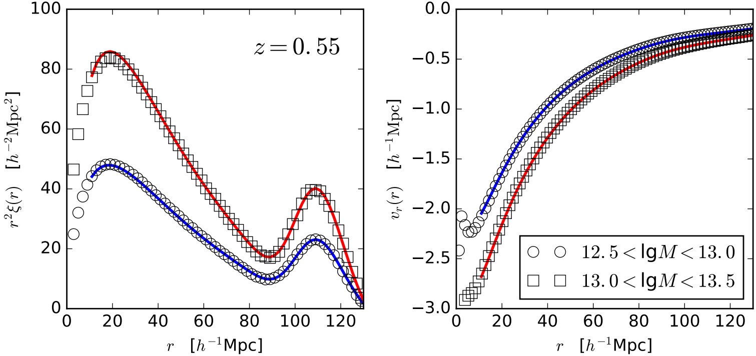
where $y = 1 + r^2 - 2rx$ and

$$Q_{s^2}(r,x) = \frac{r^2(x^2-1)(1-2r^2+4rx-3x^2)}{y^2}$$

There are similar expressions for each of the other terms in M_0, and similar expressions for v_{12} and σ_{ij}^2.

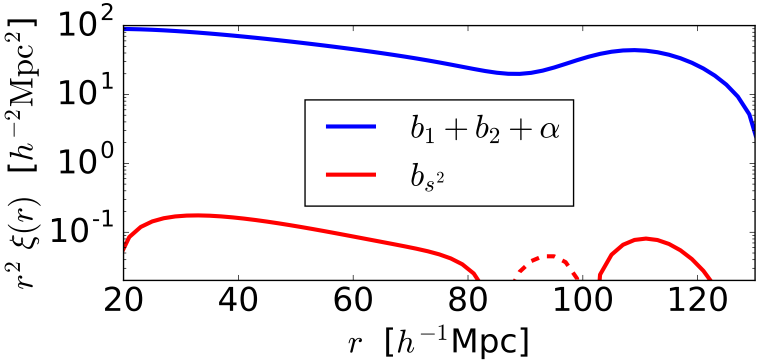
Model works well

Comparing real-space ξ and mean (infall) velocity (v_{12}) in the model to 256 $h^{-3}\mathrm{Gpc}^3$ of high-resolution N-body simulations.



Similar agreement in velocity dispersion on "large" scales – currently limited by the accuracy of N-body simulations!

Future surveys sensitive to small terms



Future surveys are sensitive to even very small contributions, especially around the BAO peak. Can also address light-cone/evolution effects, ...

Tests underway ...

- Preliminary indications are very promising.
- We are still refining the model and testing for degeneracies.
- The goal is a percent-level accurate, and highly flexible, model for the 2-point statistics in both configuration and Fourier space (and possibly hybrids).
- An LPT-based model already exists for fitting post-reconstruction BAO.

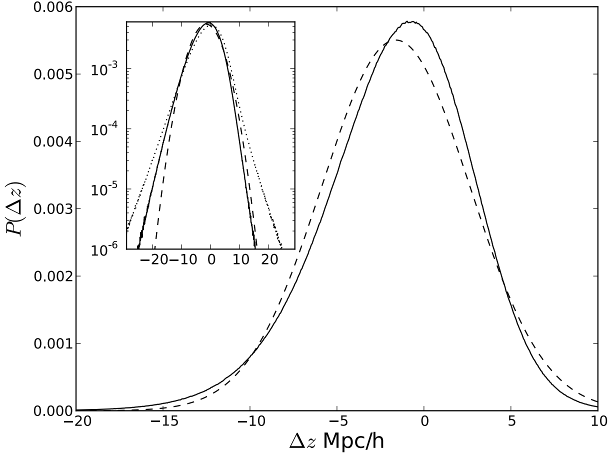
Working towards a fully Lagrangian framework for interpreting next-generation redshift-survey data, with a consistent set of parameters and assumptions.

Conclusions

- Large redshift surveys can be used to make precision tests of the ΛCDM model.
  - Expansion history (BAO)
  - Growth of structure (RSD)
  - Many other things ...
- Analytic models can shed light on the relevant physics and we hope they can be made accurate enough to fit next-generation data (on large scales).
- Modeling BAO+RSD requires beyond-linear modeling.
- Lagrangian perturbation theory (LPT) is a natural language for building such models.
- Need to go beyond straight perturbation theory and need to go beyond simple bias models.
- We are close to having a fully Lagrangian framework for interpreting next-generation redshift-survey data.

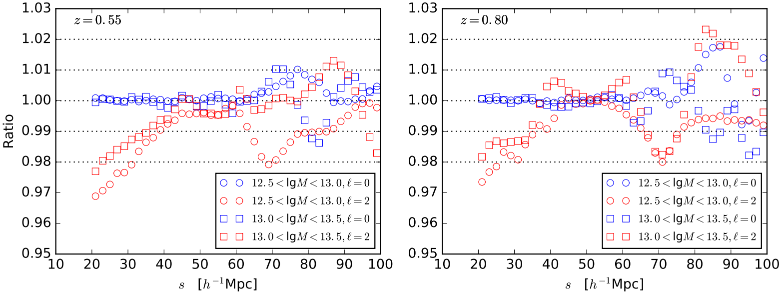
The End!

Gaussian Ansatz for $\mathcal{P}$



The distribution of velocity differences (converted to distance offsets) for pairs of halos separated by 30 h^{-1}Mpc.

Test of the GSM

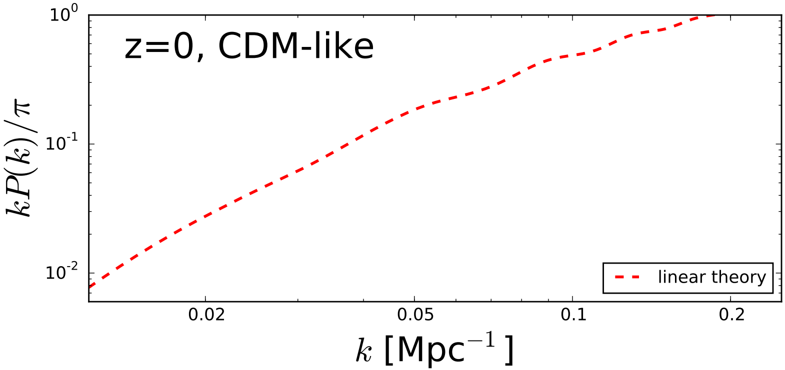


Accuracy currently limited by systematics in the N-body simulations!!

Convergence of perturbation theory

- We can compute any order in PT using a simple (1D) Fourier transform.
- It is easy to look at
  - convergence of PT (yes, it converges!)
  - common resummation schemes
  - asymptotics
- How does PT do in this simple 1D case?

Convergence of perturbation theory



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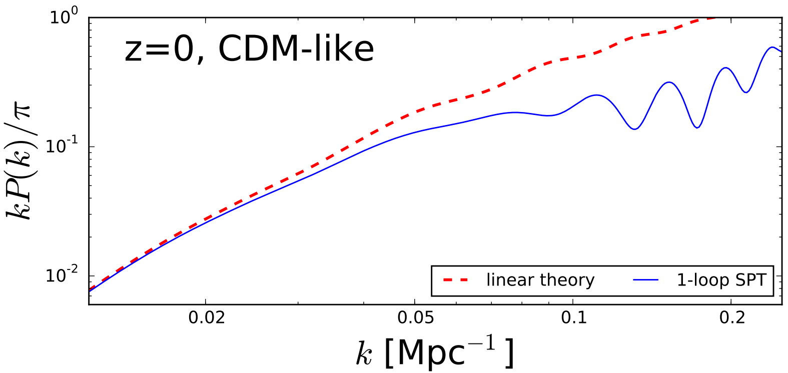

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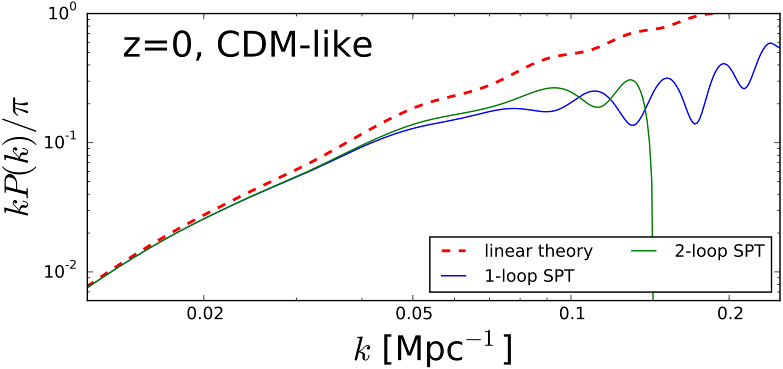

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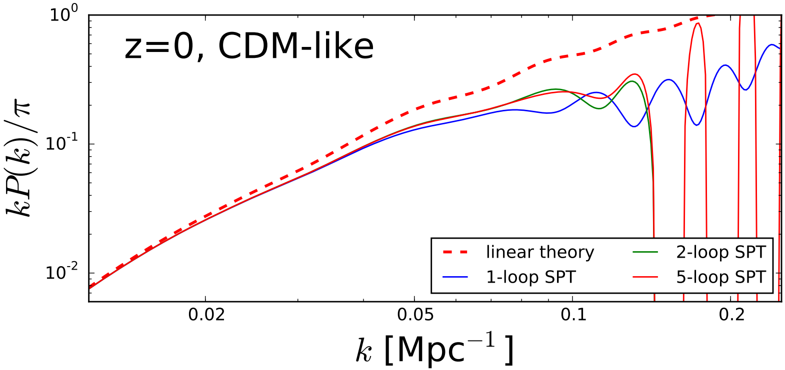

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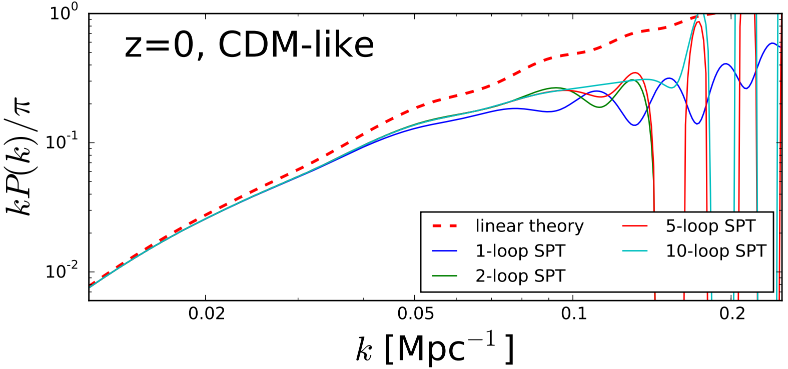

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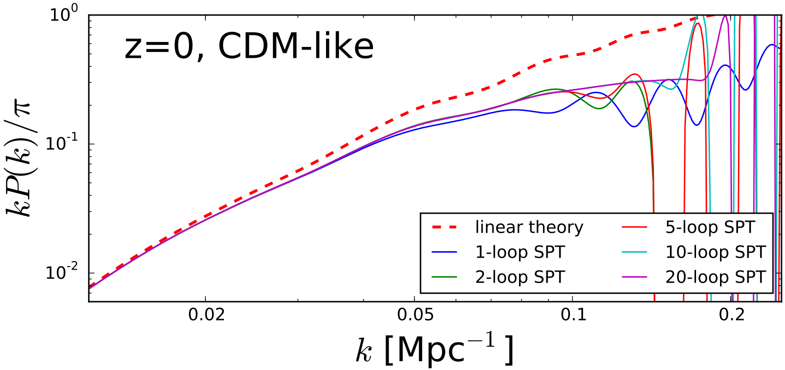

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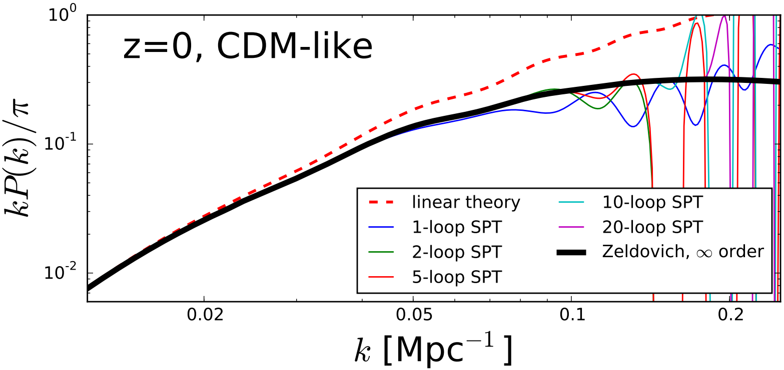

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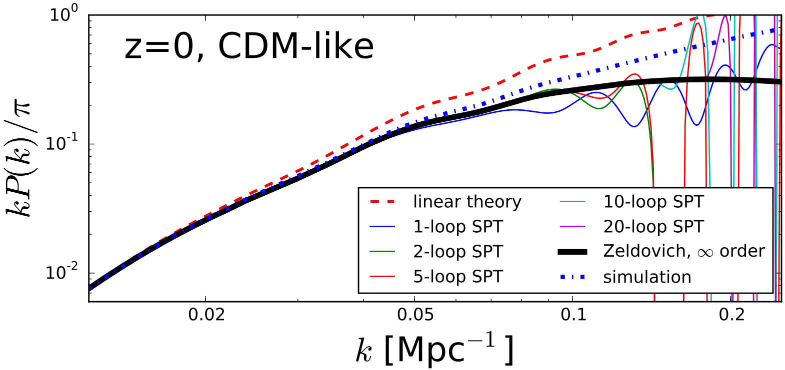

Summary

- The LPT and SPT solutions are identical to all orders, even though they describe different systems after shell crossing.
  - A perfectly cold, pressureless fluid vs. a collection of non-interacting particles/sheets.
- The perturbation theory converges smoothly to a well-defined solution.
- That solution is wrong!
- Obviously any resummation scheme based purely in perturbation theory cannot cure this problem.