Cosmology in 1 dimension

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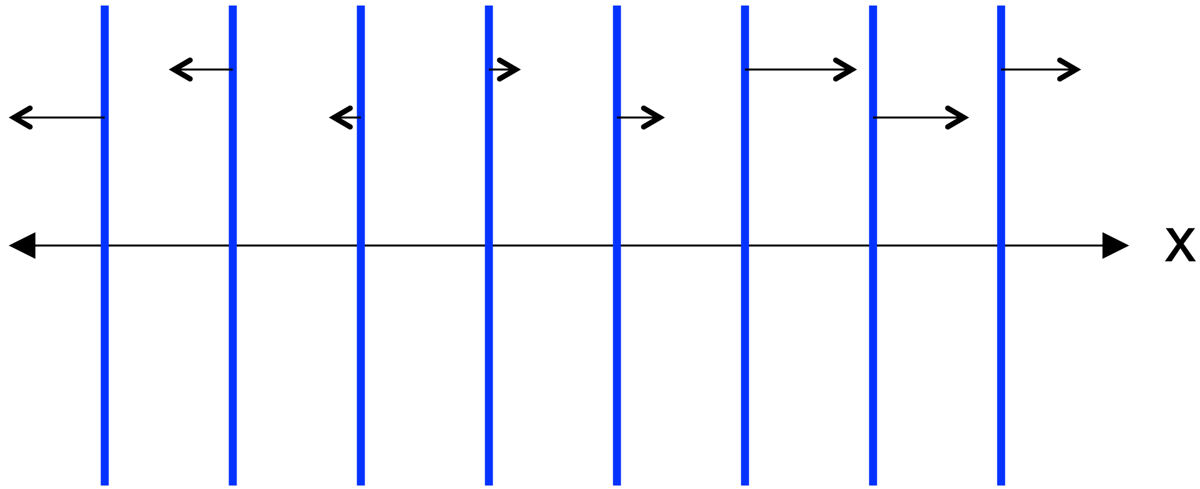
"A man grows stale if he works all the time on insoluble problems, and a trip to the beautiful world of one dimension will refresh his imagination better than a dose of LSD."
-- Freeman Dyson

Motivation

- There has been a great deal of work recently on (cosmological) perturbation theory.
  - New approaches, new resummation schemes, new renormalization techniques borrowed from QFT.
  - Growing appreciation of the uses and limitations of “standard perturbation theory” (SPT) and resummation schemes.
  - Understanding of RSD, BAO, SSC, beat-coupling, …
- Want to understand these developments (and old ideas) better in a simple context:
- Collection of uniform, parallel, 2D sheets of matter.
  - Problem becomes 1 dimensional (plus time).
  - Significant (!) analytic simplification: can do SPT to ∞ order.
  - Easier to handle numerically with high dynamic range.
  - Many of the features of 3D have close 1D analogues.

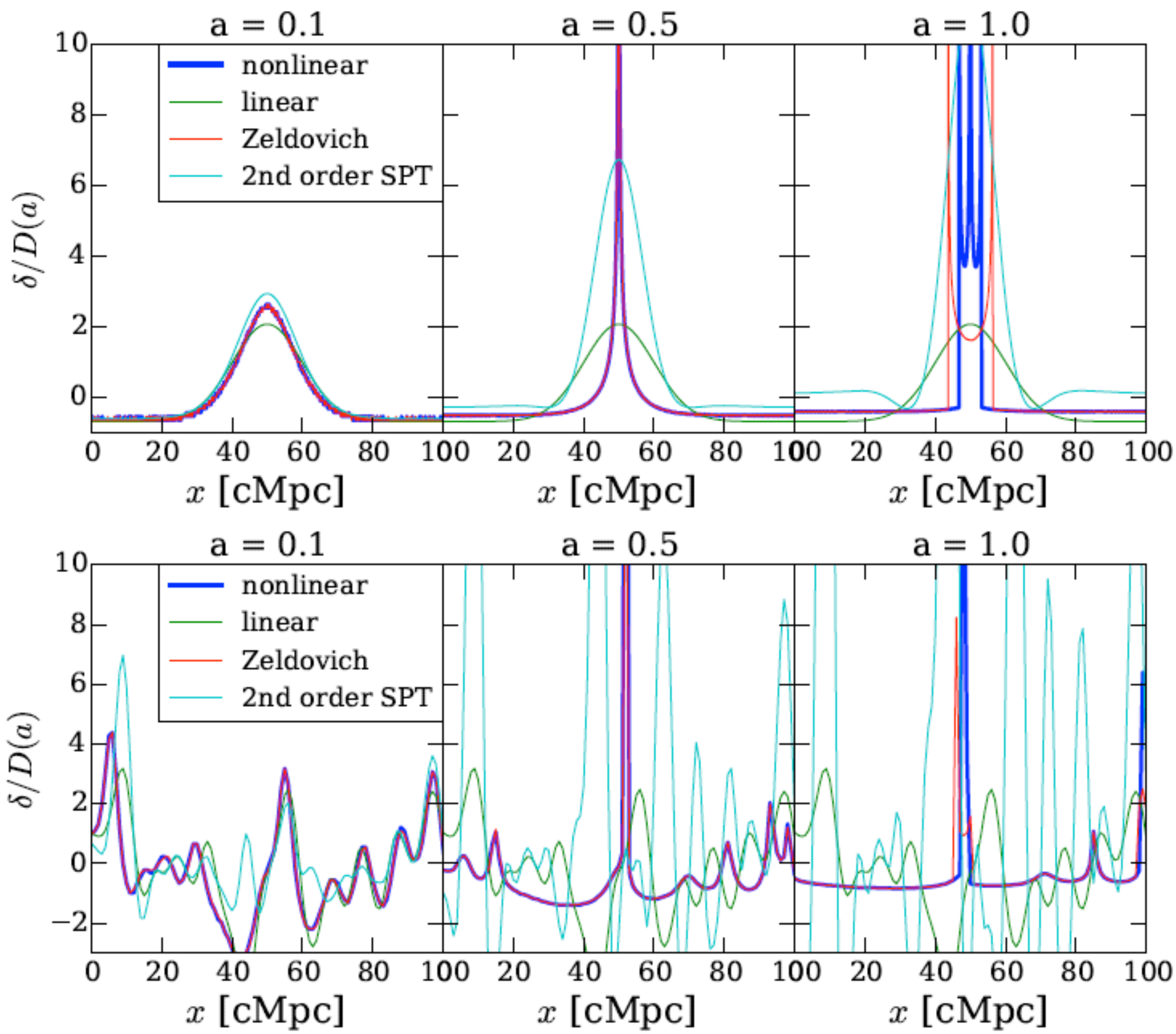
The setup
(with Ω_m=1 throughout)



$$P_{3D}(k_{||},\mathbf{k}_\perp) = (2\pi)^2\delta^{(D)}(\mathbf{k}_\perp)P_{1D}(k_{||})$$

Since the force on a particle due to a sheet is independent of the distance from the sheet, 1st order Lagrangian PT (Zeldovich) is exact until "sheet crossing" (can also show this analytically).

Evolution


a = 0.1
a = 0.5
a = 1.0
nonlinear
linear
Zeldovich
2nd order SPT
$\delta/D(a)$
x [cMpc]

Equations of Motion

- Eulerian

$$\begin{aligned} \partial_\tau \delta + \theta &= -\partial_x(\delta u) \\ \partial_\tau \theta + \mathcal{H}\theta + 4\pi G a^2 \bar{\rho}\delta &= -\partial_x(u \partial_x u) \end{aligned}$$

$$\delta^{(n)} \sim \int F_n \delta_L(k_1) \cdots \delta_L(k_n)$$

Quadratic non-linear terms generate recurrence relation for F_n …

- Lagrangian

$$\ddot{\Psi}(q) + 2H\dot{\Psi}(q) = -\partial_x \Phi(q + \Psi)$$

$$\Psi^{(n)} \sim \int L_n \delta_L(k_1) \cdots \delta_L(k_n)$$

$x=q+\Psi$

In 1D can show SPT and LPT solutions are identical, to all orders, even though the systems are different!

Power spectra and correlation functions

$$P_{\rm SPT}^{1-{\rm loop}}(k) = P_L(k) + \int_{-\infty}^{\infty} \frac{dk'}{2\pi} \left\{ 3 + 4\frac{k-k'}{k'} + \left(\frac{k-k'}{k'}\right)^2 \right\} P_L(k')P_L(k-k') - k^2 P_L(k) \int_{-\infty}^{\infty} \frac{dk'}{2\pi} \frac{P_L(k')}{k'^2}$$

$$\xi_{\rm SPT}^{1-{\rm loop}}(x) = \xi_L(x) + \overbrace{3\xi_L^2(x)}^{\rm growth} + \overbrace{4\xi'_L(x) \int_x^{\infty} dx\, \xi_L(x)}^{\rm dilation} + \overbrace{\frac{\sigma_{\rm eff}^2}{2}\xi''_L(x)}^{\rm RMS\ \ displ.} + \mathcal{O}(\xi_L^3)$$

Can look at the response of power spectra to long wavelength mode (through the gradient times the large-scale variance) and the modes themselves (super-sample covariance, beat coupling), shifts and broadening of the BAO peak, ...

Broadening of the BAO peak

- By far the dominant term (in 1D and in 3D) is the σ^2 term, which broadens the BAO peak.
- Recall near the peak, $\xi \sim 10^{-3}$, $\sigma \sim 10$Mpc, $[\xi'']^{-1/2} \sim 10$Mpc.
- Thus the ξ^2 and $\xi\xi'$ terms are small, but $\sigma^2\xi''$ is *O(*1*)*.
  - Because Lagrangian theories sum this important term to higher orders, they tend to do better near the BAO peak.
  - The situation in P(k) space is more complicated.

Shifting the BAO peak

- The "dilation" term causes a shift of the BAO peak.
  - $\xi(x[1+\alpha]) \sim \xi(x) + \alpha\, x\xi'(x) + \ldots$
- In overdense regions, the large-scale overdensity acts like a locally closed Universe remapping *r* to smaller scales.
- Since there is more growth in overdense regions than underdense ones, this leads to a net shift.
- A "separate Universe" approach can predict the coefficient of this term properly in 1D as well as in 3D.

Beat coupling/SSC

How does a high k mode respond to a long-wavelength over- or under-density? In the limit that the long wavelength is the size of the survey (or larger) this is known as a “super sample” mode.

$$
\begin{aligned}
\delta^{(2)}(k) &= \int \frac{dk'}{2\pi}\, F_2(k', k-k')\delta_L(k')\delta_L(k-k') \\
&\ni 2F_2(0,k)\,\delta_L(k) \int_{-\epsilon}^{+\epsilon} \frac{dk'}{2\pi}\delta_L(k') \\
&\simeq 2\delta_V\,\delta_L(k) \\
\Rightarrow\ \delta(k) &\simeq [1+2\delta_V]\,\delta_L(k)
\end{aligned}
$$

c.f. $[1+(34/21)\delta_V]$ in 3D.

PT issues

- Convergence rather slow.
  - Resummation schemes.
  - Beware symmetry breaking!
- Solutions only valid prior to “sheet” crossing.
- $P^{\text{1-loop}}(k)$ depends on high-k’, non-perturbative modes even at large scales.

Lagrangian theory

$$1 + \delta_{\rm LPT}(x) = \int dq\, \delta^D[x - q - \Psi(q)]$$

$$\delta_{\rm LPT}(k) = \int dq\, e^{-ikq} \left(e^{-ik\Psi(q)} - 1\right)$$

But Ψ is just a Gaussian random variable … know $\langle e^{\Psi} \rangle$

$$P_{\rm ZA}(k) = \int dq\, e^{-ik\,q} \left(e^{-k^2\sigma^2(q)/2} - 1\right)$$

$$\sigma^2(q) = \langle [\Psi_{\rm ZA}(0) - \Psi_{\rm ZA}(q)]^2 \rangle = \int_0^\infty \frac{dk}{\pi} \frac{2\,P_L(k)}{k^2} \left(1 - \cos[k\,q]\right)$$

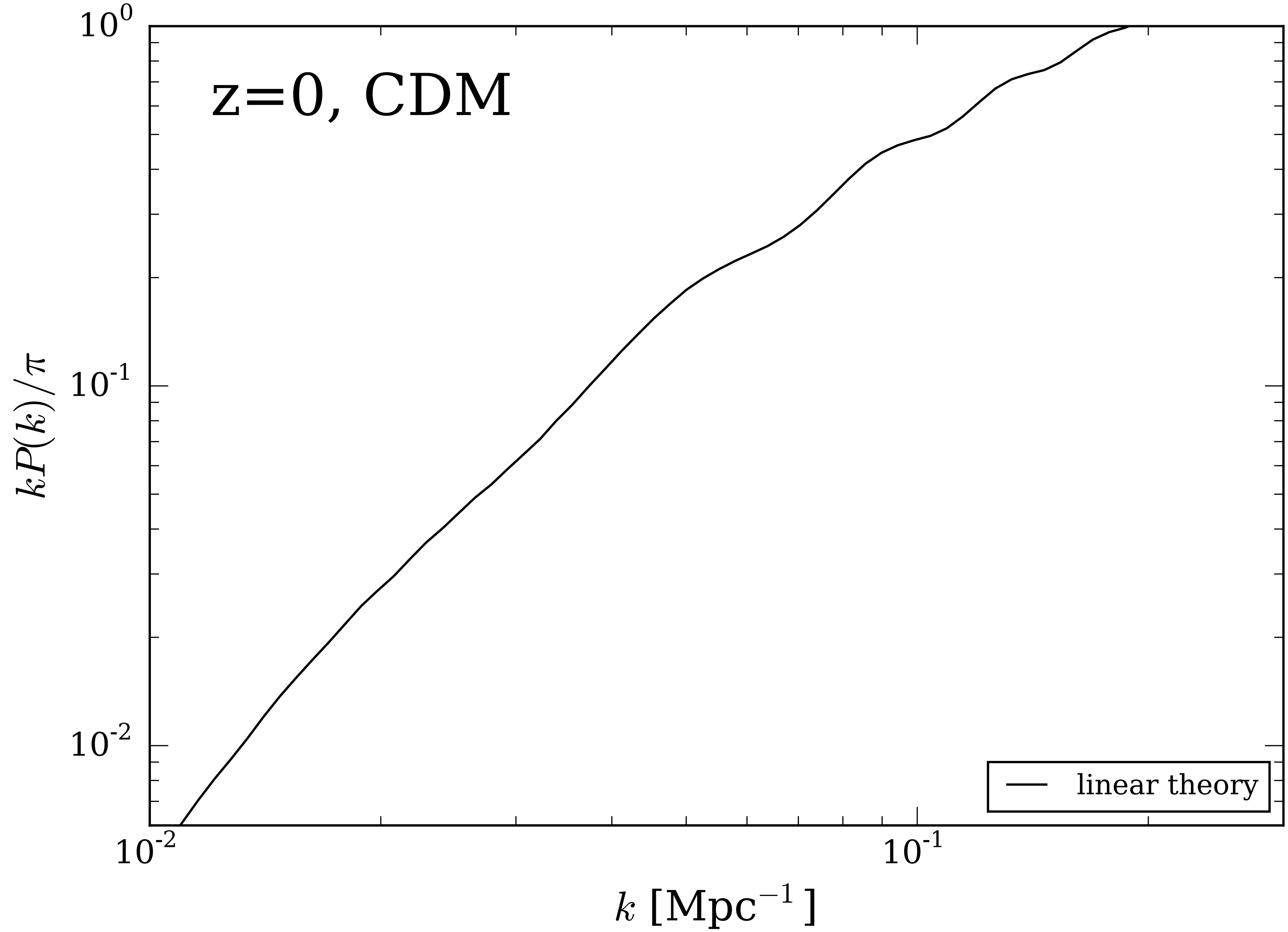
Can generate any order in PT!

$$\begin{aligned}
P_{\rm LPT}(k) &= \int dq\, e^{ik\,q}\left(-\frac{k^2}{2}\sigma^2(q)+\frac{k^4}{8}\sigma^4(q)+\cdots\right), \\
&= P_L+\frac{1}{8}\int dq\, e^{ik\,q}\,\nabla_q^4\,\sigma^4(q)+\mathcal{O}(P_L^3), \\
&= P_L+\frac{1}{8}\int dq\;e^{ik\,q}\left[6([\sigma^2]'')^2+8([\sigma^2]'[\sigma^2]''')+2[\sigma^2][\sigma^2]''''\right]+\mathcal{O}(P_L^3), \\
&= P_L+\int\frac{dk'}{2\pi}\left\{3+4\frac{k-k'}{k'}+\frac{(k-k')^2}{k'^2}\right\}P_L(k')P_L(k-k')+\cdots\,,
\end{aligned}$$

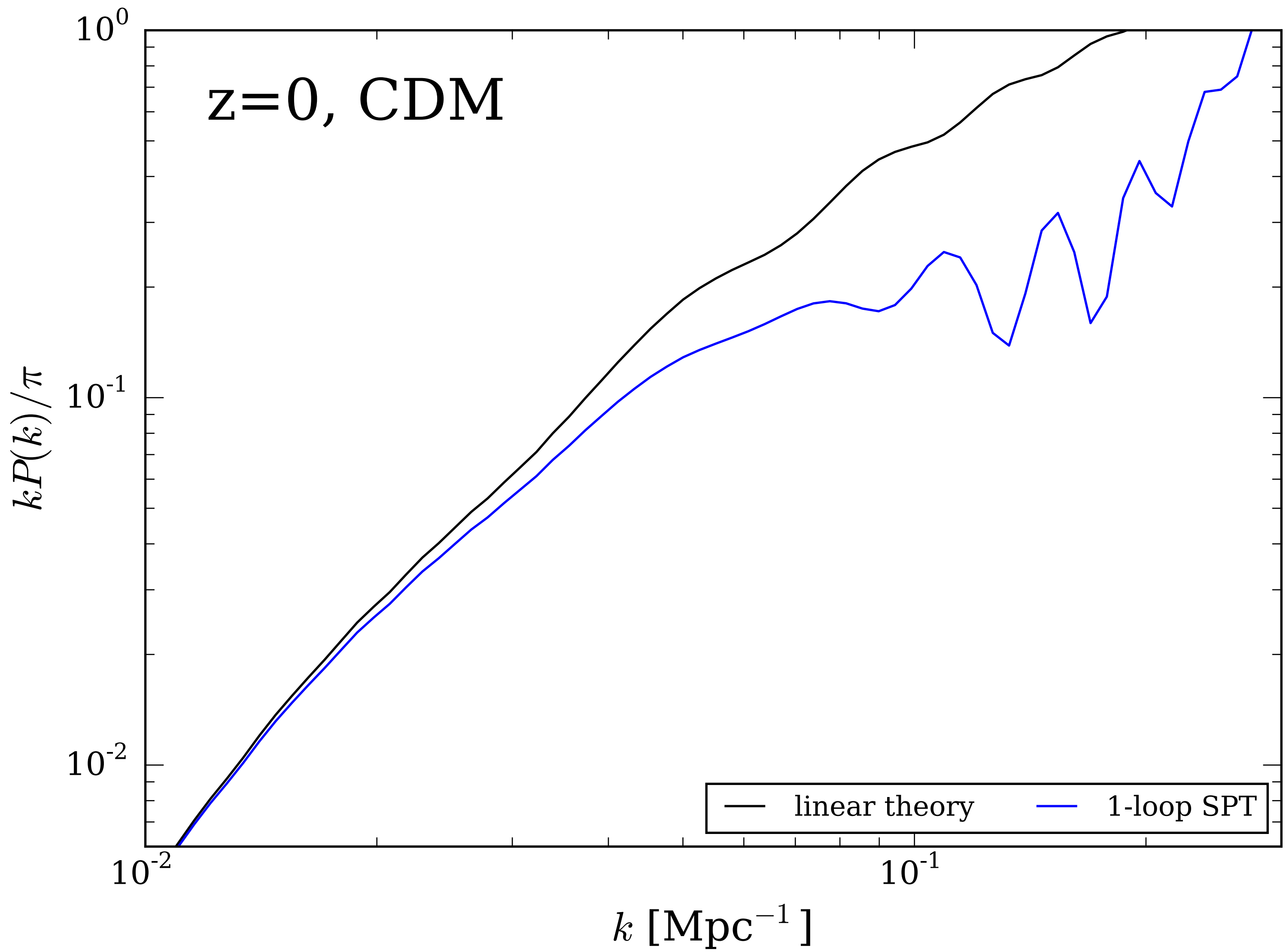
$$\boxed{F_n(k_1,\cdots,k_n)=G_n(k_1,\cdots,k_n)=\frac{1}{n!}\;\frac{k^n}{\prod_{i=1}^n k_i}}$$

Now we can understand common resummation schemes in “standard” perturbation theory, and we can look at the rate of convergence of perturbation theory.

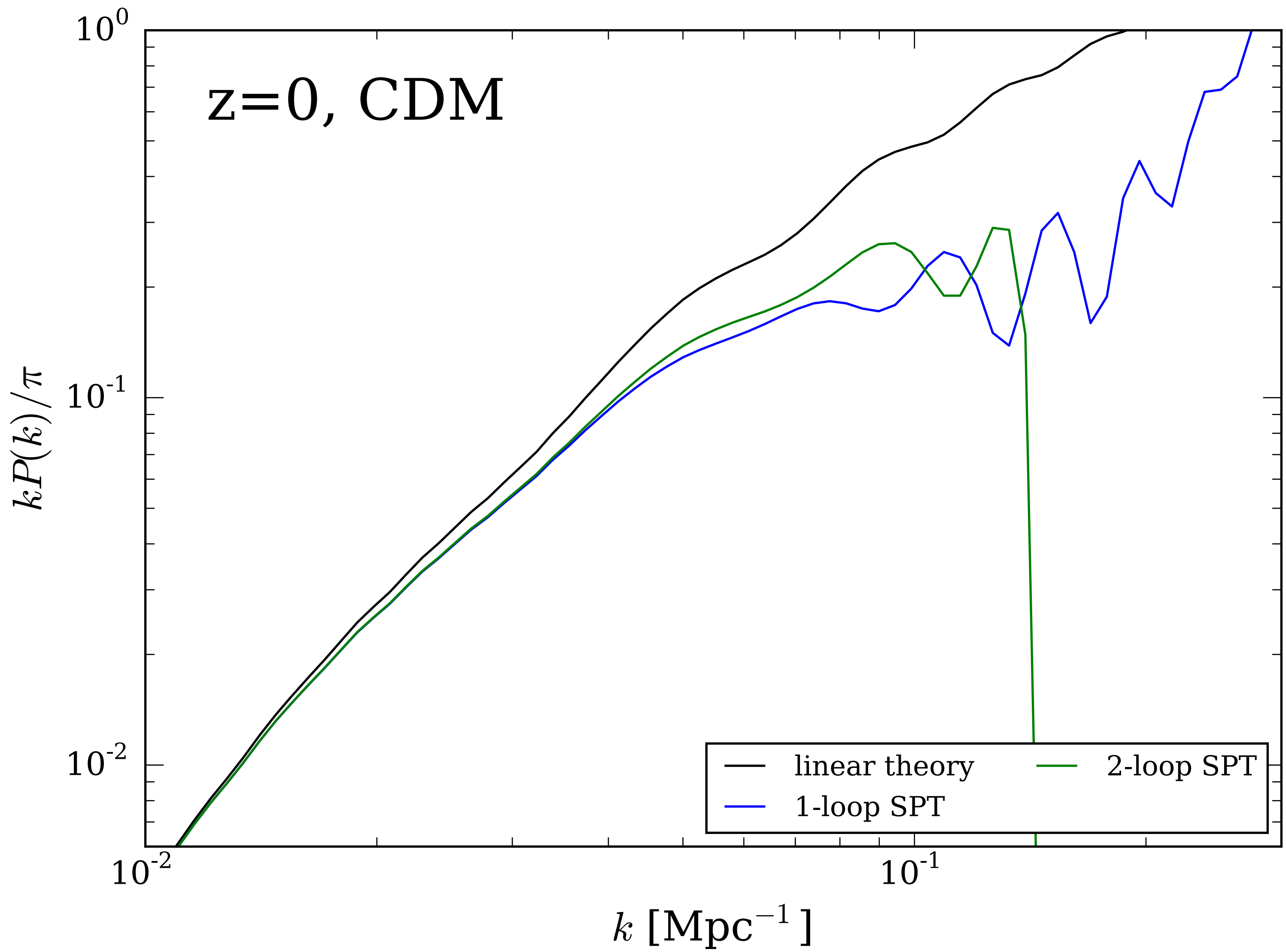
Convergence of PT



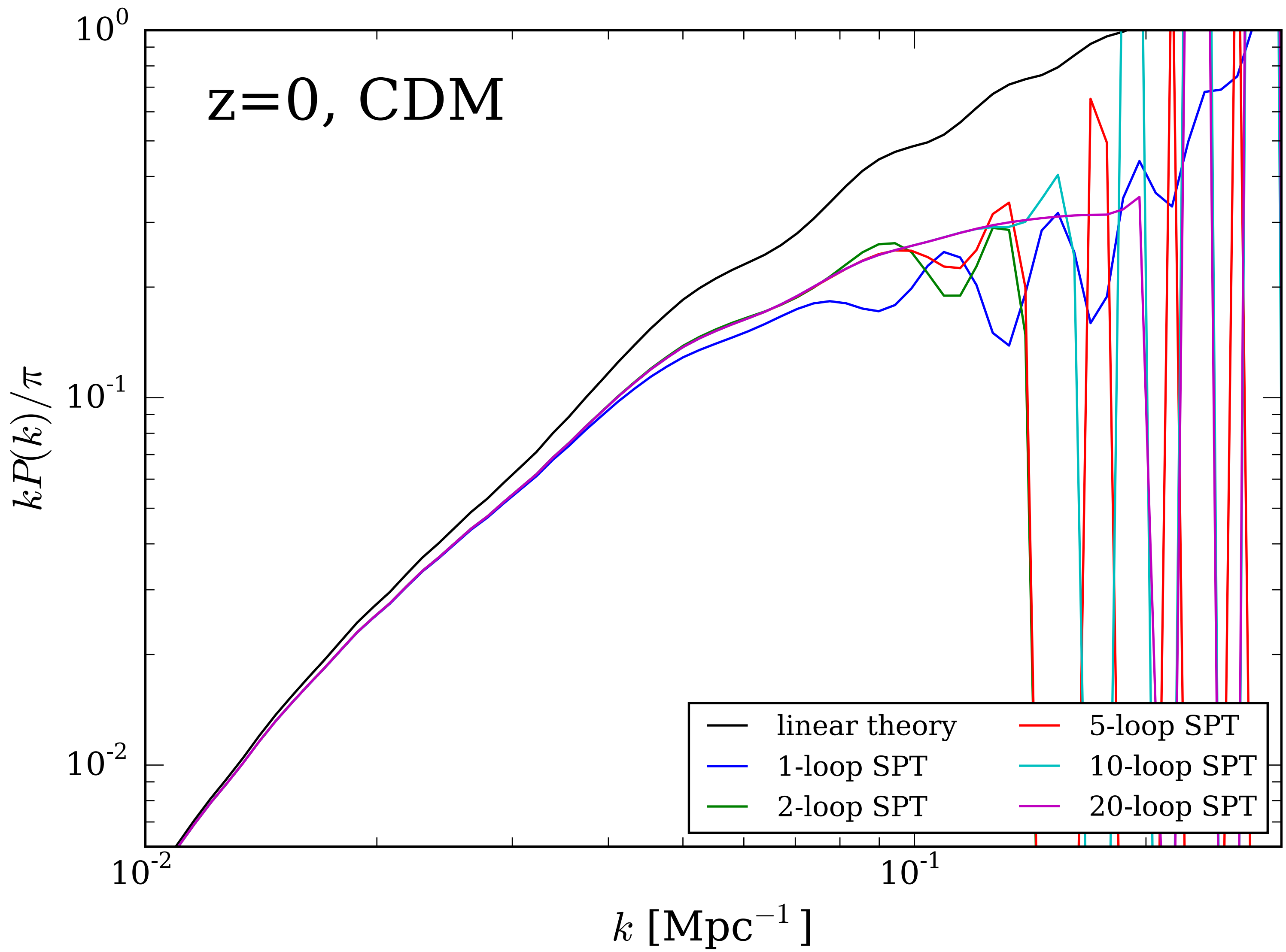
Convergence of PT



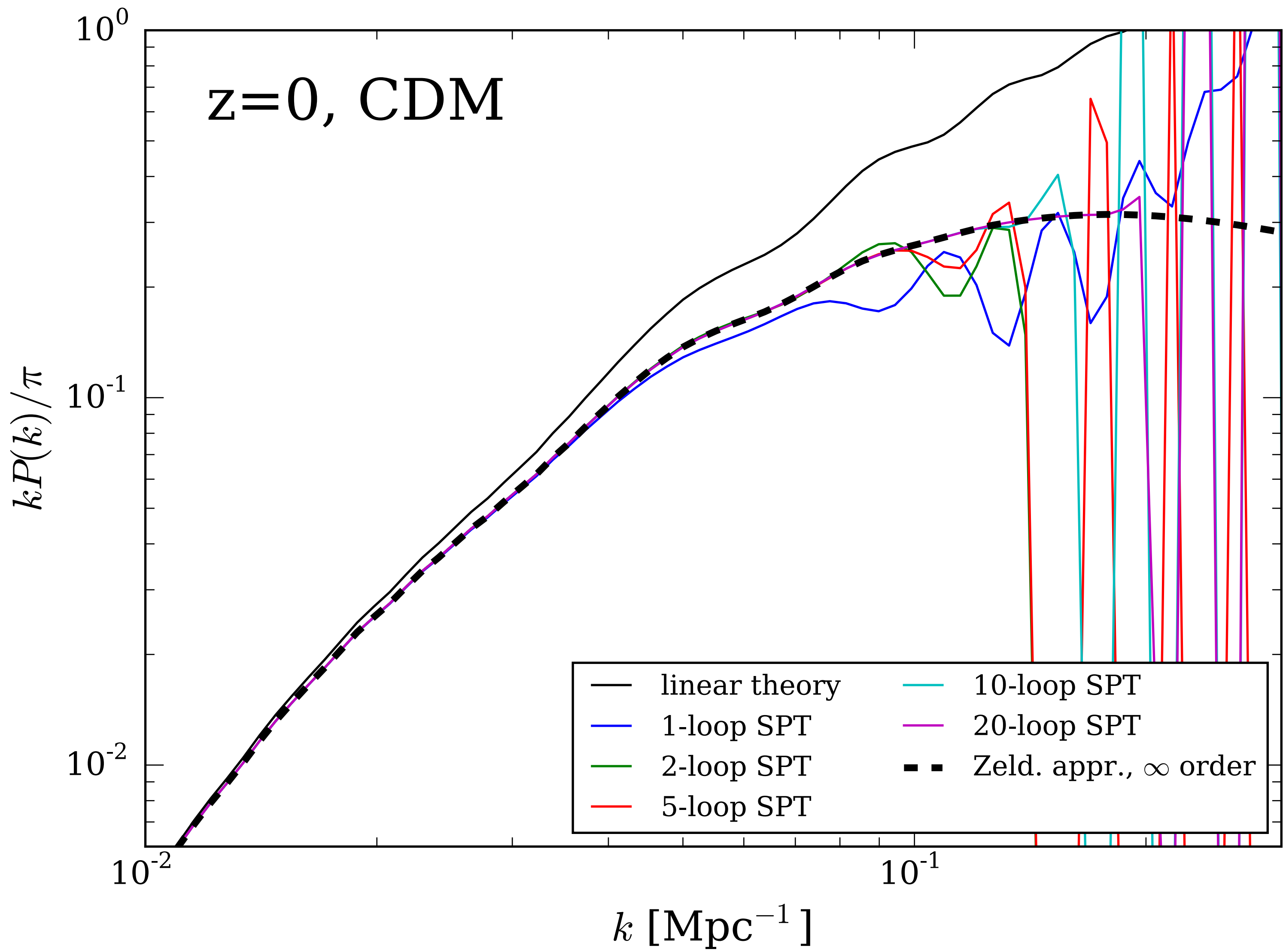
Convergence of PT



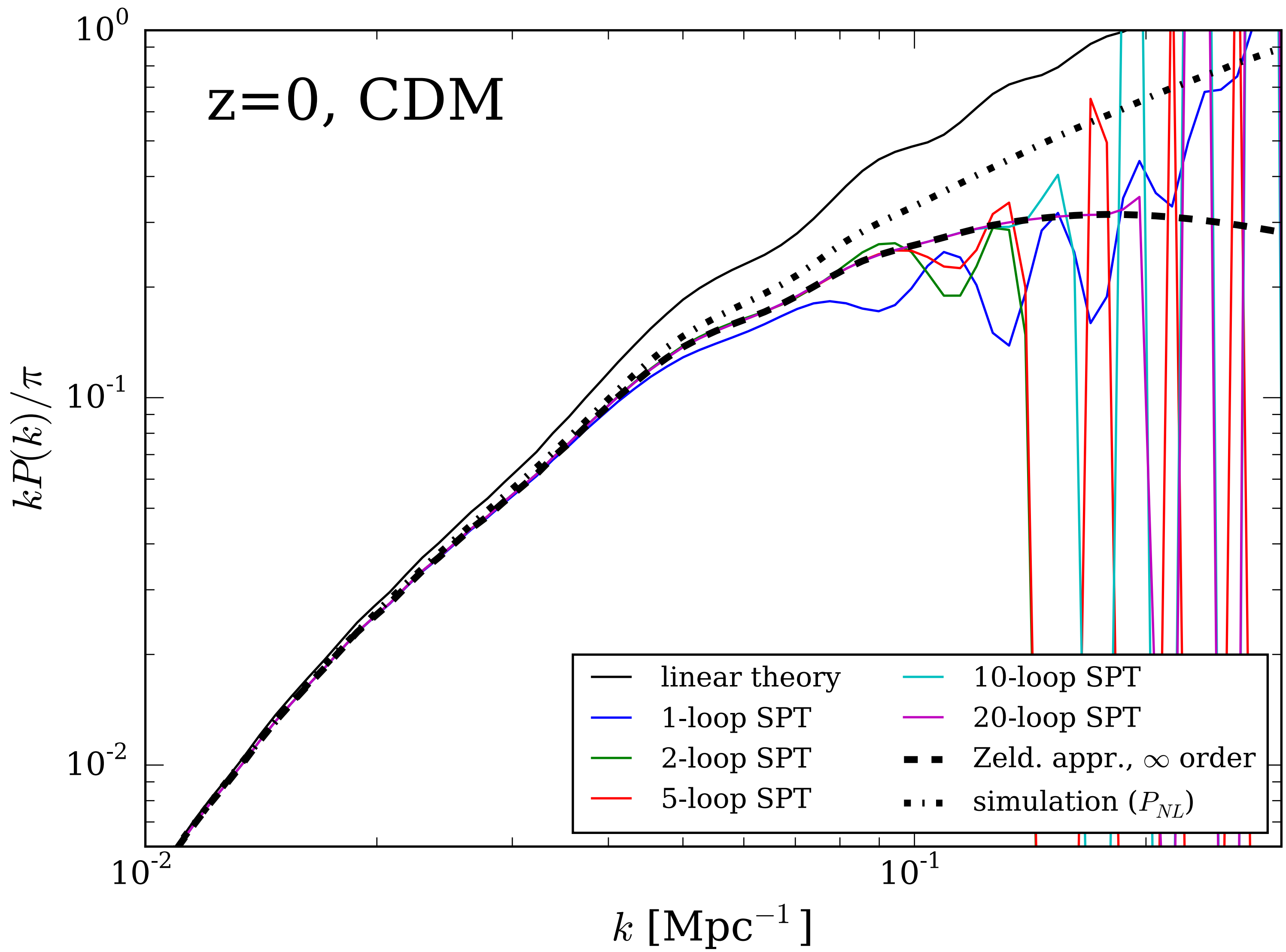
Convergence of PT



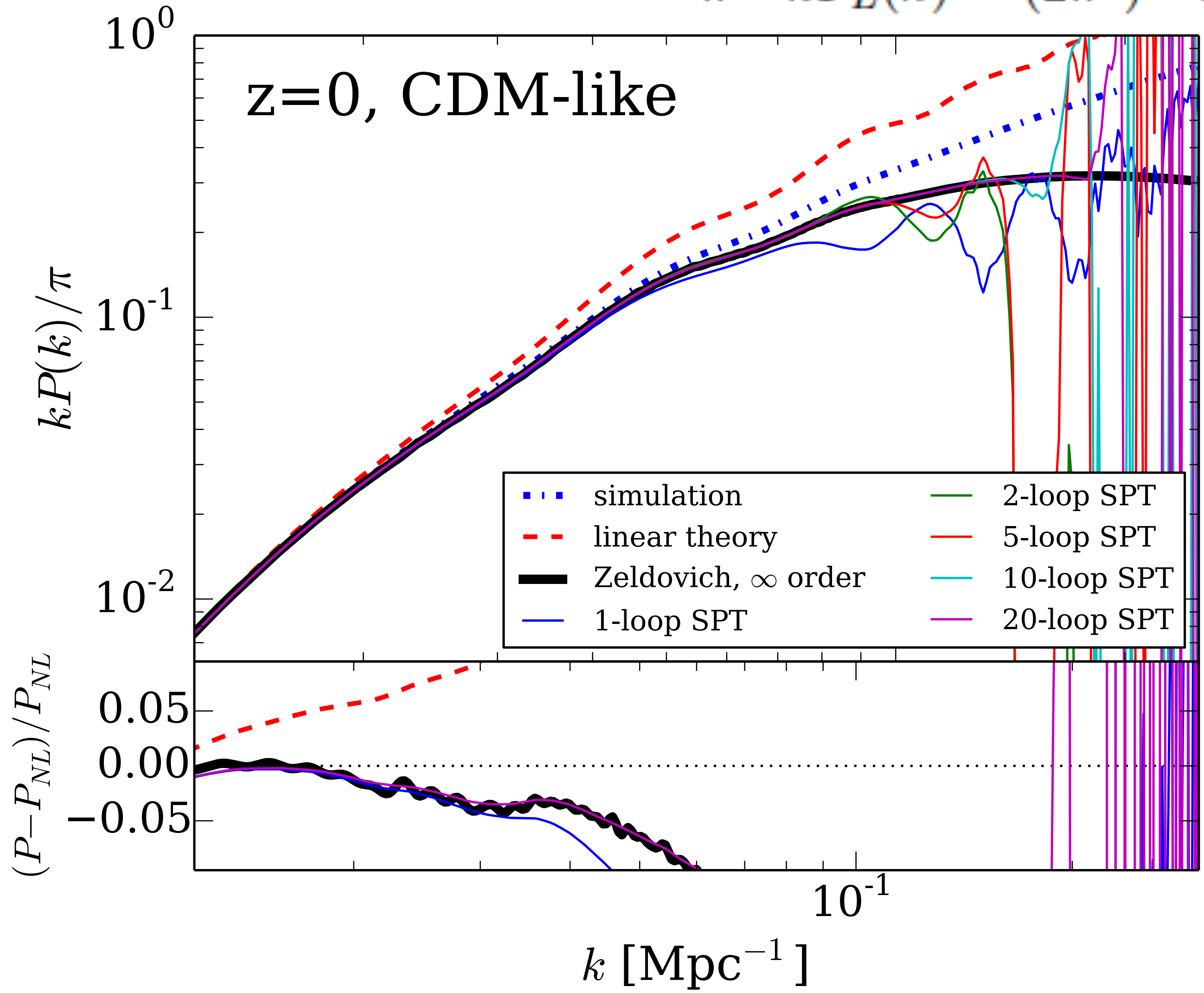
Convergence of PT



Convergence of PT



$$\pi^{-1} k P_L(k) = (2\pi^2)^{-1} k^3 P_{\mathrm{CDM}}(k; z)$$



For 1D CDM-like cosmology, standard perturbation theories do not describe evolution on any non-linear scale accurately.

Some other results

- Proof that SPT and LPT solutions identical to all orders, even though describe different physical systems!
- Easier to see the breaking of Galilean invariance that occurs in some schemes.
- Can formulate and test “effective field theory of large-scale structure” or “coarse grained perturbation theory”.
  - Correct and clarify some things in literature, give simpler derivations of some results.
  - A new pseudo-Lagrangian EFT.
  - For power-law power spectra these theories get a lot of stuff “right”. Still some issues though …

Effective field theory

Traditional perturbation theory treats all scales as if they were perturbative, and the matter field as a perfect fluid. The goal of "EFT" is to overcome these deficiencies.

- "Effective" field theory has a long history in other areas of physics.
  - But cosmology presents some unique features, so beware misleading analogies!
- Basic idea is to write equations only in terms of long-wavelength fields, with no small-scale terms explicitly involved (they've been "integrated out").
- The effects of these small-scale terms then show up as additional terms in effective equations of motion.

Trivial example: continuity

$$X_\Lambda \equiv \int dx\ W_\Lambda(x-x')X(x')$$

$$\mathrm{Velocity} = \frac{\pi_l}{\rho_l} = \frac{[(1+\delta)u]_\Lambda}{1+\delta_l} = u_l + [u(\delta-\delta_l)]_\Lambda + \cdots$$

$$\partial_\tau \delta_l + \theta_l = -\nabla\left(\delta_l u_l\right) - \bar{\rho}\nabla\left[u(\delta-\delta_l)\right]_\Lambda$$

And you can derive a similar set of “rhs terms” for the Euler equation if you start with the Poisson and Boltzmann equations and smooth them.
Terms on the rhs must obey the symmetries of the system, and this allows another route …

Procedure?

- We need to make some approximations in order to make progress …
- Combine the Euler and continuity equations into a 2nd order DE with all non-linear terms on the rhs.

$$\mathcal{D}^{(2)}_{\rm lin}\delta_l = \left(a\mathcal{H}\partial_a + \mathcal{H}\right)\nabla\left(\delta_l u_l\right) - \nabla\left(u_l \nabla u_l\right) - \bar{\rho}^{-1}\nabla^2 X_\Lambda$$

- Assume we can expand the "extra" terms on the rhs simultaneously in derivatives (powers of *k*) and powers of δ_l with unknown coefficients.
- Then integrate the source terms against the Green's function.

What terms are allowed?

- Working to lowest order, X must go as $c_1 J + c_2 k^2 \delta_l$ where J is uncorrelated with δ_l.
  - By mass and momentum conservation, the leading order expansion of J must be k^2.
- At 1-loop we simply integrate against *G(a,a')*, which gives the normal PT terms and just modifies c_i for the "extra" terms.
  - $\delta = \delta^{(1)} + \delta^{(2)} + \delta^{(3)} + c_1'J + c_2'k^2\delta^{(1)} + \ldots$
- Thus the power spectrum must look like:

$$P^{1-\mathrm{loop}}_{\mathrm{EFTLSS}}(k) = P_{11} + P_{22} + P_{13} + 2\alpha k^2 P_{11} + P_J$$

- Where α can be fit for and P_J goes as k^4 for small k, otherwise unknown (usually dropped for all k).

Limits and counter-terms

- It is easy to show that the k^2P and P_J terms asymptotically cancel any cutoff dependence in the theory.
  - The c_i are functions of Λ.
- Even SPT generates a term that goes asymptotically as k^2P, so it's reasonable to have!
- However, for reasonable k and Λ the final answers depend on Λ if we keep only 1-loop!
- Typically take the limit Λ goes to ∞.
  - This is the limit we were trying to avoid, but hope that EFTLSS is less sensitive than SPT.

Power-law cases

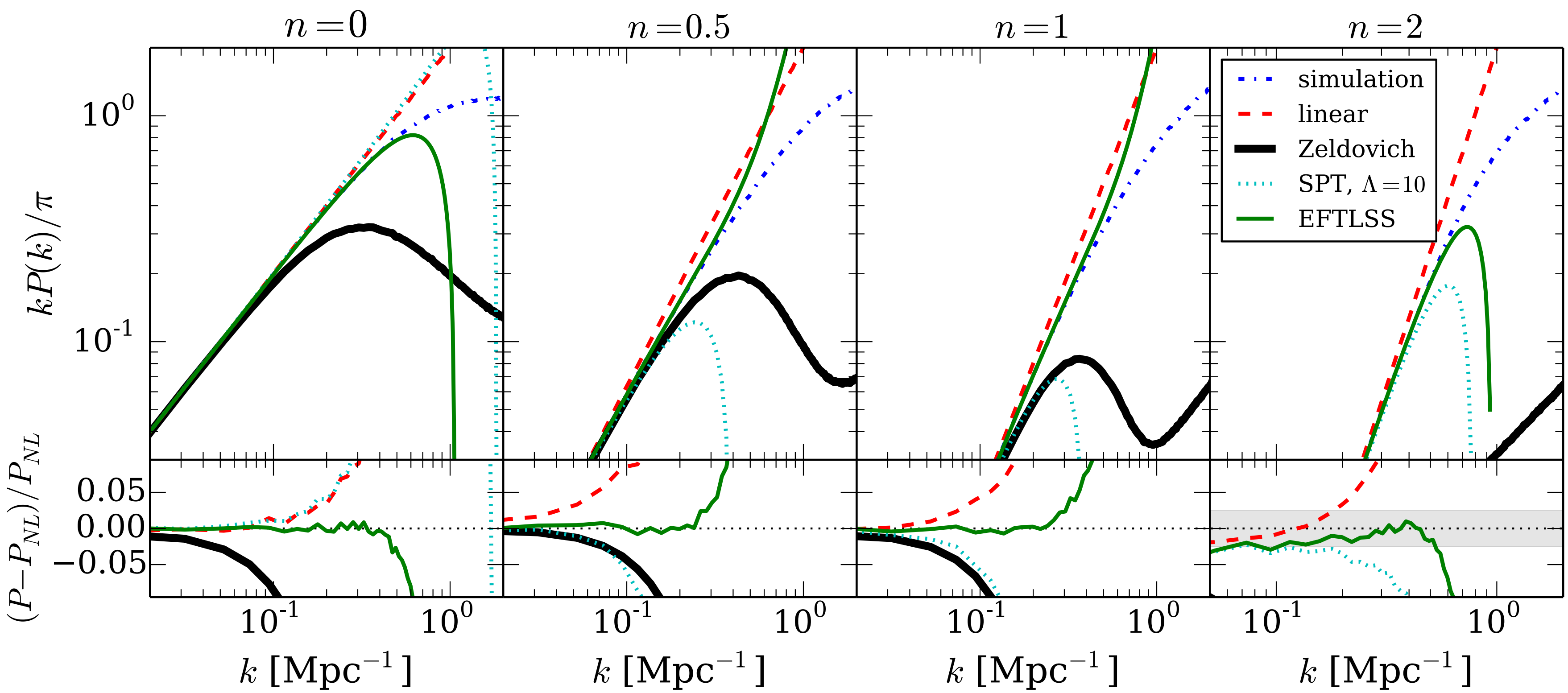
Models with an Einstein-de Sitter cosmology and a power-law power spectrum exhibit self-similar evolution. The structure of the power spectrum is easy to see, and the way the counterterms enter is much simplified.

Here are the expressions for a hard *k* cut:

$$P^{1-\text{loop}}_{\text{EFTLSS}}(k) = \begin{cases} 2\pi + 4\pi k \left(\log \frac{\Lambda}{\Lambda - k} - 2\pi k^3 \frac{1}{\Lambda(\Lambda-k)}\right) + 4\pi k^2 \alpha_{c,\Lambda} + P_{J,\Lambda} & \text{if } n = 0, \\ \boxed{2\pi k^{1/2} - 4\pi k^2 \frac{2\Lambda - k}{\sqrt{\Lambda(\Lambda-k)}} + 2\pi k^{5/2}\left(2\alpha_{c,\Lambda} - \frac{1}{\Lambda} - \frac{1}{\sqrt{\Lambda-k}}\right) + P_{J,\Lambda} \quad \text{if } n = 1/2,} \\ 2\pi k + 2\pi k^3 \left(\log\left[(\frac{k}{\Lambda})^2(1-\frac{k}{\Lambda})\right] + 2\alpha_{c,\Lambda}\right) + P_{J,\Lambda} & \text{if } n = 1, \\ 2\pi k^2 + 2\pi k^4 \left[2\alpha_{c,\Lambda} - \Lambda\right] - \pi k^5 + P_{J,\Lambda} & \text{if } n = 2. \end{cases}$$

While analytically nice, it's unclear how relevant the intuition from these models is to CDM … caveat emptor!

Power-law models



Power-law cosmologies, P(k)~k^n, have self-similar solutions. PT at 1-loop is P(k)~k^n + #k^{2n+1}. EFT adds k^{n+2} and "k^4" terms with free coefficients … these really help!

Lagrangian EFTLSS

- Like SPT, EFTLSS expands displacement in powers of δ.
- Want to avoid this with a Lagrangian scheme.

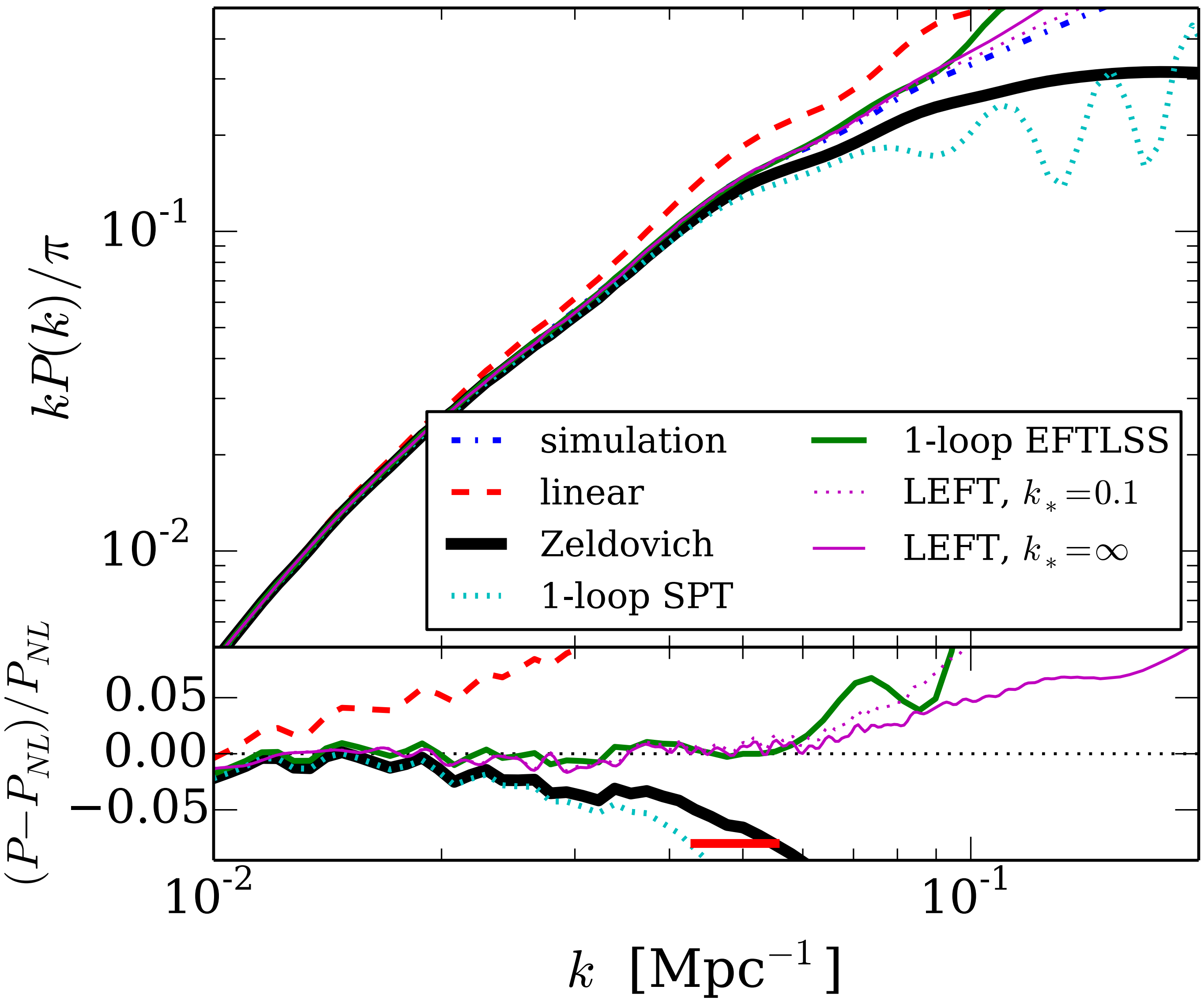
$$\Psi = \Psi_l + \Psi_s \approx \Psi_l + 2\alpha\nabla\delta_l + \nabla J$$

$$P(k) = \int dq\ e^{-ikq}\left[e^{-(1/2)k^2\sigma^2_{\rm eff}} - 1\right]$$

$$\sigma^2_{\rm eff}(q) = \sigma^2(q) + 4\alpha\left[\xi(0) - \xi(q)\right] - 2\nabla^2\xi_J(q)$$

Have choice of keeping terms exponentiated or consistently expanding by order … numerically not much difference. Expression for the correlation function is easy …

Comparison



$kP(k)/\pi$
10^{-1}
10^{-2}
simulation
linear
Zeldovich
1-loop SPT
1-loop EFTLSS
LEFT, $k_*=0.1$
LEFT, $k_*=\infty$
$(P-P_{NL})/P_{NL}$
0.05
0.00
−0.05
10^{-2}
10^{-1}
k [Mpc^{-1}]

Beyond 1-loop

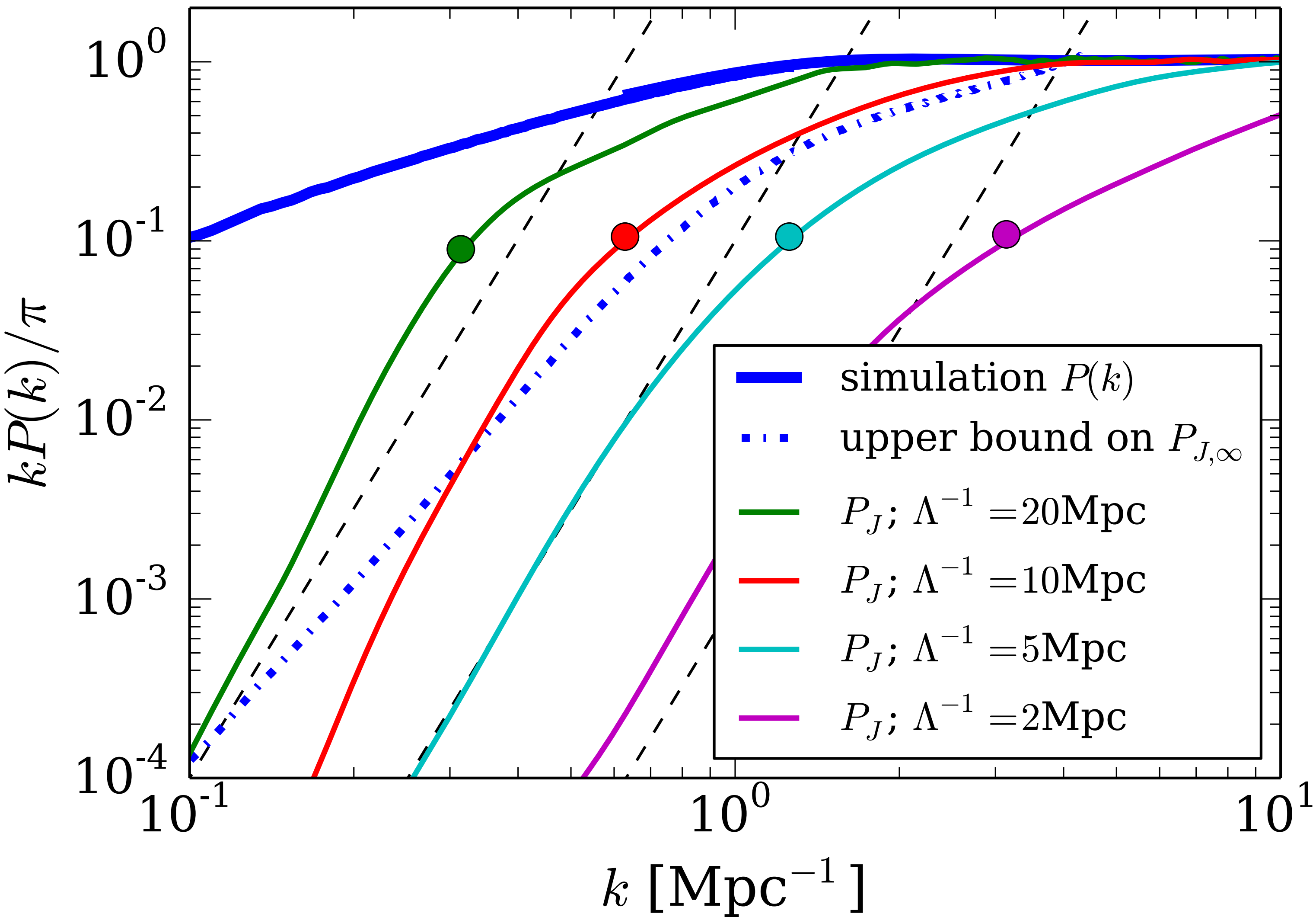
- Going beyond 1-loop is very hard.
- Unlike the 1-loop case, where integrating over *G(a,a')* simply modifies the c_i, terms at 2-loop order have different integrals which affect the *k*-dependence.
- Authors attempting 2-loop calculations need to drop many terms.

Conclusions

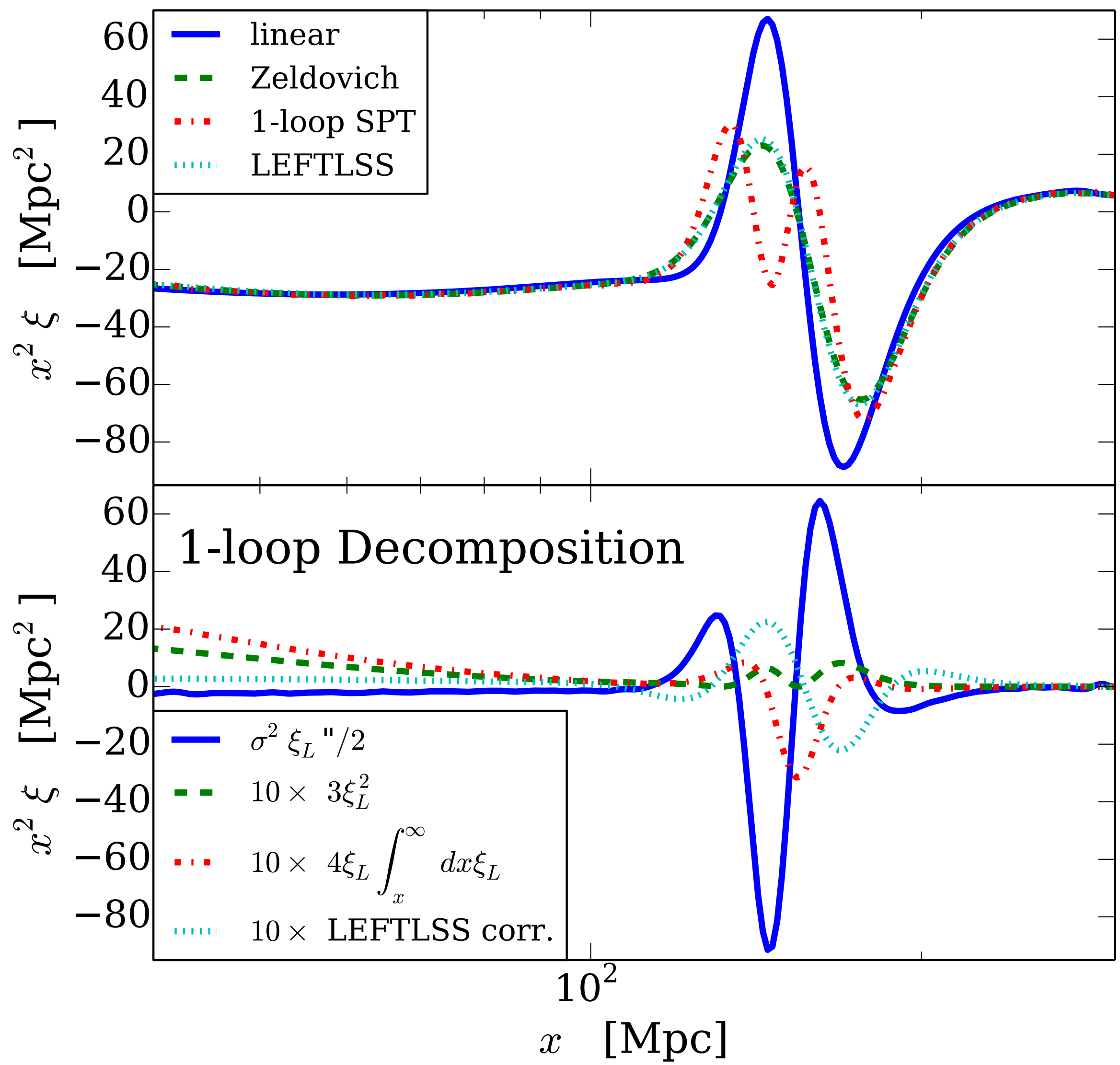
- Cosmological PT in 1D has some nice features.
  - Easy to simulate, easy to calculate.
  - Can do SPT to ∞ order.
  - Algebra for common methods easier to understand.
  - Close analogs to many 3D effects/situations.
- Can prove SPT converges … to the wrong answer.
- Can understand Fourier vs. Configuration and Euler vs. Lagrange more easily.
- EFTLSS is much simpler in 1D.
  - Easier to see analytically what’s happening.
  - Dramatic improvement for power-law models (where symmetry is really helping).
- Nice “toy” problem for understanding PT.

The End

Stochastic contribution



1-loop decomposition



1-loop decomposition