# Cosmological perturbation theory

Martin White UCB/LBNL Trieste, July 2010 (http://mwhite.berkeley.edu/Talks)

# Limited options

- Beyond a certain scale, linear perturbation theory breaks down
  - Definition of "non-linear scale"?
- At this point we have few options:
  - Analytical models of non-linear growth.
    - Zel'dovich approximation.
    - Spherical top-hat collapse.
  - Perturbation theory.
    - Realm of validity? Convergence criterion?
    - Good for small corrections to almost linear problems.
  - Direct simulation.
    - Numerical convergence.
    - What models to run?
    - Missing physics.

# Scale of non-linearity

- There are several ways to define a "scale" of non-linearity.
- Where  $\Delta^2(k)=1$  (or  $\frac{1}{2}$ , or ...).

– Dangerous when  $\Delta^2(k)$  is very flat.

• By the rms linear theory displacement.

$$R_{\rm nl} \propto \frac{1}{k_{\rm nl}^2} \propto \int \frac{dk}{k} \; \frac{\Delta^2(k)}{k^2} \propto \int dk \; P(k)$$

 Where the 2<sup>nd</sup> order correction to some quantity is 1% (10%) of the 1<sup>st</sup> order term.

# Perturbation theory

- There is no reason (in principle) to stop at linear order in perturbation theory.
  - Can expand to all orders:  $\delta = \delta^{(1)} + \delta^{(2)} + \delta^{(3)} + \dots$
  - Can sum subsets of terms.
  - Usefulness/convergence of such an expansion not always clear.
- Consider **only** dark matter and **assume** we are in the single-stream limit.

Peebles (1980), Juszkiewicz (1981), Goroff++(1986), Makino++(1992), Jain&Bertschinger(1994), Fry (1994). Reviews/comparison with N-body: Bernardeau++(2002; Phys. Rep. 367, 1). Carlson++(2009; PRD 80, 043531)

# Equations of motion

Under these approximations, and assuming  $\Omega_m = 1$ 

$$\frac{\partial \delta}{\partial \tau} + \vec{\nabla} \cdot \left[ (1+\delta)\vec{v} \right] = 0$$
  
$$\frac{\partial \vec{v}}{\partial \tau} + \mathcal{H}\vec{v} + \left( \vec{v} \cdot \vec{\nabla} \right)\vec{v} = -\vec{\nabla}\Phi$$
  
$$\nabla^2 \Phi = \frac{3}{2}\mathcal{H}^2\delta$$

Very familiar looking fluid equations

 $\circ$  means we can borrow methods/ideas from other fields.

- Note the quadratic nature of the non-linearity.
- Since equations are now non-linear, can't use superposition of (exact) solutions even if they could be found!
- Proceed by perturbative expansion.



Percival & White (2009)

### Go into Fourier space

Putting the quadratic terms on the rhs and going into Fourier space:



### Linear order

• To lowest order in  $\delta$  and  $\theta$ :

$$\delta_L(\mathbf{k}, z) = \frac{D(z)}{D(z_i)} \,\delta_i(\mathbf{k})$$
  
$$\theta_L(\mathbf{k}, z) = -f(z)\mathcal{H}(z) \,\frac{D(z)}{D(z_i)} \,\delta_i(\mathbf{k})$$

- with  $f(z) \sim \Omega_m^{0.6} = 1$  for  $\Omega_m = 1$  and  $D(a) \sim a$ .
- Decaying mode, δ~a<sup>-3/2</sup>, has to be zero for δ to be well-behaved as a->0.
- Define  $\delta_0 = \delta_L(k,z=0)$ .

### Standard perturbation theory

• Develop  $\delta$  and  $\theta$  as power series:

$$\delta(\mathbf{k}) = \sum_{n=1}^{\infty} a^n \delta^{(n)}(\mathbf{k})$$
$$\theta(\mathbf{k}) = -\mathcal{H} \sum_{n=1}^{\infty} a^n \theta^{(n)}(\mathbf{k})$$

• then the  $\delta^{(n)}$  can be written

$$\delta^{(n)}(\mathbf{k}) = \int \frac{d^3 q_1 d^3 q_2 \cdots d^3 q_n}{(2\pi)^{3n}} (2\pi)^3 \delta_D \left( \sum \mathbf{q}_i - \mathbf{k} \right) \\ \times F_n \left( \{ \mathbf{q}_i \} \right) \delta_0(\mathbf{q}_1) \cdots \delta_0(\mathbf{q}_n)$$

- with a similar expression for  $\theta^{(n)}$ .
- The F<sub>n</sub> and G<sub>n</sub> are just ratios of dot products of the *q*s and obey simple recurrence relations.

### Recurrence relations I

- Plugging the expansion into our equations and using
  - $-(d/d\tau)a^n=nHa^n$
  - $-(d/d\tau)H=(-1/2)H^2$  for EdS
- we have (canceling *H* from both sides):

$$n\delta^{(n)} + \theta^{(n)} = -\int \frac{d^3q_1}{(2\pi)^3} \frac{d^3q_2}{(2\pi)^3} (2\pi)^3 \delta(\vec{k} - \vec{q_1} - \vec{q_2}) \frac{\vec{k} \cdot \vec{q_1}}{q_1^2} \sum_{m=1}^{n-1} \theta_m(\vec{q_1}) \delta_{n-m}(\vec{q_2})$$

$$3\delta^{(n)} + (2n+1)\theta^{(n)} = -\int \frac{d^3q_1}{(2\pi)^3} \frac{d^3q_2}{(2\pi)^3} (2\pi)^3 \delta(\vec{k} - \vec{q_1} - \vec{q_2}) \frac{k^2(\vec{q_1} \cdot \vec{q_2})}{q_1^2 q_2^2} \sum_{m=1}^{n-1} \theta_m(\vec{q_1})\theta_{n-m}(\vec{q_2})$$

#### Recurrence relations II

• Which we can rewrite

$$\delta^{(n)} = \frac{(2n+1)A_n - B_n}{(2n+3)(n-1)} \quad , \quad \theta^{(n)} = \frac{-3A_n + nB_n}{(2n+3)(n-1)}$$

- where  $A_n$  and  $B_n$  are the rhs mode-coupling integrals.
- This generates recursion relations for the F<sub>n</sub> and G<sub>n</sub> (because of the sums in A<sub>n</sub> and B<sub>n</sub>)

$$F_{n} = \sum_{m=1}^{n-1} \frac{G_{m}}{(2n+3)(n-1)} \left[ (2n+1)\frac{\vec{k}\cdot\vec{k}_{1}}{k_{1}^{2}}F_{n-m} + \frac{k^{2}(\vec{k}_{1}\cdot\vec{k}_{2})}{k_{1}^{2}k_{2}^{2}}G_{n-m} \right]$$
$$G_{n} = \sum_{m=1}^{n-1} \frac{G_{m}}{(2n+3)(n-1)} \left[ 3\frac{\vec{k}\cdot\vec{k}_{1}}{k_{1}^{2}}F_{n-m} + n\frac{k^{2}(\vec{k}_{1}\cdot\vec{k}_{2})}{k_{1}^{2}k_{2}^{2}}G_{n-m} \right]$$

# Example: 2<sup>nd</sup> order

• The coupling function:

$$F_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{5}{7} + \frac{2}{7} \frac{\left(\mathbf{k}_1 \cdot \mathbf{k}_2\right)^2}{k_1^2 k_2^2} + \frac{\left(\mathbf{k}_1 \cdot \mathbf{k}_2\right)}{2} \left(k_1^{-2} + k_2^{-2}\right)$$

- where we have symmetrized the function in terms of its arguments.
  - Note: this function peaks when  $k_1 \sim k_2 \sim k/2$ .
  - This will be important later.

# Formal development

 We can make the expressions above more formal by defining η=ln(a) and

$$\left(\begin{array}{c}\phi_1\\\phi_2\end{array}\right) = e^{-\eta} \left(\begin{array}{c}\delta\\-\theta/\mathcal{H}\end{array}\right)$$

• then writing

$$\partial_{\eta}\phi_a = -\Omega_{ab}\phi_b + e^{\eta}\gamma_{abc}\phi_b\phi_c$$

- with the obvious definitions of  $\Omega$  and  $\gamma$ .
- We can also define  $P \sim \langle \phi \phi \rangle$ ,  $B \sim \langle \phi \phi \phi \rangle$  so e.g.

$$\partial_{\eta} P_{ab} = -\Omega_{ac} P_{cb} - \Omega_{bc} P_{ac} + e^{\eta} \int d^3 q \left[ \gamma_{acd} B_{bcd} + B_{acd} \gamma_{bcd} \right]$$

#### Power spectrum

• If the initial fluctuations are Gaussian only expectation values even in  $\delta_0$  survive:

$$- P(k) \sim \langle [\delta^{(1)} + \delta^{(2)} + \delta^{(3)} + \dots] [\delta^{(1)} + \delta^{(2)} + \delta^{(3)} + \dots] \rangle$$
  
= P<sup>(1,1)</sup> + 2P<sup>(1,3)</sup> + P<sup>(2,2)</sup> + \dots

• with terms like  $<\delta^{(1)}\delta^{(2)}>$  vanishing because they reduce to  $<\delta_0\delta_0\delta_0>$ .

#### Perturbation theory: diagrams

Just as there is a diagrammatic short-hand for perturbation theory in quantum field theory, so there is in cosmology:





#### Example: 2<sup>nd</sup> order

$$P^{(1,3)}(k) = \frac{1}{252} \frac{k^3}{4\pi^2} P_L(k) \int_0^\infty dr \ P_L(kr) \left[ \frac{12}{r^2} - 158 + 100r^2 - 42r^4 + \frac{3}{r^2} (r^2 - 1)^3 (7r^2 + 2) \ln \left| \frac{1 + r}{1 - r} \right| \right],$$

$$P^{(2,2)}(k) = \frac{1}{98} \frac{k^3}{4\pi^2} \int_0^\infty dr \ P_L(kr) \int_{-1}^1 dx \ P_L\left(k\sqrt{1+r^2-2rx}\right) \\ \times \frac{(3r+7x-10rx^2)^2}{(1+r^2-2rx)^2}.$$

Perturbation theory enables the generation of truly impressive looking equations which arise from simple angle integrals. Like Feynman integrals, they are simple but look erudite!

# Example: 2<sup>nd</sup> order

- At low k,  $P^{(2,2)}$  is positive and  $P^{(1,3)}$  is negative
  - Large cancellation.
- For large *k* total contribution is negative:
  - $P^{(2,2)} \sim (1/4) k^2 \Sigma^2 P_L(k)$

 $- P^{(1,3)} \sim -(1/2) k^2 \Sigma^2 P_L(k)$ 

- Here  $\Sigma$  is the rms displacement (in each component) in linear theory.
  - It will come up again!!

$$\Sigma^2 = \frac{1}{3\pi^2} \int_0^\infty dq \ P_L(q)$$



# Beyond 2<sup>nd</sup> order

- Expressions for higher orders are easy to derive, especially using computer algebra packages.
- Using rotation symmetry the N<sup>th</sup> order contribution requires mode coupling integrals of dimension 3N-1.
  - Best done using Monte-Carlo integration.
  - Prohibitive for very high orders.
  - Not clear this expansion is converging!

### Comparison with exact results



Broad-band shape of  $P_L$  has been divided out to focus on more subtle features.



# Including bias

- Perturbation theory clearly cannot describe the formation of collapsed, bound objects such as dark matter halos.
- We can extend the usual thinking about "linear bias" to a power-series in the Eulerian density field:

 $- \delta_{gal} = \Sigma b_n(\delta^n/n!)$ 

- The expressions for P(k) now involve b<sub>1</sub> to lowest order, b<sub>1</sub> and b<sub>2</sub> to next order, etc.
  - The physical meaning of these terms is actually hard to figure out, and the validity of the defining expression is dubious, but this is the standard way to include bias in Eulerian perturbation theory.

# Other methods

- Renormalized perturbation theory
  - A variant of "Dyson-Wyld" resummation.
  - An expansion in "order of complexity".
- Closure theory
  - Write expressions for  $(d/d\tau)P$  in terms of P, B, T, ...
  - Approximate B by leading-order expression in SPT.
- Time-RG theory (& RGPT)
  - As above, but assume B=0
  - Good for models with  $m_v > 0$  where linear growth is scale-dependent.
- Path integral formalism
  - Perturbative evaluation of path integral gives SPT.
  - Large N expansion, 2PI effective action, steepest descent.
- Lagrangian perturbation theory

(see Carlson++09 for references)

#### Some other theories



### Other statistics



PT makes predictions for other statistics as well. For example, the power spectra of the velocity and the density-velocity cross spectrum. Here it seems to do less well. **SPT RPT** Closure Time-RG

### Some other quantities



The propagator, or



which measures the decoherence of the final density field due to non-linear evolution.

Carlson++09

# Lagrangian perturbation theory

- A different approach to PT, which has been radically developed recently by Matsubara and is *very* useful for BAO.
  - Buchert89, Moutarde++91, Bouchet++92, Catelan95, Hivon++95.
  - Matsubara (2008a; PRD, 77, 063530)
  - Matsubara (2008b; PRD, 78, 083519)
- Relates the current (Eulerian) position of a mass element, x, to its initial (Lagrangian) position, q, through a displacement vector field, Ψ.

### Lagrangian perturbation theory

$$\delta(\mathbf{x}) = \int d^3 q \ \delta_D(\mathbf{x} - \mathbf{q} - \boldsymbol{\Psi}) - 1$$
  
$$\delta(\mathbf{k}) = \int d^3 q \ e^{-i\mathbf{k}\cdot\mathbf{q}} \left(e^{-i\mathbf{k}\cdot\boldsymbol{\Psi}(\mathbf{q})} - 1\right) .$$

$$\frac{d^2 \Psi}{dt^2} + 2H \frac{d \Psi}{dt} = -\nabla_x \phi \left[ \mathbf{q} + \Psi(\mathbf{q}) \right]$$

$$\Psi^{(n)}(\mathbf{k}) = \frac{i}{n!} \int \prod_{i=1}^{n} \left[ \frac{d^{3}k_{i}}{(2\pi)^{3}} \right] (2\pi)^{3} \delta_{D} \left( \sum_{i} \mathbf{k}_{i} - \mathbf{k} \right)$$
$$\times \mathbf{L}^{(n)}(\mathbf{k}_{1}, \cdots, \mathbf{k}_{n}, \mathbf{k}) \delta_{0}(\mathbf{k}_{1}) \cdots \delta_{0}(\mathbf{k}_{n})$$

### Kernels

$$\mathbf{L}^{(1)}(\mathbf{p}_{1}) = \frac{\mathbf{k}}{k^{2}}$$
(1)  
$$\mathbf{L}^{(2)}(\mathbf{p}_{1}, \mathbf{p}_{2}) = \frac{3}{7} \frac{\mathbf{k}}{k^{2}} \left[ 1 - \left(\frac{\mathbf{p}_{1} \cdot \mathbf{p}_{2}}{p_{1}p_{2}}\right)^{2} \right]$$
(2)  
$$\mathbf{L}^{(3)}(\mathbf{p}_{1}, \mathbf{p}_{2}, \mathbf{p}_{3}) = \cdots$$
(3)

$$\mathbf{k} \equiv \mathbf{p}_1 + \dots + \mathbf{p}_n$$

### Standard LPT

• If we expand the exponential and keep terms consistently in  $\delta_0$  we regain a series  $\delta = \delta^{(1)} + \delta^{(2)} + \dots$  where  $\delta^{(1)}$  is linear theory and e.g.

$$\begin{split} \delta^{(2)}(\mathbf{k}) &= \frac{1}{2} \int \frac{d^3 k_1 d^3 k_2}{(2\pi)^3} \delta_D(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}) \delta_0(\mathbf{k}_1) \delta_0(\mathbf{k}_2) \\ &\times \left[ \mathbf{k} \cdot \mathbf{L}^{(2)}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}) + \mathbf{k} \cdot \mathbf{L}^{(1)}(\mathbf{k}_1) \mathbf{k} \cdot \mathbf{L}^{(1)}(\mathbf{k}_2) \right] \end{split}$$

- which regains "SPT".
  - The quantity in square brackets is  $F_2$ .

$$F_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{5}{7} + \frac{2}{7} \frac{\left(\mathbf{k}_1 \cdot \mathbf{k}_2\right)^2}{k_1^2 k_2^2} + \frac{\left(\mathbf{k}_1 \cdot \mathbf{k}_2\right)}{2} \left(k_1^{-2} + k_2^{-2}\right)$$

# LPT power spectrum

- Alternatively we can use the expression for  $\delta_{\textbf{k}}$  to write

$$P(k) = \int d^3q \ e^{-i\vec{k}\cdot\vec{q}} \left( \left\langle e^{-i\vec{k}\cdot\Delta\vec{\Psi}} \right\rangle - 1 \right)$$

- where  $\Delta \Psi = \Psi(\mathbf{q}) \Psi(0)$ .
- Expanding the exponential and plugging in for  $\Psi^{(n)}$  gives the usual results.
- **BUT** Matsubara suggested a different and very clever approach.

# Cumulants

- The cumulant expansion theorem allows us to write the expectation value of the exponential in terms of the exponential of expectation values.
- Expand the terms  $(\mathbf{k}\Delta\Psi)^N$  using the binomial theorem.
- There are two types of terms:
  - Those depending on  $\Psi$  at same point.
    - This is independent of position and can be factored out of the integral.
  - Those depending on  $\Psi$  at different points.
    - These can be expanded as in the usual treatment.

# Example

- Imagine  $\Psi$  is Gaussian with mean zero.
- For such a Gaussian:  $\langle e^{\chi} \rangle = \exp[\sigma^2/2]$ .

$$P(k) = \int d^3 q e^{-i\mathbf{k}\cdot\mathbf{q}} \left( \left\langle e^{-ik_i \Delta \Psi_i(\mathbf{q})} \right\rangle - 1 \right)$$

$$\left\langle e^{-i\mathbf{k}\cdot\Delta\Psi(q)}\right\rangle = \exp\left[-\frac{1}{2}k_ik_j\left\langle\Delta\Psi_i(\mathbf{q})\Delta\Psi_j(\mathbf{q})\right\rangle\right]$$

$$k_i k_j \left\langle \Delta \Psi_i(\mathbf{q}) \Delta \Psi_j(\mathbf{q}) \right\rangle = 2k_i^2 \left\langle \Psi_i^2(\mathbf{0}) \right\rangle - 2k_i k_j \xi_{ij}(\mathbf{q})$$

$$\uparrow$$
Keep exponentiated. Expand

# Resummed LPT

• The first corrections to the power spectrum are then:

$$P(k) = e^{-(k\Sigma)^2/2} \left[ P_L(k) + P^{(2,2)}(k) + \widetilde{P}^{(1,3)}(k) \right],$$

- where P<sup>(2,2)</sup> is as in SPT but part of P<sup>(1,3)</sup> has been "resummed" into the exponential prefactor.
- The exponential prefactor is identical to that obtained from
  - The peak-background split (Eisenstein++07)
  - Renormalized Perturbation Theory (Crocce++08).

# Beyond real-space mass

- One of the more impressive features of Matsubara's approach is that it can gracefully handle both biased tracers and redshift space distortions.
- In redshift space  $\Psi \rightarrow \Psi + \frac{\widehat{\mathbf{z}} \cdot \dot{\Psi}}{H} \widehat{\mathbf{z}}$
- For bias local in Lagrangian space:

$$\delta_{\rm obj}(\mathbf{x}) = \int d^3 q \ F[\delta_L(\mathbf{q})] \, \delta_D(\mathbf{x} - \mathbf{q} - \mathbf{\Psi})$$

• we obtain

$$P(k) = \int d^3q \ e^{-i\mathbf{k}\cdot\mathbf{q}} \left[ \int \frac{d\lambda_1}{2\pi} \frac{d\lambda_2}{2\pi} F(\lambda_1) F(\lambda_2) \left\langle e^{i[\lambda_1\delta_L(\mathbf{q}_1) + \lambda_2\delta_L(\mathbf{q}_2)] + i\mathbf{k}\cdot\Delta\Psi} \right\rangle - 1 \right]$$

- which can be massaged with the same tricks as we used for the mass.
- If we assume halos/galaxies form at peaks of the initial density field ("peaks bias") then explicit expressions for the integrals of F exist.



#### Non-linearities and BAO

# Effects of non-linearity on BAO

- Non-linear evolution has 3 effects on the power spectrum:
  - It generates "excess" high k power, reducing the contrast of the wiggles.
  - It damps the oscillations.
  - It generates an out-of-phase component.
- In configuration space:
  - Generates "excess" small-scale power.
  - Broadens the peak.
  - Shifts the peak.

### Non-linearities smear the peak



# Mode-coupling terms

Recall in PT we can write  $\delta = \delta^{(1)} + \delta^{(2)} + \dots$  or P = {P<sub>11</sub> + P<sub>13</sub> + P<sub>15</sub> + ...} + {P<sub>22</sub> + ...} = P<sub>1n</sub> + P<sub>mn</sub>.

- The P<sub>1n</sub> terms are benign.
- By contrast the P<sub>mn</sub> terms involve integrals of products of P<sub>L</sub>s times peaked kernels.
- Example:  $P_{22} \sim \int P_L P_L F_2$  and  $F_2$  is sharply peaked around  $q_1 \approx q_2 \approx k/2$ .
- Thus the  $\int P_L P_L$  term contains an out-of-phase oscillation

 $- P_L \sim \dots + \sin(kr)$ :  $P_L P_L F_2 \sim \sin^2(kr/2) \sim 1 + \cos(kr)$ 

 Since cos(x)~d/dx sin(x) this gives a "shift" in the peak

-  $P(k/\alpha) \sim P(k) - (\alpha-1) dP/dlnk + ...$ 

#### Mode-coupling approximates derivative



Up to an overall factor the modecoupling term,  $P_{22}$ , is well approximated by  $dP_{\rm L}/d\ln k$ .

# Modified template

• This discussion suggests a modified template, which has just as many free parameters as our old template:

$$P_{\rm w}(k,\alpha) = \exp\left(-\frac{k^2\Sigma^2}{2}\right) P_L(k/\alpha) + \exp\left(-\frac{k^2\Sigma_1^2}{2}\right) P_{22}(k/\alpha).$$

• This removes most of the shift.

Z	DM	$x\delta_L$	w/P <sub>22</sub>
0.0	$2.91 \pm 0.20$	-0.2 ±0.1	$-0.03 \pm 0.16$
0.3	$1.88 \pm 0.12$	-0.2 ±0.1	$-0.38 \pm 0.09$
0.7	$1.17\pm0.07$	-0.1 ±0.1	$-0.12 \pm 0.05$
1.0	$0.88 \pm 0.06$	-0.1 ±0.1	$-0.04 \pm 0.04$

#### Biased tracers?

- In order to remove the shift we needed to know the relative amplitude of P<sub>11</sub> and P<sub>22</sub>.
   – For the mass, this is known.
- What do we do for biased tracers?

$$- \text{ Eulerian bias} P_{h} = (b_{1}^{E})^{2} (P_{11} + P_{22}) + b_{1}^{E} b_{2}^{E} \left(\frac{3}{7}Q_{8} + Q_{9}\right) + \frac{(b_{2}^{E})^{2}}{2}Q_{13} + \cdots - \text{ Lagrangian bias} P_{h} = \exp\left[-\frac{k^{2}\Sigma^{2}}{2}\right] \left\{\left(1 + b_{1}^{L}\right)^{2} P_{11} + P_{22} + b_{1}^{L} \left[\frac{6}{7}Q_{5} + 2Q_{7}\right] + b_{2}^{L} \left[\frac{3}{7}Q_{8} + Q_{9}\right]\right\}$$

+ 
$$(b_1^L)^2 [Q_9 + Q_{11}] + 2b_1^L b_2^L Q_{12} + \frac{1}{2} (b_2^L)^2 Q_{13}$$
 + ...

# Mode-coupling integrals

$$Q_n(k) = \frac{k^3}{4\pi^2} \int_0^\infty dr \, P_L(kr) \int_{-1}^1 dx \, P_L(k\sqrt{1+r^2-2rx}) \widetilde{Q}_n(r,x)$$

$$\begin{split} \widetilde{Q}_{1} &= \frac{r^{2}(1-x^{2})^{2}}{y^{2}}, \quad \widetilde{Q}_{2} &= \frac{(1-x^{2})rx(1-rx)}{y^{2}}, \\ \widetilde{Q}_{3} &= \frac{x^{2}(1-rx)^{2}}{y^{2}}, \quad \widetilde{Q}_{4} &= \frac{1-x^{2}}{y^{2}}, \\ \widetilde{Q}_{5} &= \frac{rx(1-x^{2})}{y}, \quad \widetilde{Q}_{6} &= \frac{(1-3rx)(1-x^{2})}{y}, \\ \widetilde{Q}_{7} &= \frac{x^{2}(1-rx)}{y}, \quad \widetilde{Q}_{8} &= \frac{r^{2}(1-x^{2})}{y}, \\ \widetilde{Q}_{9} &= \frac{rx(1-rx)}{y}, \quad \widetilde{Q}_{10} &= 1-x^{2}, \\ \widetilde{Q}_{11} &= x^{2}, \quad \widetilde{Q}_{12} &= rx, \quad \widetilde{Q}_{13} &= r^{2} \end{split}$$

(Matsubara 2008)



The numerous combinations that come in are also well approximated by the (log-)derivative of  $P_{11}$ ! All of these terms can be effectively written as:

$$P_h = \exp\left(-\frac{k^2\Sigma^2}{2}\right) \left[\mathcal{B}_1 P_L + \mathcal{B}_2 P_{22}\right].$$

### Size of the shifts?

- Simple model explains  $B_1$ - $B_2$  relation.
  - True for a variety of cosmologies, including  $\Lambda$ CDM.
  - Can also be measured from simulations (using some tricks).
- For  $\Lambda$ CDM the shifts are:
  - $\alpha$ -1~0.5% x D<sup>2</sup> x  $B_2/B_1$



Shifts at *z*=0 for

Halos of mass M Halos above M N~[1+M/M<sub>1</sub>]

At higher z the shift decreases as  $D^2$ .

Recall, the final error in BAO scale is the *uncertainty* in this correction, not the size of the correction itself!

# Redshift space

- In resummed LPT we can also consider the redshift space power spectrum for biased tracers.
- For the isotropic P(k) find a similar story though now the scaling coefficients depend on *f*~dD/dln*a*.
  - Expressions become more complex, but the structure is unchanged.
- The amplitude of the shift increases slightly.

# Perturbation theory & BAO

- Meiksin, White & Peacock, 1999
  - Baryonic signatures in large-scale structure
- Crocce & Scoccimarro, 2007
  - Nonlinear Evolution of Baryon Acoustic Oscillations
- Nishimichi et al., 2007
  - Characteristic scales of BAO from perturbation theory
- Matsubara, 2007, 2008
- Jeong & Komatsu, 2007, 2008
  - Perturbation theory reloaded I & II
- Pietroni, 2008
  - Flowing with time
- Padmanabhan & White (2009)
  - Calibrating the baryon oscillation ruler for matter and halos
- Padmanabhan et al., 2009; Noh et al. 2009
  - Reconstructing baryon oscillations: A Lagrangian theory perspective
  - Reconstructing baryon oscillations.
- Taruya et al., 2009
  - Non-linear Evolution of Baryon Acoustic Oscillations from Improved Perturbation Theory in Real and Redshift Spaces

