Redshift space distortions for biased tracers

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Outline

- Introduction
- RSD in configuration space.
 - Difficulties in modeling RSD.
 - Insights from a toy model.
 - Convolution LPT (CLPT)
- Conclusions.

RSD: Why

- What you observe in a redshift survey is the density field in redshift space!
 - A combination of density and velocity fields.
- Tests GI.
 - Structure growth driven by motion of matter and inhibited by expansion.
- Constrains GR.
 - Knowing a(t) and ρ_i , GR provides prediction for growth rate.
 - In combination with lensing measures Φ and Ψ .
- Measures "interesting" numbers.
 - Constrains H(z), DE, m_v , etc.
- Surveys can make percent level measurements would like to have theory to compare to!
- Fun problem!

Simplify ...

- We will work in the distant observer, planeparallel approximation(s).
- All velocities will be expressed in units of the Hubble expansion.
 - i.e. in distance units.
- Use polar coordinates.



Two dimensional clustering (BOSS; Reid++12)

Line-of-sight picks out a preferred direction inducing anisotropy in the 2-point function – measures the growth of structure and tests GR.



In configuration space

- Kaiser's pioneering work was done in Fourier space.
- There are valuable insights to be gained by working in configuration, rather than Fourier, space.
- We begin to see why this is a hard problem ...

$$1 + \xi^{s}(R, Z) = \left\langle \int dy \ (1 + \delta_{1})(1 + \delta_{2})\delta^{(D)}(Z - y - v_{12}) \right\rangle$$
$$1 + \xi^{s}(R, Z) = \left\langle \int dy \ (1 + \delta_{1})(1 + \delta_{2}) \int \frac{d\kappa}{2\pi} e^{i\kappa(Z - y - v_{12})} \right\rangle$$

• Note all powers of the velocity field enter.

Gaussian limit (Fisher, 1995, ApJ 448, 494)

If δ and v are Gaussian can do all of the expectation values.

$$1 + \xi^{s}(R, Z) = \int \frac{dy}{\sqrt{2\pi\sigma_{12}^{2}(y)}} \exp\left[-\frac{(Z-y)^{2}}{2\sigma_{12}^{2}(y)}\right] \times \left[1 + \xi^{r}(r) + \frac{y}{r} \frac{(Z-y)v_{12}(r)}{\sigma_{12}^{2}(y)} - \frac{1}{4} \frac{y^{2}}{r^{2}} \frac{v_{12}^{2}(r)}{\sigma_{12}^{2}(y)} \left(1 - \frac{(Z-y)^{2}}{\sigma_{12}^{2}(y)}\right)\right]$$

Expanding around y=Z:

$$\xi^{s}(R,Z) = \xi^{r}(s) - \left. \frac{d}{dy} \left[v_{12}(r) \frac{y}{r} \right] \right|_{y=Z} + \frac{1}{2} \left. \frac{d^{2}}{dy^{2}} \left[\sigma_{12}^{2}(y) \right] \right|_{y=Z}$$

Linear theory: configuration space (Fisher, 1995, ApJ 448, 494)

- One can show that this expansion agrees with the Kaiser formula.
- Two important points come out of this way of looking at the problem:
 - Correlation between δ and v leads to v₁₂.
 - Overdensities will fall towards each other.
 - The μ^2 term is a $\langle v \delta \rangle$ correlation as for Kaiser.
 - LOS velocity dispersion is scale- and orientation-dependent.
- ξ^s depends on the 1st and 2nd derivatives of velocity statistics.

Two forms of non-linearity

- Part of the difficulty is that we are dealing with two forms of non-linearity/non-perturbative behavior.
 - The velocity field is non-linear.
 - The mapping from real- to redshift-space is "non-linear".
- These two forms of non-linearity interact, and can partially cancel.
- They also depend on parameters differently!



Non-linear mapping?



Want a fully non-linear "toy model", like spherical top-hat collapse, to gain some intuition ...

A model for the redshift-space clustering of halos

- We would like to develop a model capable of describing the redshift space clustering of halos.
 - This will form the 1st step in a model for galaxies, but it also interesting in its own right.
- The model should try to treat the "non-linear mapping" part of the problem non-perturbatively.
- We will start with a toy model and then add realism/ dynamics ...

The correlation function of halos



The correlation function of halo centers doesn't have strong fingers of god, but still has "squashing" at large scales.

Scale-dependent Gaussian streaming model

Let's go back to the exact result for a Gaussian field, a la Fisher:

$$1 + \xi^{s}(R, Z) = \int \frac{dy}{\sqrt{2\pi\sigma_{12}^{2}(y)}} \exp\left[-\frac{(Z-y)^{2}}{2\sigma_{12}^{2}(y)}\right] \times \left[1 + \xi^{r}(r) + \frac{y}{r} \frac{(Z-y)v_{12}(r)}{\sigma_{12}^{2}(y)} - \frac{1}{4} \frac{y^{2}}{r^{2}} \frac{v_{12}^{2}(r)}{\sigma_{12}^{2}(y)} \left(1 - \frac{(Z-y)^{2}}{\sigma_{12}^{2}(y)}\right)\right]$$

Looks convolution-like, but with a scale-dependent v_{12} and σ (also, want to resum v_{12} into the exponential ...)

Scale-dependent Gaussian
streaming model/ansatz
$$1 + \xi(R, Z) = \int dy \ [1 + \xi(r)] \mathcal{P}(v = Z - y, \mathbf{r})$$

Note: *not* a convolution because of (important!) *r* dependence or kernel.

Non-perturbative mapping.

If lowest moments of *P* set by linear theory, agrees at linear order with Kaiser. Approximate *P* as Gaussian ...



Gaussian ansatz



Testing the ansatz



The mapping



Note, the behavior of the quadrupole is particularly affected by the non-linear mapping. The effect of non-linear velocities is to suppress ξ_2 (by ~10%, significant!). The mapping causes the enhancement. This effect is tracer/ bias dependent!

The "b³" term?

- One of the more interesting things to come out of this ansatz is the existence of a "b³" term.
 - Numerically quite important when b~2.
 - Another reason why mass results can be very misleading.
 - But hard to understand (naively) from

$$1 + \xi^s(R, Z) = \left\langle \int dy \ (1 + \delta_1)(1 + \delta_2) \int \frac{d\kappa}{2\pi} e^{i\kappa(Z - y - v_{12})} \right\rangle$$

- Where does it come from?

Lagrangian perturbation theory

- A different approach to PT.
 - Buchert89, Moutarde++91, Bouchet++92, Catelan95, Hivon++95.
- Relates the current (Eulerian) position of a mass element, x, to its initial (Lagrangian) position, q, through a displacement vector field, Ψ.
- Has been radically extended recently by Matsubara:
 - Matsubara (2008a; PRD, 77, 063530)
 - Matsubara (2008b; PRD, 78, 083519)
- (and is *very* useful for BAO)

Lagrangian perturbation theory

$$\begin{split} \delta(\mathbf{x}) &= \int d^3 q \, \, \delta_D(\mathbf{x} - \mathbf{q} - \boldsymbol{\Psi}) - 1 \\ \delta(\mathbf{k}) &= \int d^3 q \, \, e^{-i\mathbf{k}\cdot\mathbf{q}} \left(e^{-i\mathbf{k}\cdot\boldsymbol{\Psi}(\mathbf{q})} - 1 \right) \, . \end{split}$$

$$\frac{d^2 \Psi}{dt^2} + 2H \frac{d \Psi}{dt} = -\nabla_x \phi \left[\mathbf{q} + \Psi(\mathbf{q}) \right]$$

$$\Psi^{(n)}(\mathbf{k}) = \frac{i}{n!} \int \prod_{i=1}^{n} \left[\frac{d^{3}k_{i}}{(2\pi)^{3}} \right] (2\pi)^{3} \delta_{D} \left(\sum_{i} \mathbf{k}_{i} - \mathbf{k} \right)$$
$$\times \mathbf{L}^{(n)}(\mathbf{k}_{1}, \cdots, \mathbf{k}_{n}, \mathbf{k}) \delta_{0}(\mathbf{k}_{1}) \cdots \delta_{0}(\mathbf{k}_{n})$$

Beyond real-space mass

- One of the more impressive features of Matsubara's LPT approach is that it can gracefully handle both biased tracers and redshift space distortions.
- In redshift space, in the plane-parallel limit,

$$\Psi \to \Psi + \frac{\widehat{\mathbf{z}} \cdot \dot{\Psi}}{H} \ \widehat{z} = R \Psi$$

• In PT
$$\Psi^{(n)} \propto D^n \Rightarrow R_{ij}^{(n)} = \delta_{ij} + nf \, \hat{z}_i \hat{z}_j$$

• For bias local in Lagrangian space:

$$\delta_{\rm obj}(\mathbf{x}) = \int d^3 q \ F[\delta_L(\mathbf{q})] \,\delta_D(\mathbf{x} - \mathbf{q} - \mathbf{\Psi})$$

 If we assume halos/galaxies form at peaks* of the initial density field ("peaks bias") then explicit expressions exist for the integrals of F that we will need.

*...and assume the peak-background split.

Configuration-space result

• The density of objects can be written:

$$1 + \delta_{\rm obj}(\mathbf{x}) = \int d^3q \int \frac{d\lambda}{2\pi} \ \widetilde{F}(\lambda) \int \frac{d^3k}{(2\pi)^3} \ e^{i\{\lambda\delta(\mathbf{q}) + \mathbf{k}\cdot[\mathbf{x}-\mathbf{q}-\boldsymbol{\Psi}]\}}$$

• so the 2-point function is

$$1 + \xi_{\rm obj}(\mathbf{r}) = \int d^3q \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot(\mathbf{q}-\mathbf{r})} \int \frac{d\lambda_1}{(2\pi)} \int \frac{d\lambda_2}{(2\pi)} \widetilde{F}(\lambda_1) \widetilde{F}(\lambda_2) K(\mathbf{q},\mathbf{k},\lambda_1,\lambda_2)$$

where we have written

$$K(\mathbf{q}, \mathbf{k}, \lambda_1, \lambda_2) = \left\langle e^{i\{\lambda_1\delta_1 + \lambda_2\delta_2 + \mathbf{k} \cdot (\boldsymbol{\Psi}_1 - \boldsymbol{\Psi}_2)\}} \right\rangle$$

• This is the configuration-space analog of Matsubara's Fourier-space expression.

Example: Zel'dovich

- Let's consider the lowest order expression
 - Zel'dovich approximation.

$$\Psi(\mathbf{q}) = \Psi^{(1)}(\mathbf{q}) = \int \frac{d^3k}{(2\pi)^3} \ e^{i\mathbf{k}\cdot\mathbf{q}} \ \frac{i\mathbf{k}}{k^2} \ \delta_0(\mathbf{k})$$

• Since δ_0 is Gaussian

$$K = \exp\left[-\frac{1}{2}\left(\lambda_1^2 + \lambda_2^2\right)\sigma^2 - \frac{1}{2}A_{ij}k_ik_j - \lambda_1\lambda_2\xi - (\lambda_1 + \lambda_2)U_ik_i\right]$$

where we have defined

and Δ=Ψ₁-Ψ₂. The matrix A_{ij} can be decomposed into pieces going as δ_{ij} and q_iq_j

Matter & Zel'dovich approximation

$$A_{ij} = \langle \Delta \Psi_i \Delta \Psi_j \rangle = B + C = 2\sigma^2 \delta_{ij} + C$$

Biased tracers & Zel'dovich

- For biased tracers Taylor expand terms going as ξ and *U* but keep σ and *A* terms exponentiated.
 - Both ξ and *U* vanish as q-> ∞ but σ and *A* do not.
 - Note our result is not simply the FT of Matsubara's expression b/c he keeps only constant piece of A exponentiated while we keep all of it.
- Have to plug this into 1+ ξ formula, do λ integrals, ...

$$1 + \xi_X(r) = \int \frac{d^3 q}{(2\pi)^{3/2} |A|^{1/2}} e^{-\frac{1}{2}(q-r)^T A^{-1}(q-r)} \\ \times \left[1 - \cdots 2\langle F' \rangle \langle F'' \rangle \xi_R U_i g_i + \cdots \right]$$

Peaks bias

- Our final expression contains terms with averages of F' and F'' over the density distribution.
- These take the place of "bias" terms
 - b₁ and b₂ in standard perturbation theory^{*}.
- If we assume halos form at the peaks of the initial density field and use the peak-background split we can obtain:

$$b_1 = \frac{\nu^2 - 1}{\delta_c} \quad , \quad b_2 = \frac{\nu^4 - 3\nu^2}{\delta_c^2} \approx b_1^2 \qquad \mbox{ for large ν}$$

• so <F'><F''>~b³.

*but "renormalized".

Convolution LPT?

- Can go beyond 1LPT (Zel'dovich) and do perturbative expansion.
- Keep all of $<\Delta \Psi_i \Delta \Psi_i >$ (and σ_R) exponentiated.
 - Expand the rest.
 - Do some algebra.
 - Evaluate convolution integral numerically.
 - This is a partial resummation of Matsubara's expression.
- Guarantees we recover the Zel'dovich limit as 0th order CLPT (for the matter).
 - Eulerian and LPT require an ∞ number of terms.
 - Many advantages: as emphasized recently/independently by Tassev & Zaldarriaga

Matter: Real: Monopole



Matter: Red: Monopole



Matter: Quadrupole









Halos: Red: Monopole





A combination of approaches? $Z(r,J) = \int d^3q \int \frac{d^3k}{(2\pi)^3} e^{ik \cdot (q-r)} \int \frac{d\lambda_1}{2\pi} \frac{d\lambda_2}{2\pi} \tilde{F}(\lambda_1) \tilde{F}(\lambda_2) K(q,k,\lambda_1,\lambda_2,J)$

$$K = \left\langle e^{i(\lambda_1 \delta_1 + \lambda_2 \delta_2 + k \cdot \Delta + J \cdot \dot{\Delta})} \right\rangle$$

$$1 + \xi(r) = Z(r, J = 0) \equiv Z_0(r),$$
$$v_{12,\alpha}(r) = \frac{\partial Z}{\partial J_\alpha} \Big|_{J=0} \equiv Z_{0,\alpha}(r),$$
$$D_{\alpha\beta}(r) = \frac{\partial^2 Z}{\partial J_\alpha \partial J_\beta} \Big|_{J=0} \equiv Z_{0,\alpha\beta}(r)$$

... plus streaming model ansatz.

From halos to galaxies

- In principle just another convolution
 - Intra-halo PDF.
- In practice need to model cs, ss^(1h) and ss^(2h).
- A difficult problem in principle, since have fingers-ofgod mixing small and large scales.
 - Our model for ξ falls apart at small scales...
- On quasilinear scales things simplify drastically.
 - Classical FoG unimportant.
 - Remaining effect can be absorbed into a single Gaussian dispersion which can be marginalized over.

Conclusions

- Redshift space distortions arise in a number of contexts in cosmology.
 - Fundamental questions about structure formation.
 - Constraining cosmological parameters.
 - Testing the paradigm.
- Linear theory doesn't work very well.
- Two types of non-linearity.
 - Non-linear dynamics and non-linear maps.
- Bias dependence can be complex.
- We are developing a new formalism for handling the redshift space correlation function of biased tracers.
 - Stay tuned!

