Wide angle effects in redshift surveys (a.k.a. “What the heck are we measuring?”)

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Redshift space

- The observed redshift of a cosmological object has contributions from the Hubble expansion *and* the peculiar velocity.
- We convert z to a distance using a distance-redshift relation.
- Thus in redshift surveys we measure not the true position of objects but their redshift-space position:

$$\mathbf{s} = \mathbf{r} + \hat{r} \cdot \mathbf{v}/(aH)\,\hat{r}$$

- This is both a blessing and a curse:
  - it makes the analysis more complicated, but
  - it gives access to more information.

[Throughout all my 3D positions will be in redshift space. Real-space quantities will be called out specifically.]

Plane-parallel approximation

- Each redshift-space position depends upon 1 component of velocity.
- For narrow fields or objects at high redshift, the plane-parallel or distant observer approximation(s) are valid:
  - we can replace $\hat{s}$ with a fixed vector (conventionally $\hat{z}$).
  - all objects 'probe' same component of velocity.
- This is the standard assumption/approximation in the field, and is nearly ubiquitous in theory and data analysis.
- This assumption restores a symmetry which is absent in the full calculation.

We are not *always* in that limit, and it is interesting to ask 'fundamental' questions about what we're measuring and how that limit is approached.

History

This is an **old** question, which has been explored since the first papers on redshift-space distortions. We are treading some new ground, but building upon a vast amount of earlier work!

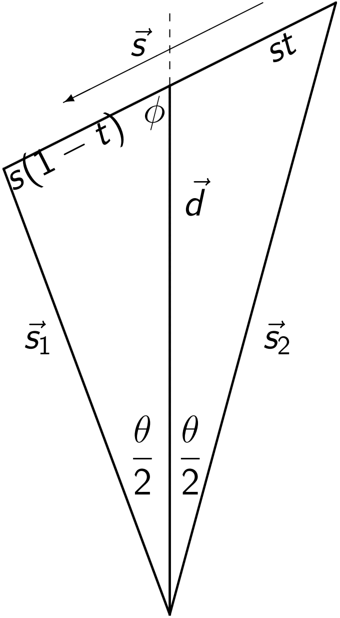
Hamilton (1992); Hamilton & Culhane (1996); Zaroubi & Hoffman (1996); Szalay et al. (1998); $\cdots$; Raccanelli et al. (2014); Yoo & Seljak (2015); Samushia et al. (2015); Reimberg et al. (2016); Castorina & White (2018a,b).

[I will concentrate on 'physical' wide angle effects, not those induced by the survey selection function – see paper for the latter. Ditto for lightcone effects and h.o. functions.]

The two-point function

- Observer viewing two galaxies, at $\vec{s}_1$ and $\vec{s}_2$.
- Separation vector $\vec{s} = \vec{s}_1 - \vec{s}_2$.
- Opening angle θ.
- Line of sight = angle bisector ($\vec{d}$)
- Define $\mu = \cos\phi = \hat{s} \cdot \hat{d}$.

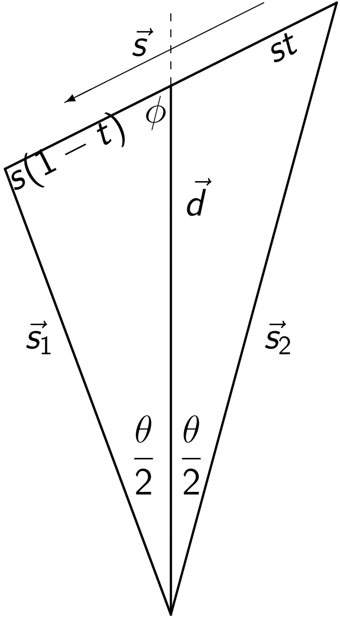
The BAO scale at $z = 0.2$ has $\theta \simeq 0.2$; at $z = 1$ have $\theta \simeq 0.05$.



The two-point function

- $\hat{s}$ is different for the two legs, we've lost translational invariance – the observation picks a preferred origin (us).
- Breaks translational invariance down into remaining rotation of $\hat{d}$ and rotation about $\hat{d}$.
- Clustering depends on triangle:

  $$\xi = \xi(\triangle) \neq \xi(\mathbf{s})$$



Triangles!

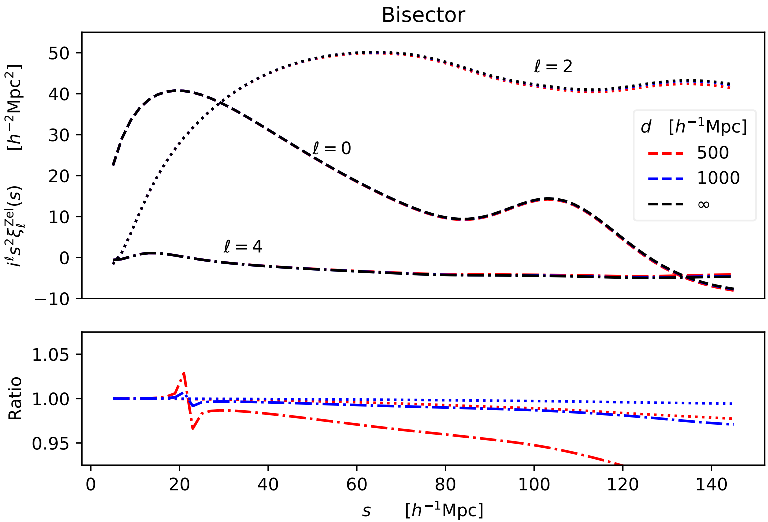
Several ways to specify the triangle, starting from $\mathbf{s}_1$ and $\mathbf{s}_2$ and allowing rotations of $\hat{d}$ and around $\hat{d}$:

- Use angle bisector, $\mathbf{d}$, and relative coordinate, $\mathbf{s} = \mathbf{s}_1 - \mathbf{s}_2$ to specify $\xi(\triangle) = \xi(s, d, \mu = \hat{d} \cdot \hat{s})$.
- Same but using the mid-point as $\mathbf{d}$.
- Same but using an end-point $\mathbf{s}_1$ (or $\mathbf{s}_2$) as $\mathbf{d}$.
- Give two side lengths and enclosed angle: $\xi(s_1, s_2, \cos\theta)$.
- Several others we won't need.

Can convert different conventions using Euclidean geometry theorems about triangles: *kind of fun if you're into that kind of thing*.

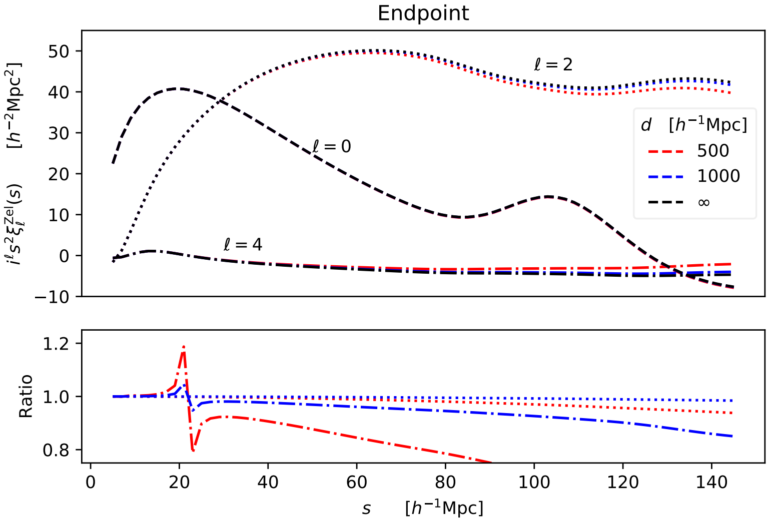
Results - bisector

For the angle-bisector definition of **d** the effects in configuration space are small, $\mathcal{O}(\theta^2)$, but not negligible at high ℓ:



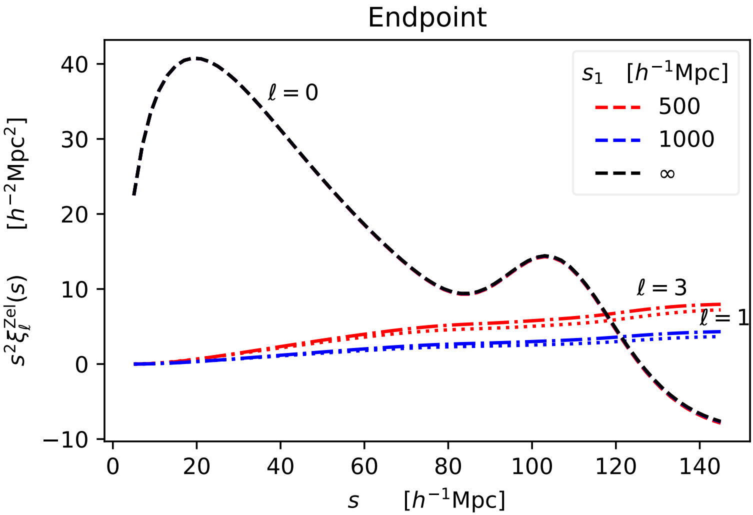
Results - endpoint

The effects are much larger, $\mathcal{O}(\theta)$, for the endpoint definition:



Odd multipoles

...and the endpoint definition introduces odd multipoles:



Power spectrum

How do I Fourier transform $\xi(\triangle)$?

- Since we've lost the translational symmetry of the underlying problem, there is no longer an obvious $P(\mathbf{k})$:

$$\langle \delta(\mathbf{k}_1)\delta^\star(\mathbf{k}_2)\rangle \neq (2\pi)^3\delta^{(D)}(\mathbf{k}_1 - \mathbf{k}_2)P(\mathbf{k}_1)$$

- We need to take care in specifying what we mean by **"the"** power spectrum
  - Zarroubi & Hoffman 1996; Szalay++98; $\cdots$
- So what is it that we've measured when we quote $P(\mathbf{k})$ in a redshift survey?
- What would a sensible definition look like?

Power spectrum

- We need to take care in specifying what we mean by "the" power spectrum
  - Zarroubi & Hoffman 1996; Szalay++98; $\cdots$
- A fruitful definition is a 'mixed' statistic:

$$P(\mathbf{k},\mathbf{d}) = \int d^3s\ e^{-i\mathbf{k}\cdot\mathbf{s}}\xi(\mathbf{s},\mathbf{d}) = \sum_{\ell n}(kd)^{-n}P_\ell^{(n)}(k)\mathcal{P}_\ell(\mu_k)$$

  and the plane-parallel limit is $kd \to \infty$.
- The multipoles of P and ξ are related by a Hankel transform:

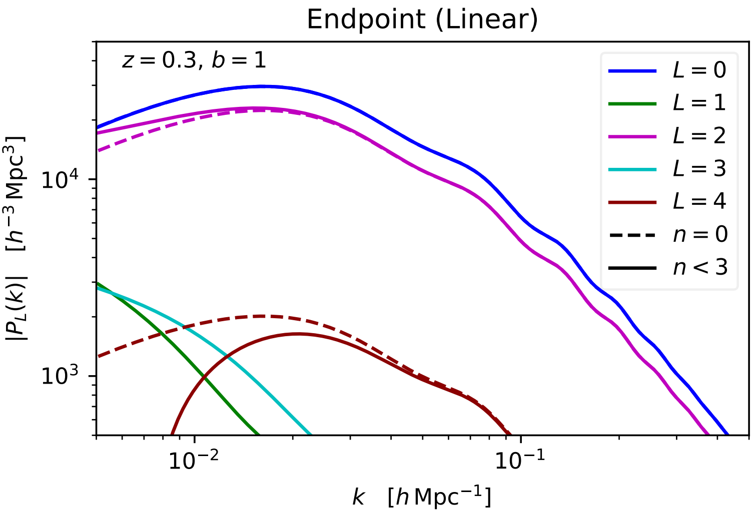
$$P_\ell(k,d) = 4\pi(-i)^\ell \int s^2\,ds\ \xi_\ell(s,d)j_\ell(ks)$$

Most other definitions lead to difficulties...

Yamamoto estimator

- People usually compute $P(\mathbf{k})$ using the 'Yamamoto' estimator.
- This is an approximation to $\int (d^3d/V) P(\mathbf{k}; d)$.
- P_ℓ can be computed by summing over pairs, but this is slow and 'expensive'.
- People usually use end-point definition: $\hat{d} = \hat{s}_1$, then the $P_\ell(k)$ estimator factorizes and can use FFTs ...
- ... but it gives corrections at $\mathcal{O}(\theta)$ rather than $\mathcal{O}(\theta^2)$ (and mixes Hankel transforms in a very non-trivial way).
- Corrections grow with ℓ: possibly important if computing wedges or treating fiber assignment this way.

Power spectra



Masks

We are familiar with a mask in a survey generating a 'window function'. Specifically if our density field is multiplied by $W(\mathbf{x})$ (usually 0 or 1) then

$$P(\mathbf{k}) = \int \frac{d^3q}{(2\pi)^3} P(\mathbf{q}) \left| \widetilde{W}(\mathbf{k}-\mathbf{q}) \right|^2$$

However, this transition from multiplication to convolution assumes translational invariance – which is violated in wide-angle.

Leading order wide angle terms become important at the same scale as the mask does.

[Indications that this could matter for BOSS and future surveys]

Masks

Need to follow an approach which is a generalization of the "pseudo-C_ℓ" method used in CMB. For bisector-Yamamoto

$$\left\langle \hat{P}_L(k) \right\rangle \propto \sum_\ell \int \frac{d^3s_1\, d^3s_2}{V}\; W_\ell^L(s,d)\; \underbrace{\xi_\ell(s,d)}_{\int P(k')j_\ell(k's)}\; j_L(ks)$$

Note $\xi_\ell(s) j_L(ks)$ is *not* a Hankel transform!

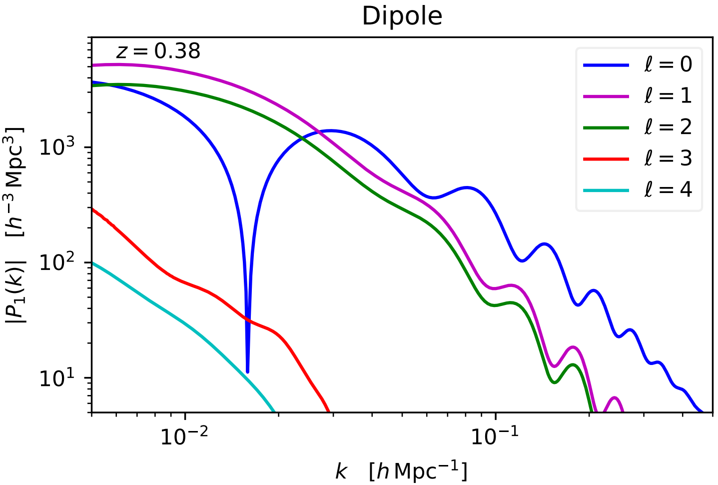
One can compute W_ℓ^L efficiently using randoms:

$$W_\ell^L(s,d) = \int \frac{d\Omega_1 d\Omega_2}{4\pi} W_1 W_2 \mathcal{P}_\ell(\hat{s}\cdot\hat{d})\mathcal{P}_L(\hat{s}\cdot\hat{d})$$

(sum over pairs of randoms, binning the integrand in bins of s,d).

[For the FFT estimator there are a few extra terms.]

Power spectra - BOSS window



Aside:

- This formalism could be useful for 21 cm interferometers which observe the sky with wide primary beams.
- It could be useful for 'nearby' surveys aimed at BAO or large-scale velocities.
- Also it offers a way of immunizing yourself against systematics which vary slowly with distance, e.g. purely angular systematics and a principled way to marginalize at the 2-point function level.
- It overcomes a big issue with the sFB formalism: covariances.

Fisher et al. (1994); Heaves & Taylor (1995); Tadros et al. (1999); Taylor et al. (2001); Padmanabhan et al. (2001); Percival et al. (2004); Padmanabhan et al. (2007); $\cdots$; Yoo & Desjacques (2013); Pratten & Munshi (2013); Nicola et al. (2014); Liu et al. (2016); Passaglia et al. (2017).

Conclusions

- Beyond plane-parallel what it is we're measuring requires some thought.
  - Outside of the distant observer limit $\xi = \xi(\triangle)$ not $\xi(\mathbf{s})$.
  - There is no 'natural' $P(\mathbf{k})$ definition, or even obvious plane-parallel limit.
  - Symmetry breaking!
- Effects are small, but not negligible.
  - Grow with ℓ and s/d (i.e. large scale).
  - Analytic models work well, highlight issues.
- Standard FFT $P_L(k)$ estimator has $\mathcal{O}(\theta)$ corrections.
- Formalism for masks becomes more complex.
  - Could be seeing this already.
- Interesting connections with sFB, intensity mapping, mode marginalization, ...

The End!

Plane parallel or distant observer limit

- The plane-parallel limit is regained in the limit $d \to \infty$.
- In the angle bisector definition we usually define $x = s/d$, so that $\xi = \xi(s, x, \mu)$. Expanding in power of x and in multipoles:

$$\xi(s, x, \mu) = \sum_{\ell n} x^n \, \xi_\ell^{(n)}(s) \mathcal{P}_\ell(\mu)$$

- The plane-parallel limit is

$$\xi^{\rm pp} = \lim_{x \to 0} \xi(s, x, \mu) = \sum_\ell \xi_\ell^{(0)}(s) \mathcal{P}_\ell(\mu)$$

The BAO scale at $z = 0.2$ has $\theta \simeq 0.2$; at $z = 1$ have $\theta \simeq 0.05$.

Plane parallel or distant observer limit

In linear theory

$$\xi_\ell^{(n)}(s,d) = \int \frac{k^2\,dk}{2\pi^2}(kd)^{-n}P_{\rm lin}(k)j_\ell(ks) \sim x^n\xi_\ell^{(0)}$$

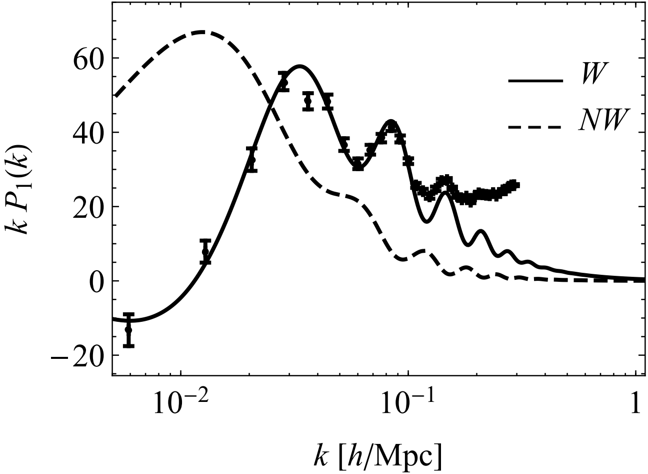
- For the angle bisector or midpoint definitions, which are 'symmetric', the first correction comes in at $\mathcal{O}(\theta^2)$, or $\mathcal{O}(x^2)$.
- For the endpoint convention the first correction is $\mathcal{O}(\theta)$, however the correction can be expressed in terms of known terms – it contains no further signal.
  - e.g. $\xi_1(s,s_1) = -(3/5)(s/s_1)\,\xi_2^{\rm pp}(s) + \cdots$
  - e.g. $\xi_3(s,s_1) = \quad (3/5)(s/s_1)\,\xi_2^{\rm pp}(s) - (10/9)(s/s_1)\,\xi_4^{\rm pp}(s) + \cdots$

Zeldovich and wide angle effects

- Conveniently, wide-angle effects occur at large scales where we can use analytic techniques.
- All analytical results on wide-angle effects have assumed linear theory (with linear bias).
  - Reproduced and extended those results.
- Can compute $\xi(\triangle)$ precisely within the Zeldovich approximation, including complex bias modeling.
  - Tassev 2014 states this, but didn't pursue it.
- In Castorina & White (2018b) we give the full expressions and various limits.
- This allows one to beyond $\ell = 4$, and to include advection properly.

Window function effects

Window vs. no-window at $z = 0.38$ for BOSS:



Courtesy: E. Castorina & F. Beutler

MAPS I

- The last convention (side lengths and included angle) is the most obvious generalization of CMB formalism, and the most directly connected to observables:
  - s_i are related to redshift and hence frequency,
  - θ is the observed angular separation.
- Normally you Legendre transform in angle to get the multi-frequency angular power spectrum (MAPS):

$$\xi(s_1, s_2, \cos\theta) = \sum_\ell \frac{2\ell+1}{4\pi} C_\ell(s_1, s_2)\mathcal{P}_\ell(\cos\theta)$$

- Alternatively you can think of spherical transforming the slices at s_1 and s_2 into $a_{\ell_1 m_1}(s_1)$ and $a_{\ell_2 m_2}(s_2)$ then

$$\left\langle a^\star_{\ell_1 m_1}(s_1) a_{\ell_2 m_2}(s_2)\right\rangle = C_\ell(s_1, s_2)\delta_{\ell_1\ell_2}\delta_{m_1 m_2}$$

- A further Hankel transform in the length of the legs gives $C_\ell(k_1, k_2)$ [connection to sFB formalism of Heavens & Taylor (1995)].

MAPS: plane parallel limit

In the plane-parallel limit, C_ℓ is a bit of a hybrid. Changing variables from s_1 and s_2 to $d = (s_1 + s_2)/2$ and $s_\parallel = s_1 - s_2$ we have

$$
\begin{aligned}
C_\ell(s_\parallel, d) &= 2\pi \int d(\cos\theta)\ \xi(s, d, \mu)\mathcal{P}_\ell(\cos\theta) \\
&\simeq 2\pi \int \tilde{\omega} d\tilde{\omega}\ \xi(s_\perp, s_\parallel, d) J_0(\ell\tilde{\omega}) \qquad \left[\tilde{\omega} = 2\sin\left(\frac{\theta}{2}\right)\right] \\
&\simeq \int \frac{d^2 s_\perp}{d^2}\ \xi(s_\parallel, s_\perp, d) e^{i\mathbf{k}_\perp \cdot \mathbf{s}_\perp} \\
&\simeq \int_0^\infty \frac{dk_\parallel}{\pi\, d^2} P(k_\perp = \ell/d, k_\parallel) \cos(k_\parallel s_\parallel)
\end{aligned}
$$

with $s_\parallel = s\mu$ to lowest order in s/d (and $\ell = k_\perp d$).

MAPS: beyond plane-parallel

- Interestingly the high-ℓ moments come from aliasing of low L power through geometry, in the same way that high ℓ in the CMB angular power spectrum can come from the density ($L = 0$) and velocity ($L = 1$) of the photon fluid.
- In Castorina & White (2018) we worked through the angular momentum addition to find the exact relation beyond plane-parallel:

$$C_\ell(\underbrace{k_1, k_2}_{1D}) = \sum_{nL} \mathcal{M}_{\ell L}^{(n)}(k_1, k_2) P_L \underbrace{(k_2)}_{3D}$$

- $\mathcal{M}_{\ell L}^{(n)}$ depends only on geometry, not cosmology.

Yamamoto estimator

- In surveys $P_\ell(k)$ is usually computed using "the Yamamoto estimator":

$$P_L^Y(k) = (2L+1)\int \frac{d\Omega_k}{4\pi}\frac{d^3s_1\,d^3s_2}{V}\delta(\mathbf{s}_1)\delta(\mathbf{s}_2)\,e^{-i\mathbf{k}\cdot\mathbf{s}}\mathcal{P}_L\left(\hat{k}\cdot\hat{d}\right)$$

- One can show that $P_\ell^Y = \int(d^3d/V)P_L(k,d)$.
- The 'full' Yamamoto estimator is very expensive to compute, so people usually use end-point definition: $\hat{d} = \hat{s}_1$.
- This allows the use of FFTs, ...
- ... but it gives corrections at $\mathcal{O}(\theta)$ rather than $\mathcal{O}(\theta^2)$ and mixes Hankel transforms in a very non-trivial way.
- Corrections grow with L: possibly important if computing wedges or treating fiber assignment this way.