Cosmology in 1 dimension

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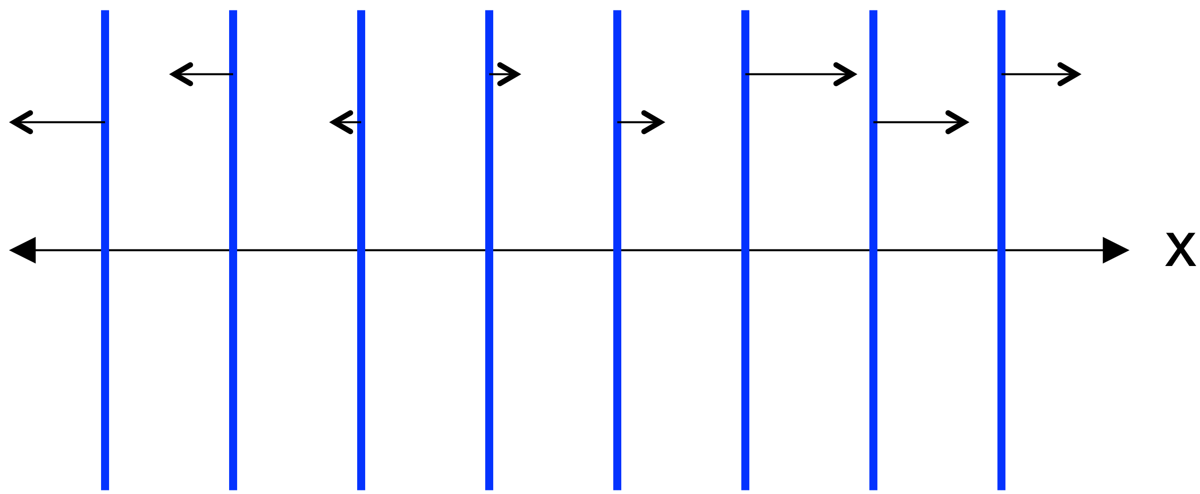
"A man grows stale if he works all the time on insoluble problems, and a trip to the beautiful world of one dimension will refresh his imagination better than a dose of LSD."
-- Freeman Dyson

Motivation

- There has been a great deal of work recently on (cosmological) perturbation theory.
  - New approaches, new resummation schemes, new renormalization techniques borrowed from QFT.
  - Growing appreciation of the uses and limitations of "standard perturbation theory" (SPT) and resummation schemes.
  - Understanding of RSD, BAO, SSC, beat-coupling, …
- Want to understand these developments (and old ideas) better in a simple context:
- Collection of uniform, parallel, 2D sheets of matter.
  - Problem becomes 1 dimensional (plus time).
  - Significant (!) analytic simplification: can do SPT to ∞ order.
  - Easier to handle numerically with high dynamic range.
  - Many of the features of 3D have close 1D analogues.

The setup

(with Ω_m=1 throughout)



$$P_{3D}(k_{\parallel}, \mathbf{k}_{\perp}) = (2\pi)^2 \delta^{(D)}(\mathbf{k}_{\perp}) P_{1D}(k_{\parallel})$$

Since the force on a particle due to a sheet is independent of the distance from the sheet, 1st order Lagrangian PT (Zeldovich) is exact until “sheet crossing” (can also show this analytically).

Power spectra and correlation functions

$$P_{\rm SPT}^{1-\rm loop}(k) = P_L(k) + \int_{-\infty}^{\infty}\frac{dk'}{2\pi}\left\{3 + 4\frac{k-k'}{k'} + \left(\frac{k-k'}{k'}\right)^2\right\} P_L(k')P_L(k-k') - k^2 P_L(k)\int_{-\infty}^{\infty}\frac{dk'}{2\pi}\frac{P_L(k')}{k'^2}$$

$$\xi_{\rm SPT}^{1-\rm loop}(x) = \xi_L(x) + \overbrace{3\xi_L^2(x)}^{\rm growth} + \overbrace{4\xi_L'(x)\int_x^{\infty} dx\,\xi_L(x)}^{\rm dilation} + \overbrace{\frac{\sigma_{\rm eff}^2}{2}\xi_L''(x)}^{\rm RMS\ \ displ.} + \mathcal{O}(\xi_L^3)$$

Can look at the response of power spectra to long wavelength mode (through the gradient times the large-scale variance) and the modes themselves (super-sample covariance, beat coupling), shifts and broadening of the BAO peak, ...

PT issues

- Convergence rather slow.
  - Resummation schemes.
  - Beware symmetry breaking!
- Solutions only valid prior to “sheet” crossing.
- $P^{1\text{-loop}}(k)$ depends on high-k’, non-perturbative modes even at large scales.

Lagrangian theory

$$1 + \delta_{\rm LPT}(x) = \int dq\, \delta^D[x - q - \Psi(q)]$$

$$\delta_{\rm LPT}(k) = \int dq\, e^{-ikq} \left(e^{-ik\Psi(q)} - 1\right)$$

But Ψ is just a Gaussian random variable … know <e$^{\Psi}$>

$$P_{\rm ZA}(k) = \int dq\, e^{-ik\,q} \left(e^{-k^2\sigma^2(q)/2} - 1\right)$$

$$\sigma^2(q) = \langle [\Psi_{\rm ZA}(0) - \Psi_{\rm ZA}(q)]^2 \rangle = \int_0^\infty \frac{dk}{\pi} \frac{2\, P_L(k)}{k^2} \left(1 - \cos[k\,q]\right)$$

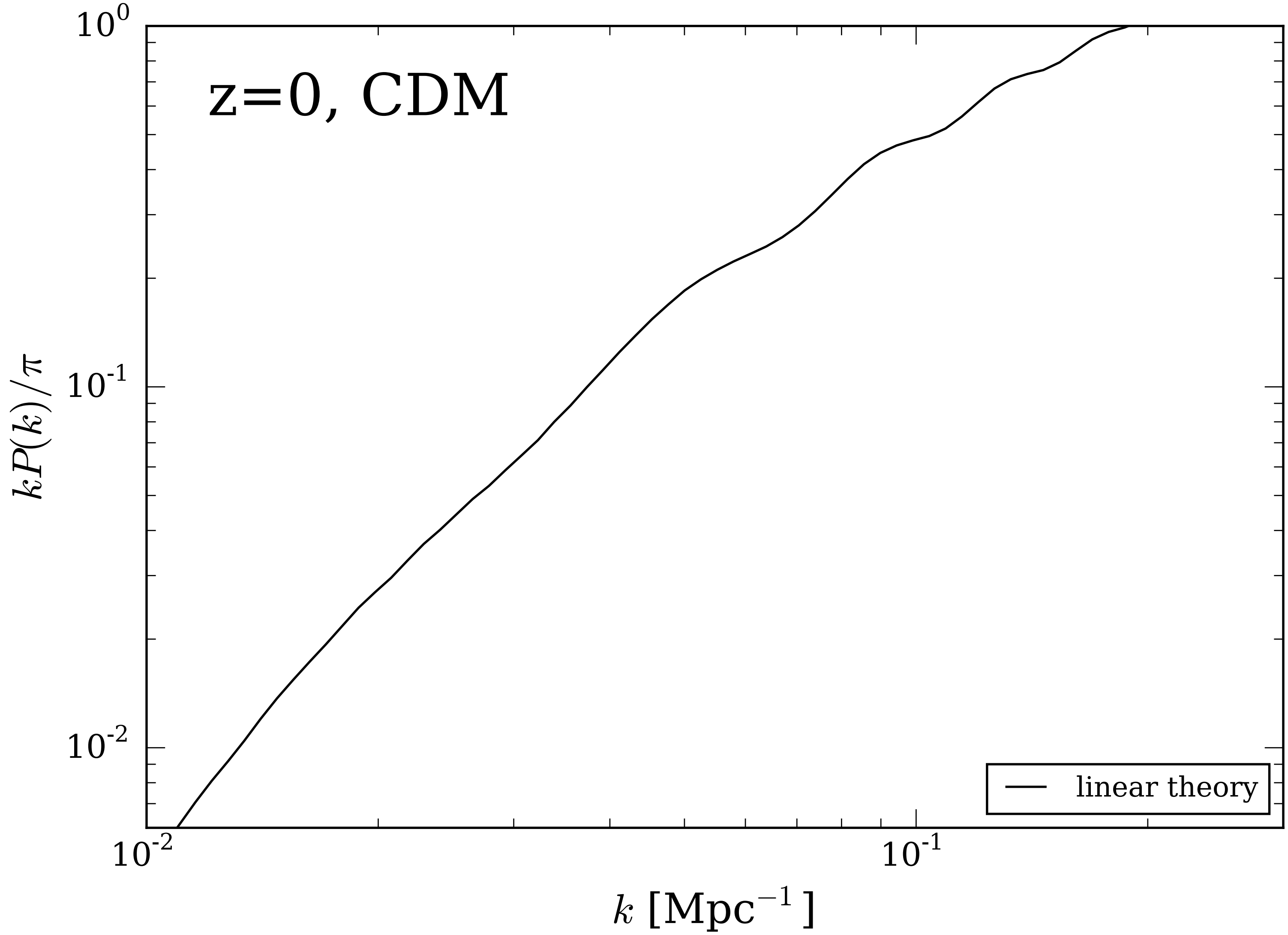
Can generate any order in PT!

$$\begin{aligned}
P_{\rm LPT}(k) &= \int dq\, e^{ik\,q} \left(-\frac{k^2}{2}\sigma^2(q) + \frac{k^4}{8}\sigma^4(q) + \cdots \right), \\
&= P_L + \frac{1}{8}\int dq\, e^{ik\,q}\, \nabla_q^4\, \sigma^4(q) + \mathcal{O}(P_L^3), \\
&= P_L + \frac{1}{8}\int dq\;\, e^{ik\,q} \left[6([\sigma^2]'')^2 + 8([\sigma^2]'[\sigma^2]''') + 2[\sigma^2][\sigma^2]'''' \right] + \mathcal{O}(P_L^3), \\
&= P_L + \int \frac{dk'}{2\pi} \left\{ 3 + 4\frac{k-k'}{k'} + \frac{(k-k')^2}{k'^2} \right\} P_L(k')P_L(k-k') + \cdots ,
\end{aligned}$$

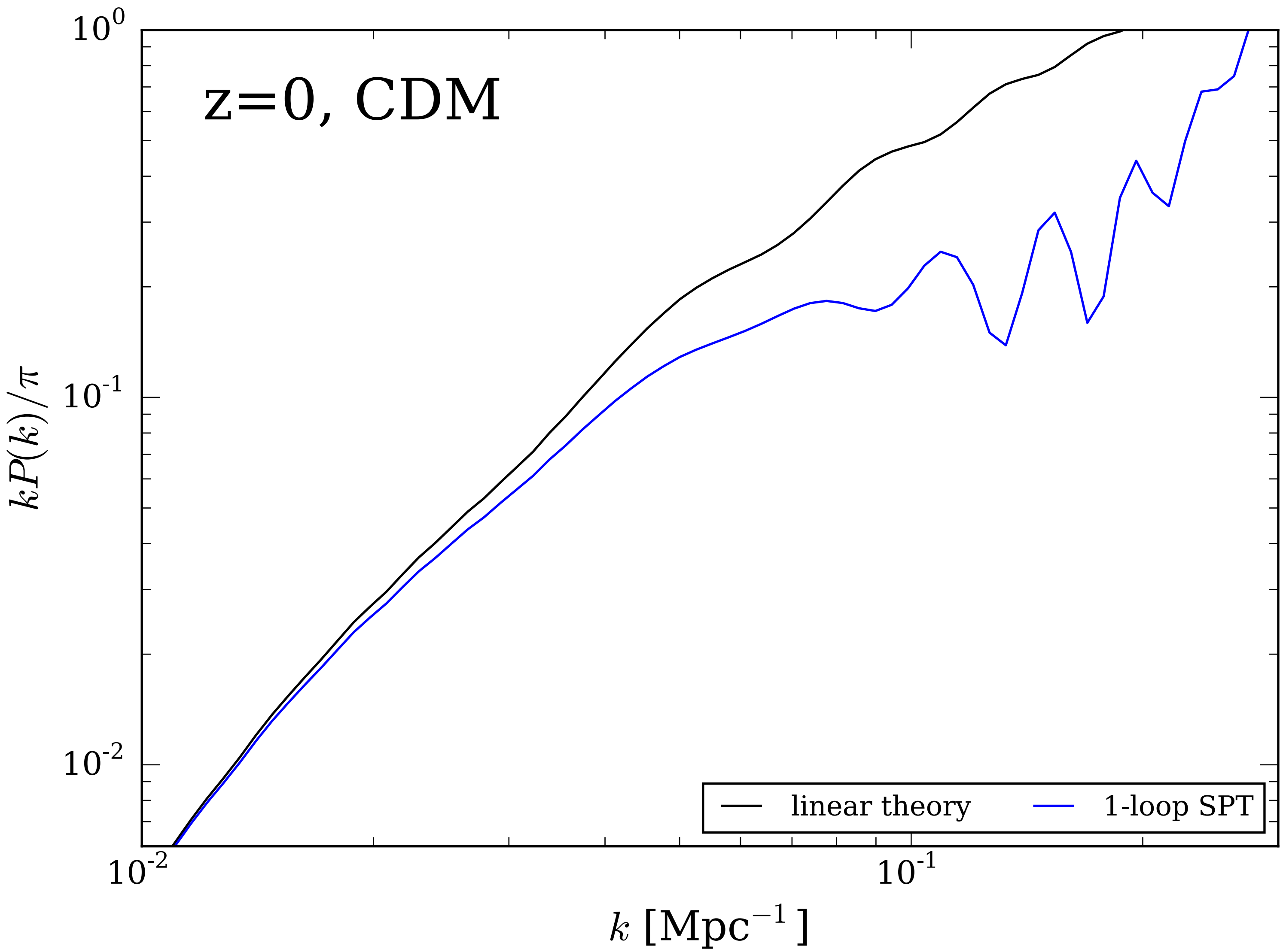
$$\boxed{F_n(k_1, \cdots, k_n) = G_n(k_1, \cdots, k_n) = \frac{1}{n!}\; \frac{k^n}{\prod_{i=1}^n k_i}}$$

Now we can understand common resummation schemes in “standard” perturbation theory, and we can look at the rate of convergence of perturbation theory.

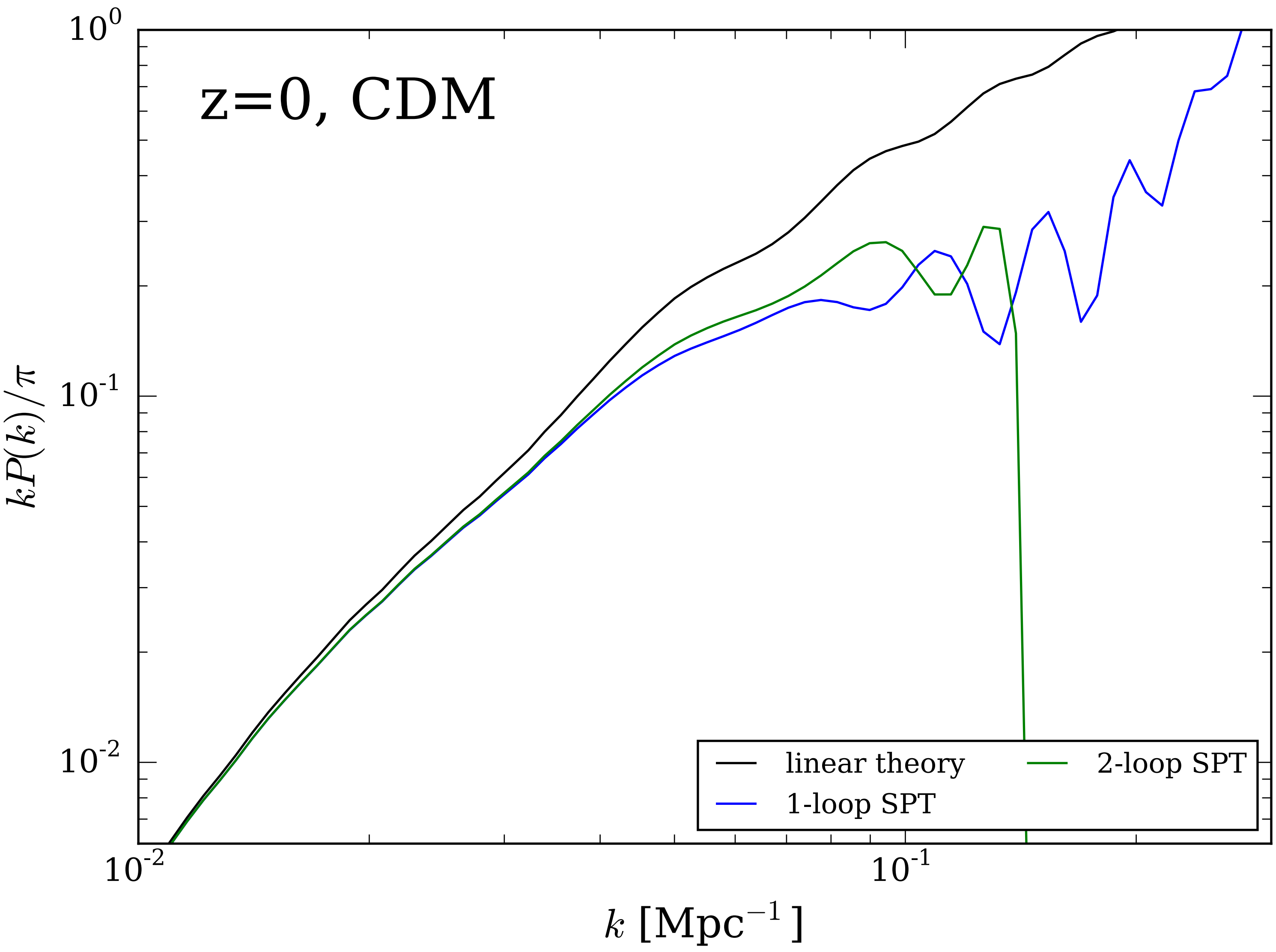
Convergence of PT



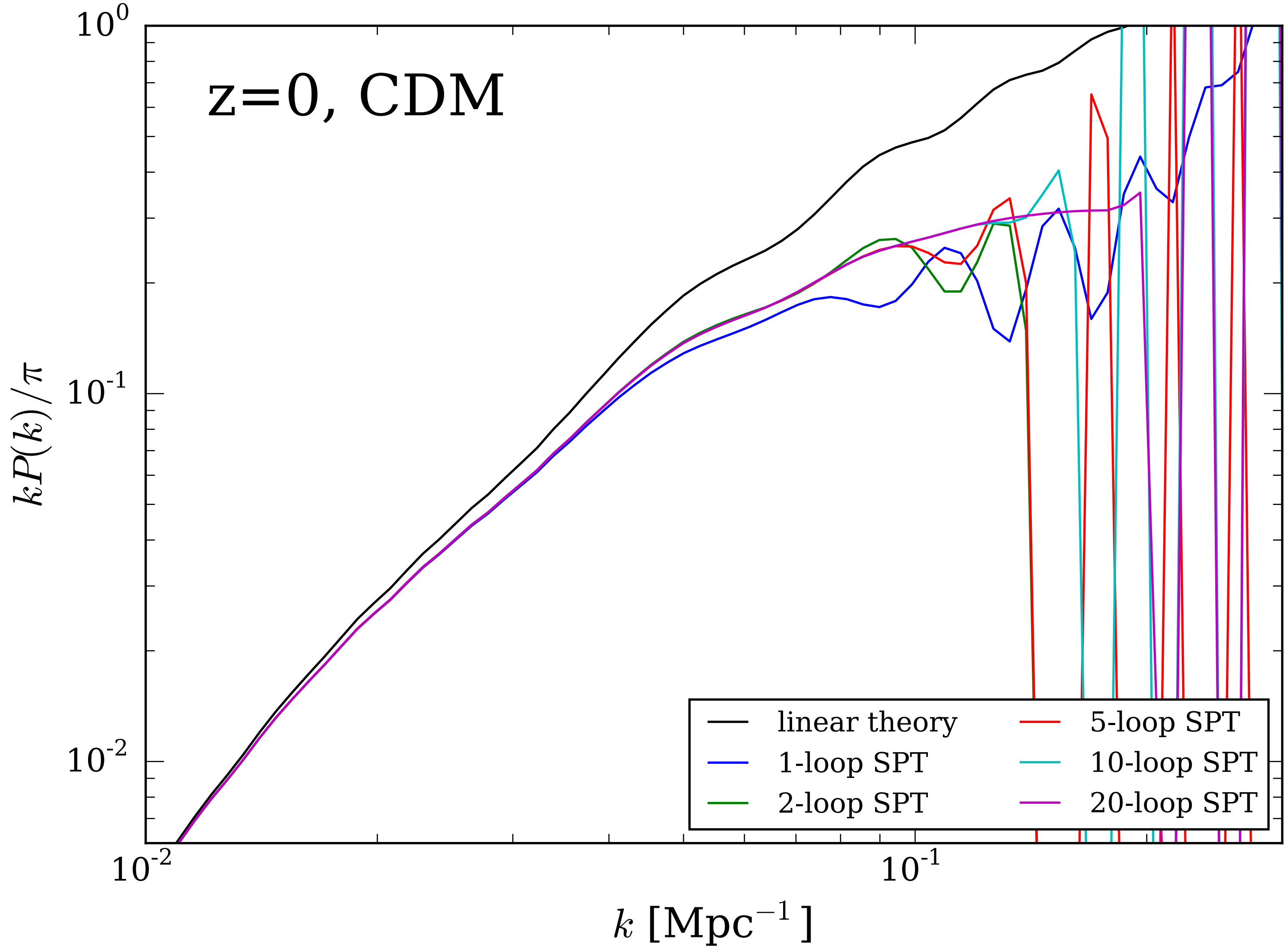
Convergence of PT



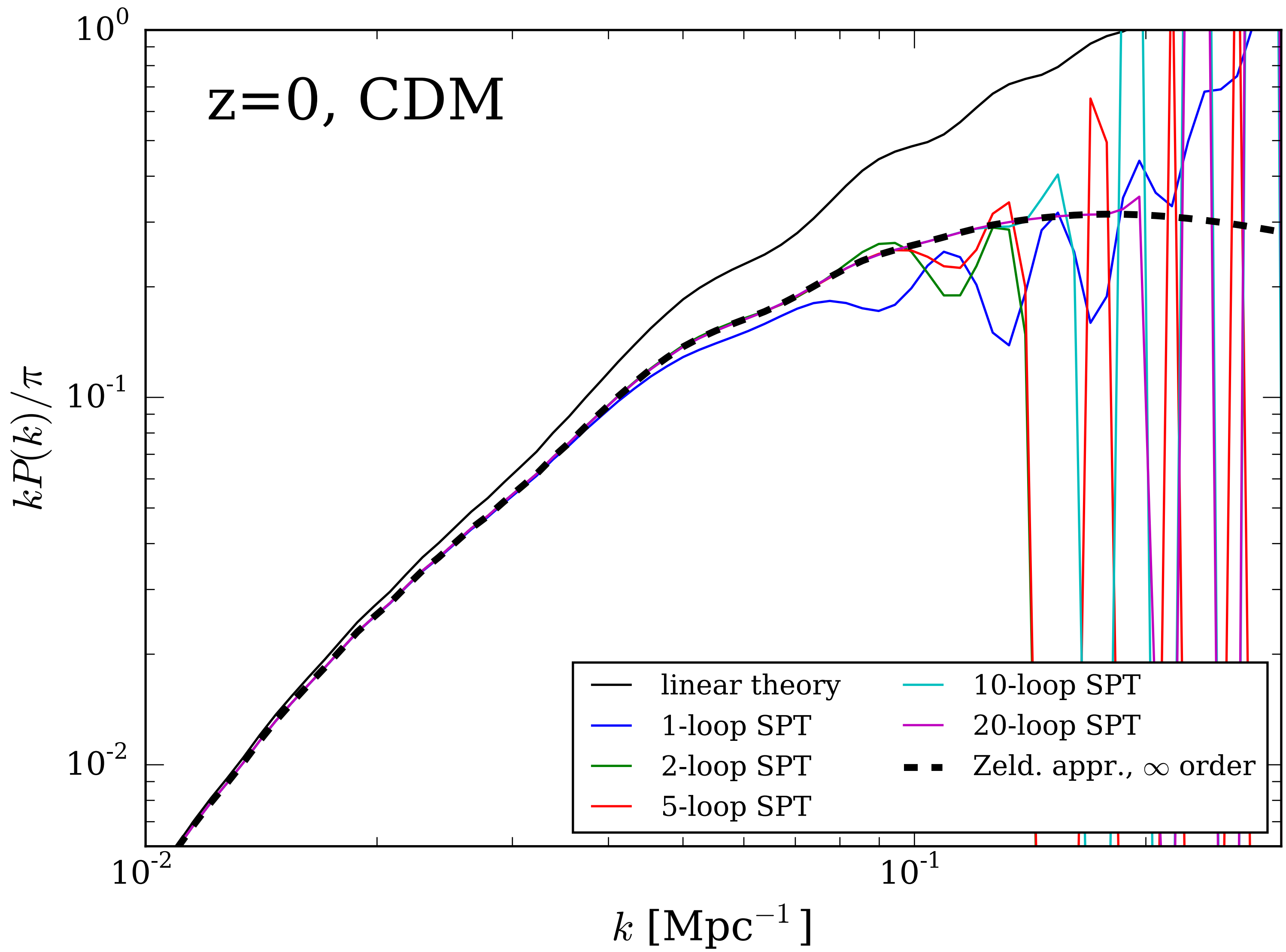
Convergence of PT



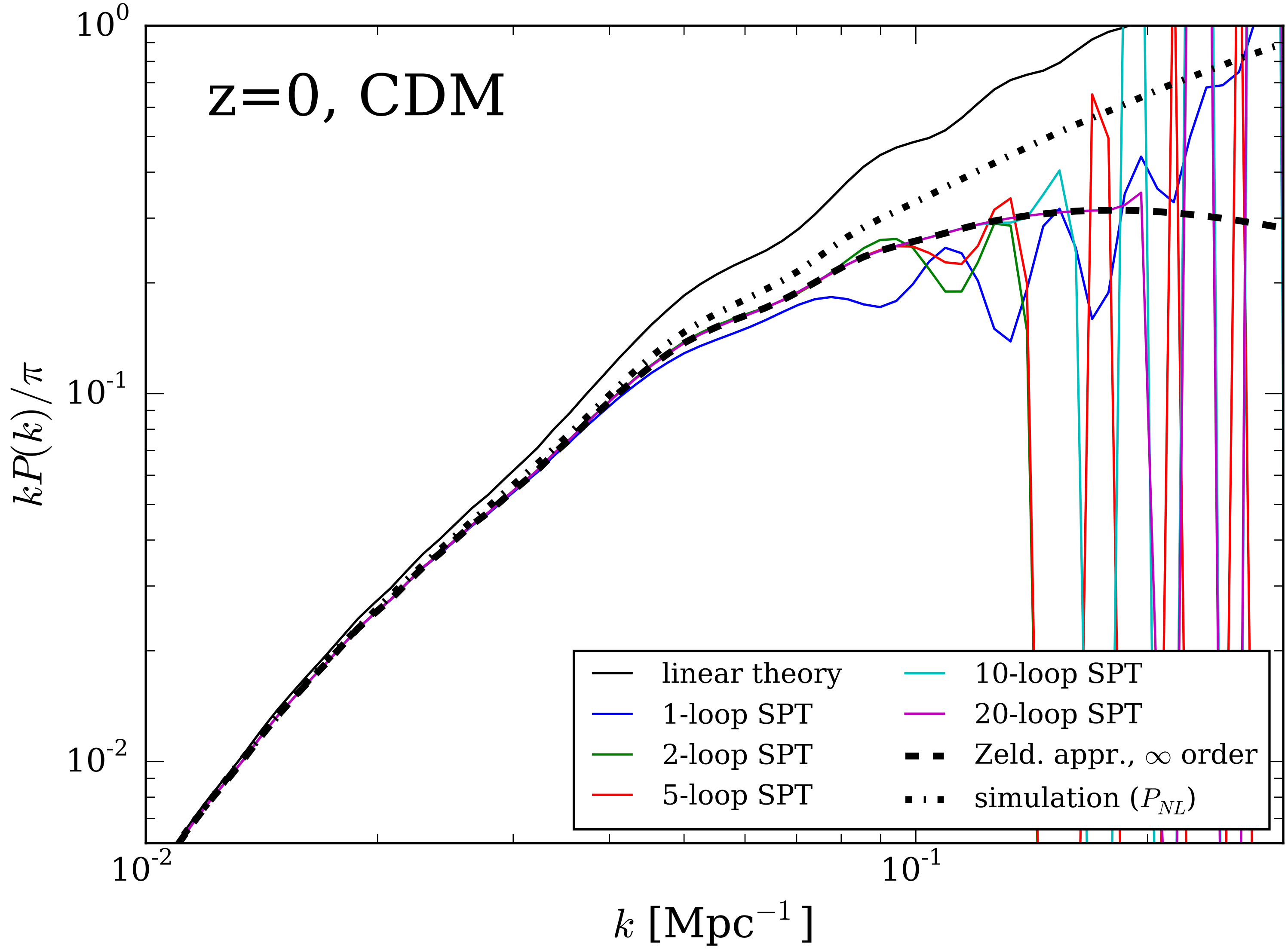
Convergence of PT



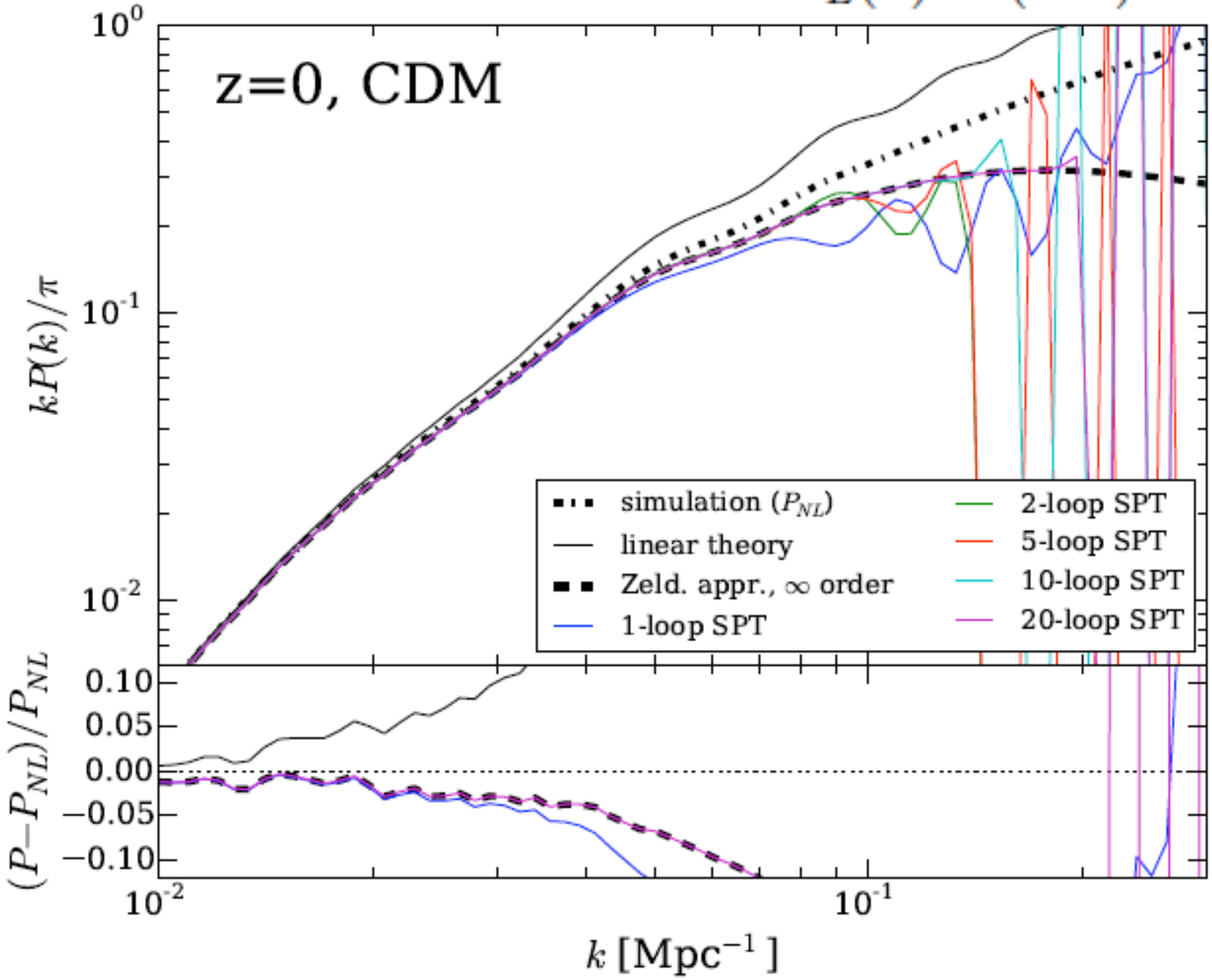
Convergence of PT



Convergence of PT



$$\pi^{-1} k P_L(k) = (2\pi^2)^{-1} k^3 P_{\text{CDM}}(k; z)$$



For 1D CDM-like cosmology, standard perturbation theories do not describe evolution on any non-linear scale accurately.

Some other results & future directions

- We have movies of the evolution of $\delta(x)$ which can be helpful for gaining intuition.
- Proof that SPT and LPT solutions identical to all orders, even though describe different physical systems!
- Easier to see the breaking of Galilean invariance that occurs in some schemes.
- Can formulate and test "effective field theory of large-scale structure" or "coarse grained perturbation theory".
  - Correct and clarify some things in literature, give simpler derivations of some results.
  - A new Lagrangian EFT.
  - For power-law power spectra these theories get a lot of stuff "right". Still some issues though …
- Want to study bias/halos and redshift-space distortions.

The End