Large-scale structure

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Linear PT

- For many scales and most of age of Universe linear perturbation theory is valid.
- Focus here on matter (not radiation).
- Transfer function, *T(k)*, encodes 14Gyr of evolution.
  - $\delta_{today}(k) \sim (\text{growth}) \times T(k)\delta_{init}(k)$.
  - Main features RD->MD->ΛD.
  - Structure only grows when matter dominates energy density of Universe.

Notation

$$\delta(\mathbf{x}) = \frac{\rho(\mathbf{x}) - \bar{\rho}}{\bar{\rho}} = \frac{\delta\rho}{\rho}(\mathbf{x})$$

$$\delta(\mathbf{k}) = \int d^3x \; \delta(\mathbf{x})\, e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$\langle \delta(\mathbf{k})\delta^\star(\mathbf{k}') \rangle = (2\pi)^3 \delta_D(\mathbf{k} - \mathbf{k}') P(k)$$

$$\Delta^2(k) = \frac{k^3 P(k)}{2\pi^2}$$

$$\xi(x) = \int \frac{d^3k}{(2\pi)^3} P(k) e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$= \int \frac{dk}{k} \; \Delta^2(k) j_0(kr)$$

Which gauge is an N-body simulation defining?

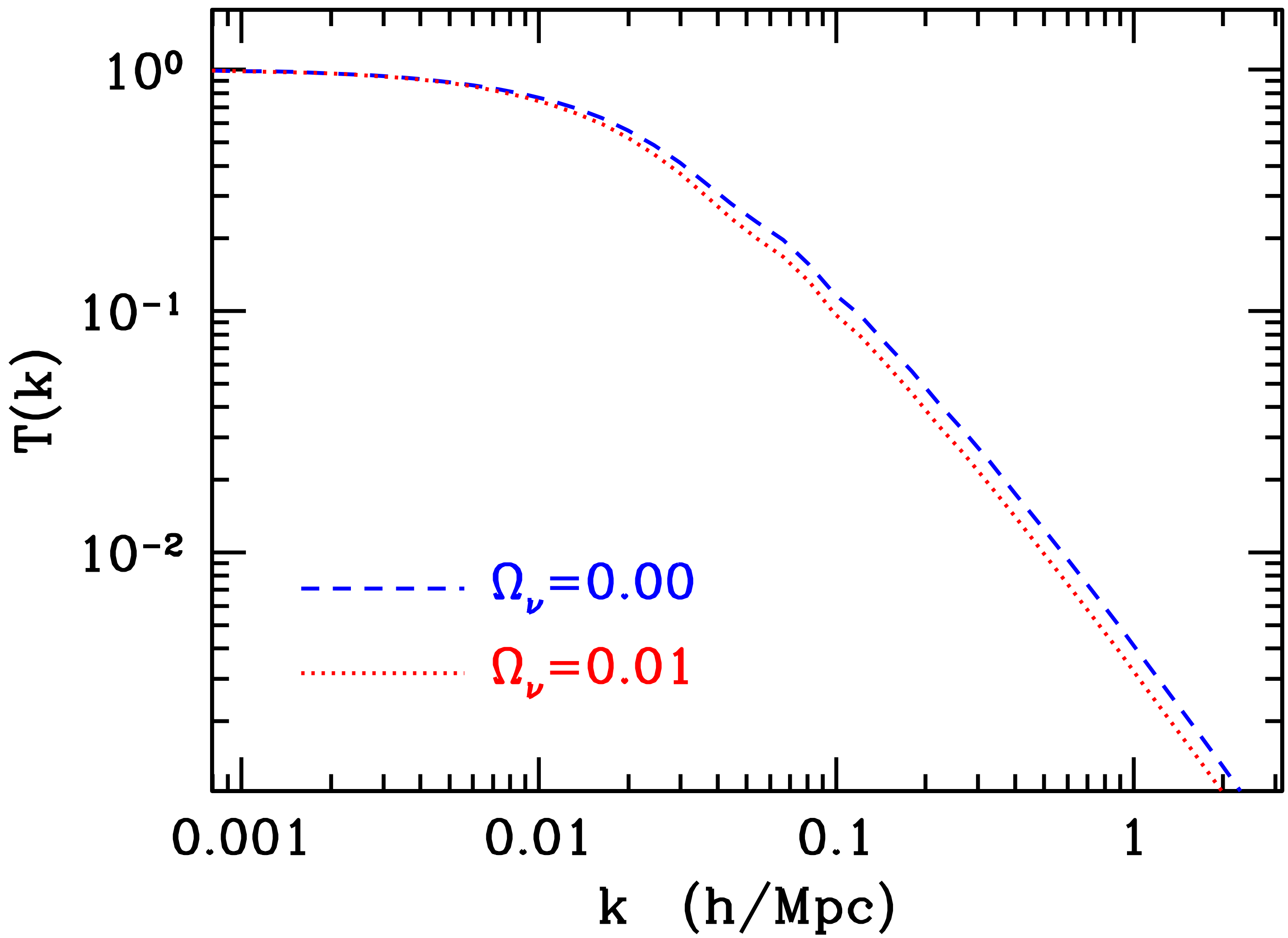
Growth of perturbations

$$
\begin{aligned}
ds^2 &= -(1+2\Phi)dt^2 + a^2 d\vec{x}^2 \\
L &= \frac{1}{2}a^2\dot{x}^2 - \Phi
\end{aligned}
$$

$$
\begin{aligned}
\frac{d}{dt}\left[a^2\dot{x}\right] &= -\nabla\Phi \\
\ddot{x} + 2\frac{\dot{a}}{a}\dot{x} &= -a^{-2}\nabla\Phi \\
&= -a^{-2}\nabla\nabla^{-2}4\pi G\rho a^2\delta \\
&= -\frac{3}{2}\Omega_m H^2\nabla\nabla^{-2}\delta
\end{aligned}
$$

$$
\begin{aligned}
\delta &\sim -\nabla\cdot\vec{x} \\
\ddot{\delta} + 2H\dot{\delta} &= \frac{3}{2}\Omega_m H^2\delta \\
\nabla\cdot\nabla P &\rightarrow k^2 c_s^2\delta
\end{aligned}
$$

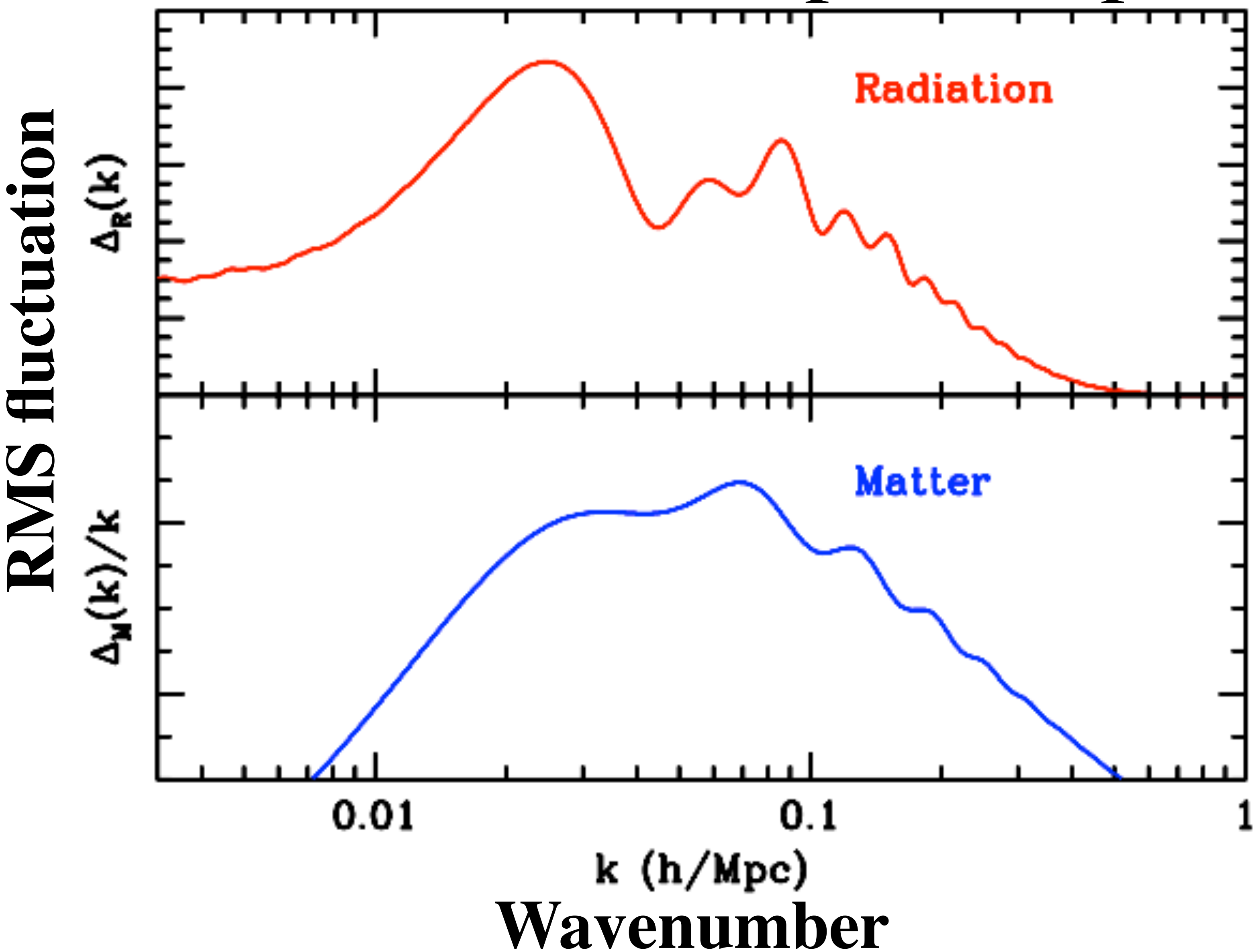
Massive neutrinos



Small-scale suppression of power in neutrino models:

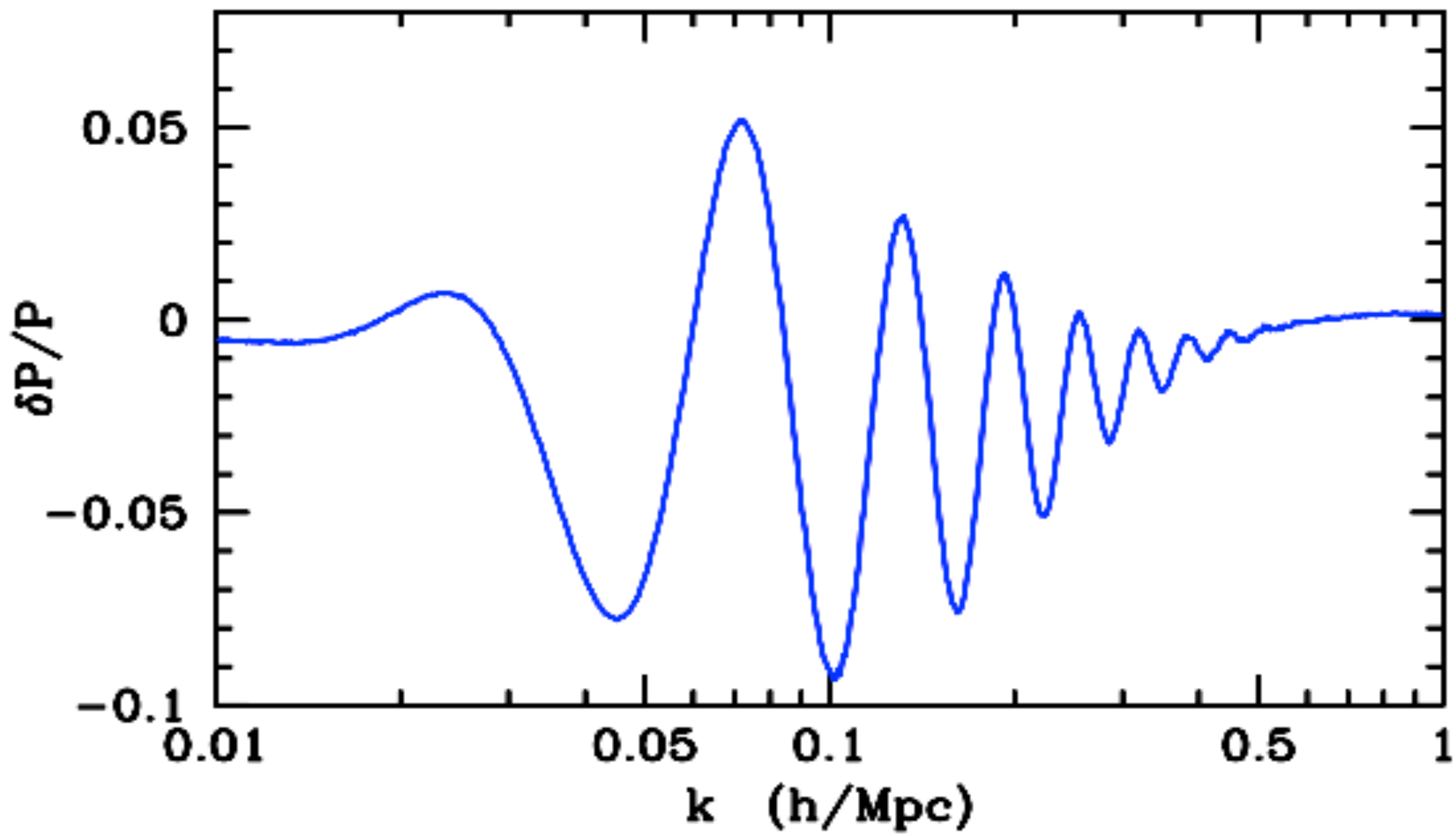
$$\frac{\Delta P(k)}{P(k)} \approx -8\frac{\Omega_\nu}{\Omega_m}$$

Matter and radiation power spectra



Divide out the gross trend …

A damped, almost harmonic sequence of “wiggles” in the power spectrum of the mass perturbations of amplitude *O*(10%).



Configuration space

In configuration space on uses a Green's function method to solve the equations, rather than expanding *k*-mode by *k*-mode. (…, Bashinsky & Bertschinger 2000, …)

To linear order Einstein's equations look similar to Poisson's equation relating ϕ and δ, but upon closer inspection one finds that the equations are hyperbolic: they describe traveling waves.

[effects of local stress-energy conservation, causality, …]

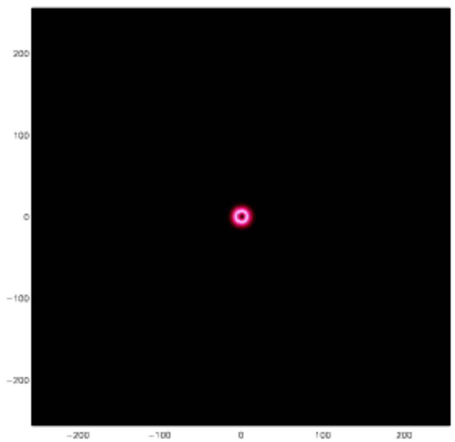
e.g. RD: $$\ddot{\phi} + \frac{2}{\eta}\dot{\phi} + \frac{1}{3}k^2\phi = 0 \quad , \quad \ddot{\phi} + \frac{2}{\eta}\dot{\phi} = \frac{1}{3}\nabla^2\phi$$

$$\phi_k \propto J_{3/2} \propto \frac{j_1(kc_s\eta)}{kc_s\eta} \quad , \quad G_\phi = -\frac{1}{2\pi r}\frac{3}{4}\frac{\partial}{\partial r}\left[\frac{(c_s\eta)^2 - r^2}{(c_s\eta)^3}\Theta(c_s\eta - r)\right]$$

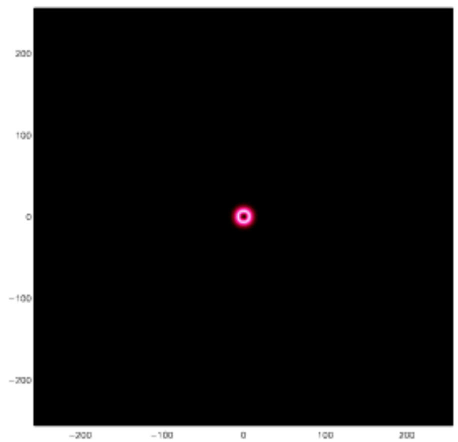
The acoustic wave

Start with a single perturbation. The plasma is totally uniform except for an excess of matter at the origin.

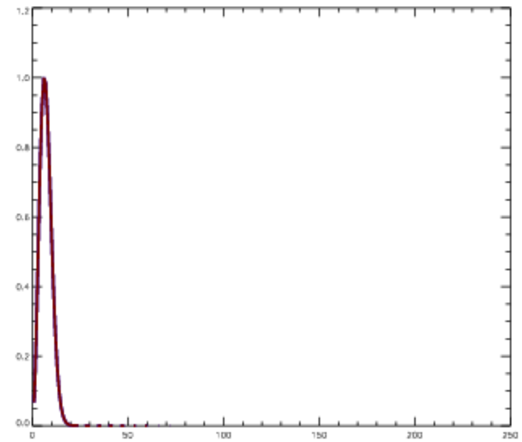
High pressure drives the gas+photon fluid outward at speeds approaching the speed of light.



Baryons



Photons

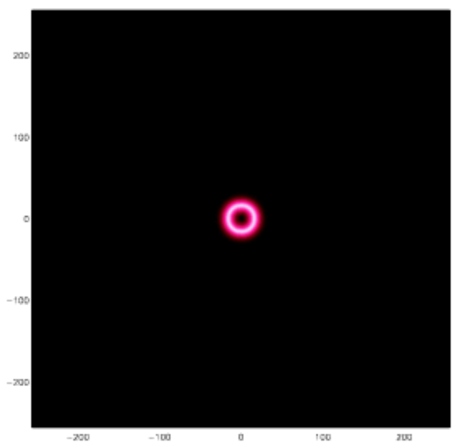


Mass profile

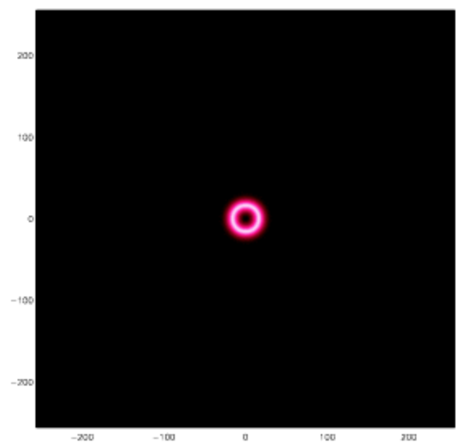
Eisenstein, Seo & White (2006)

The acoustic wave

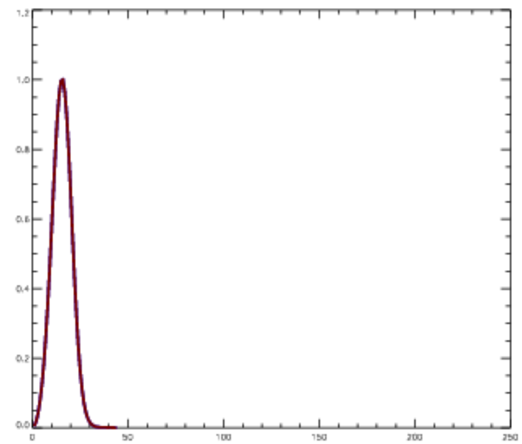
Initially both the photons and the baryons move outward together, the radius of the shell moving at over half the speed of light.



Baryons

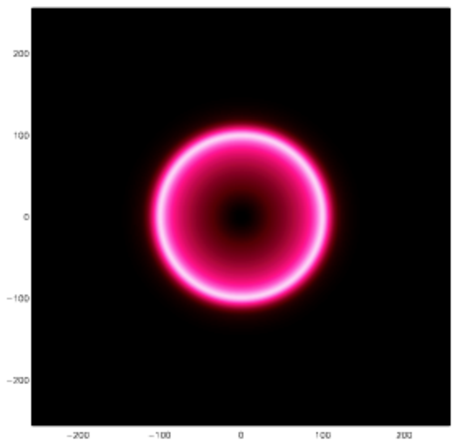


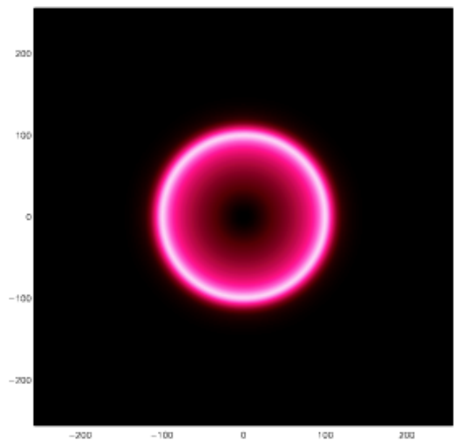
Photons

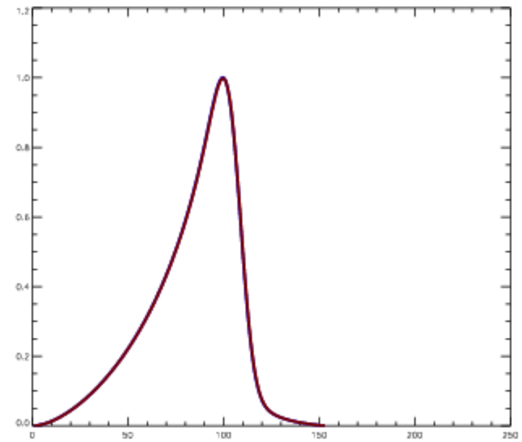


The acoustic wave

This expansion continues for 10^5 years

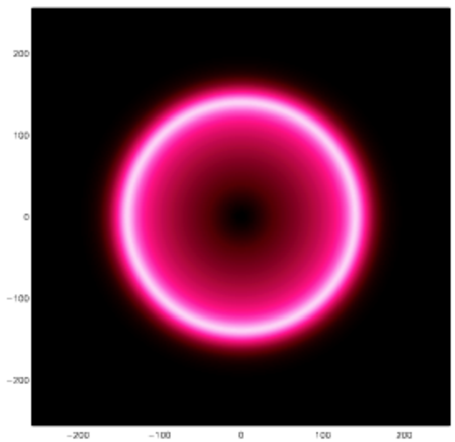


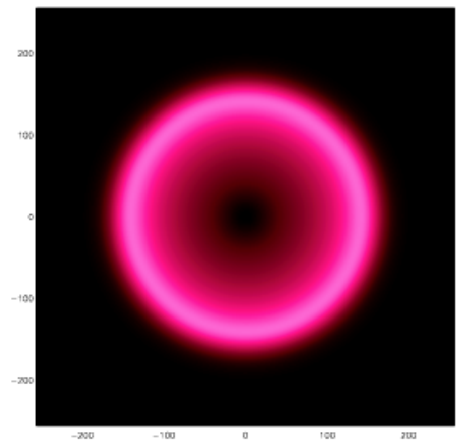


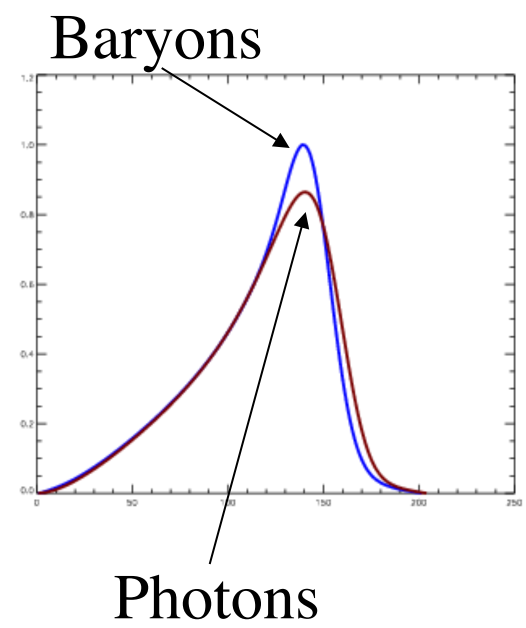


The acoustic wave

After 10^5 years the universe has cooled enough the protons capture the electrons to form neutral Hydrogen. This decouples the photons from the baryons. The former quickly stream away, leaving the baryon peak stalled.

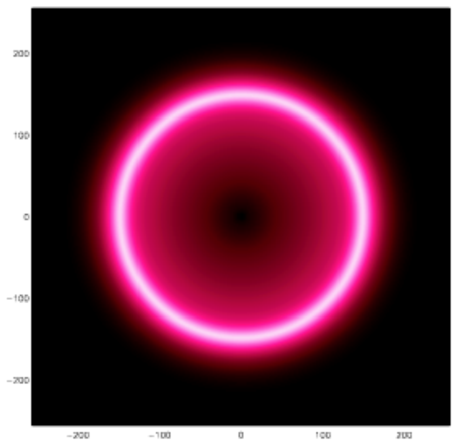


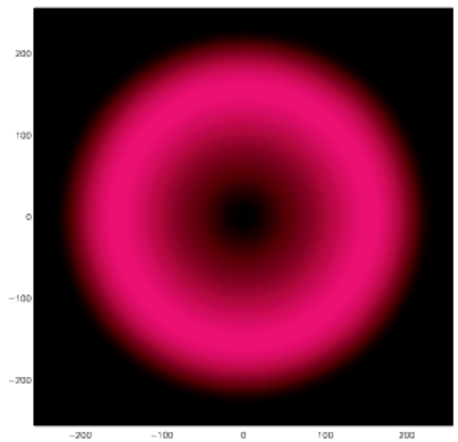


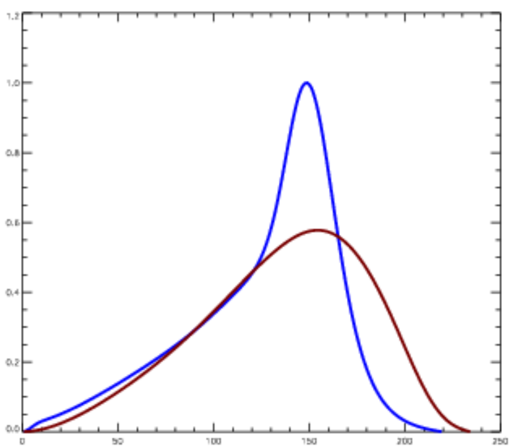


The acoustic wave

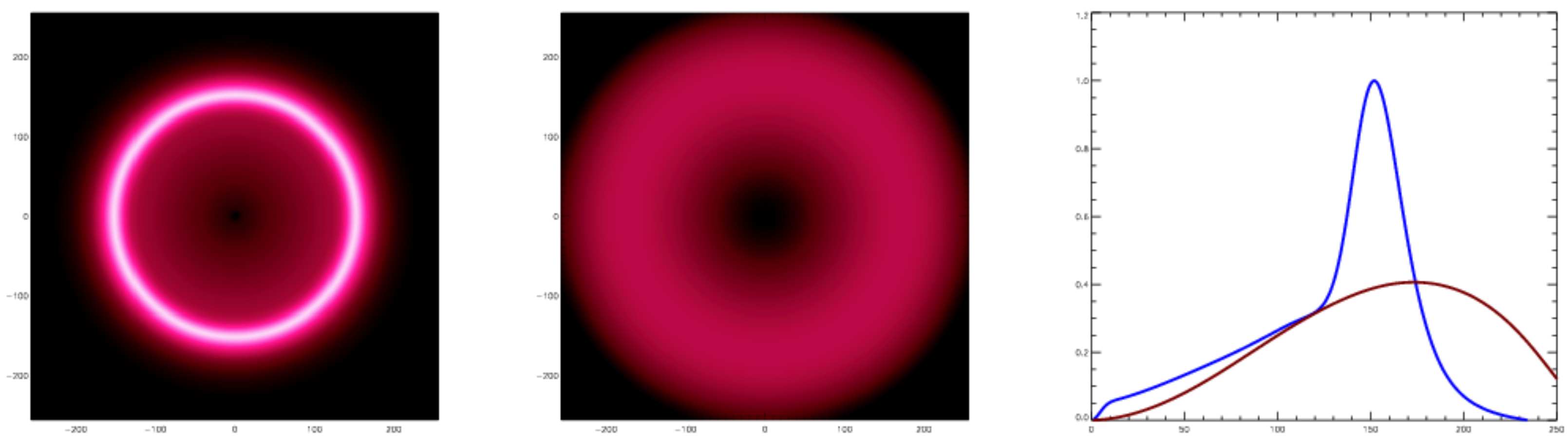
The photons continue to stream away while the baryons, having lost their motive pressure, remain in place.







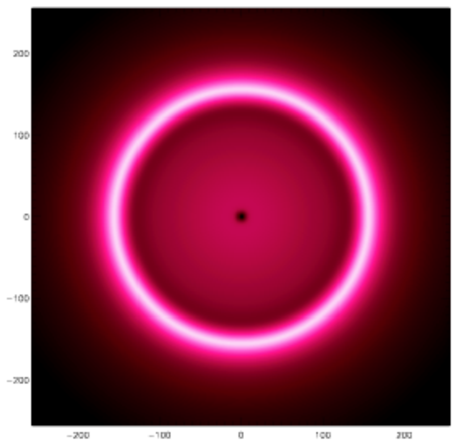
The acoustic wave

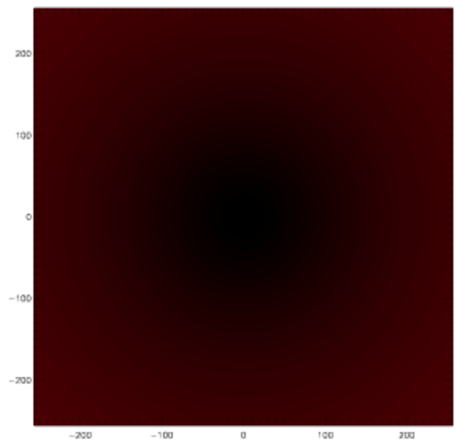


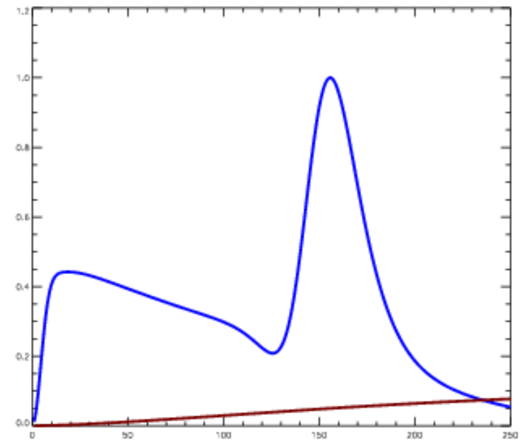
The acoustic wave

The photons have become almost completely uniform, but the baryons remain overdense in a shell 100Mpc in radius.

In addition, the large gravitational potential well which we started with starts to draw material back into it.



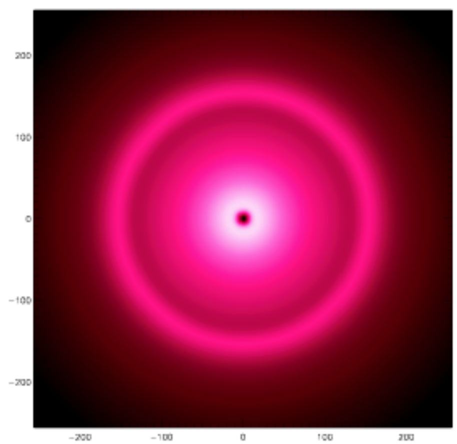


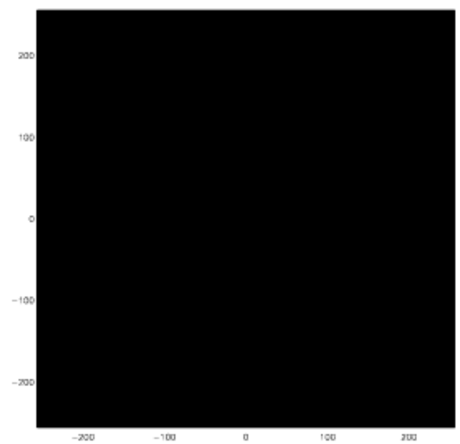


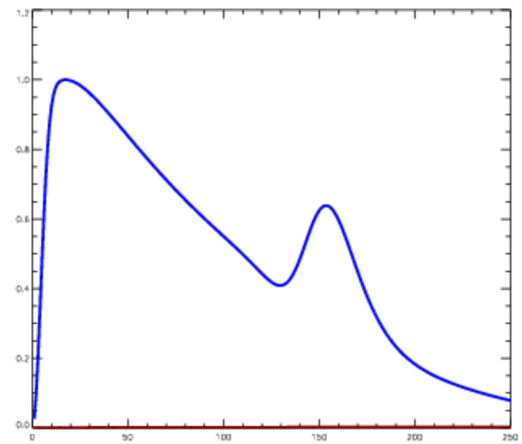
The acoustic wave

As the perturbation grows by $\sim 10^3$ the baryons and DM reach equilibrium densities in the ratio Ω_b/Ω_m.

The final configuration is our original peak at the center (which we put in by hand) and an “echo” in a shell roughly 100Mpc in radius.





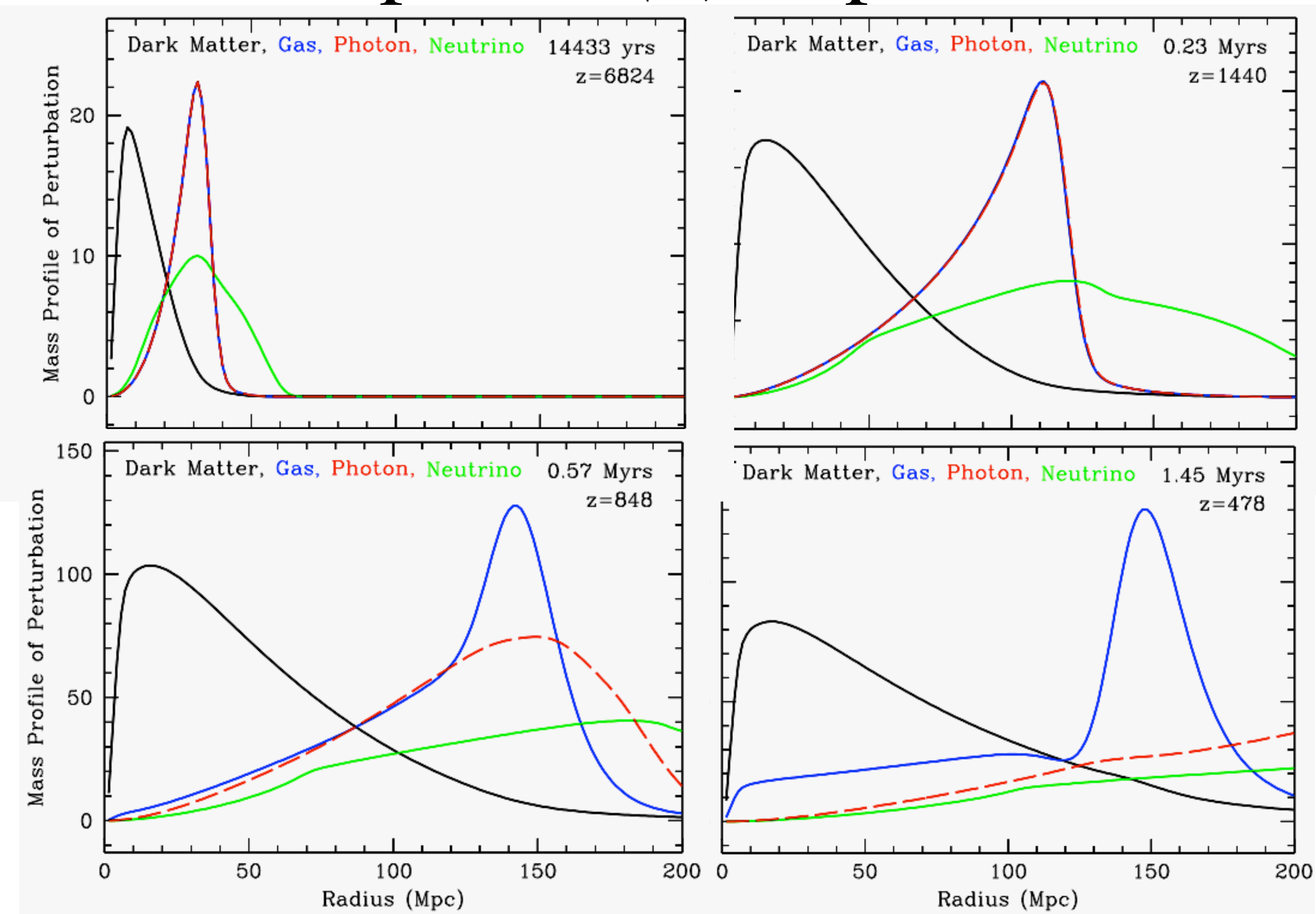


Further (non-linear) processing of the density field acts to broaden and very slightly shift the peak.

Broad-band shape of *P(k)*

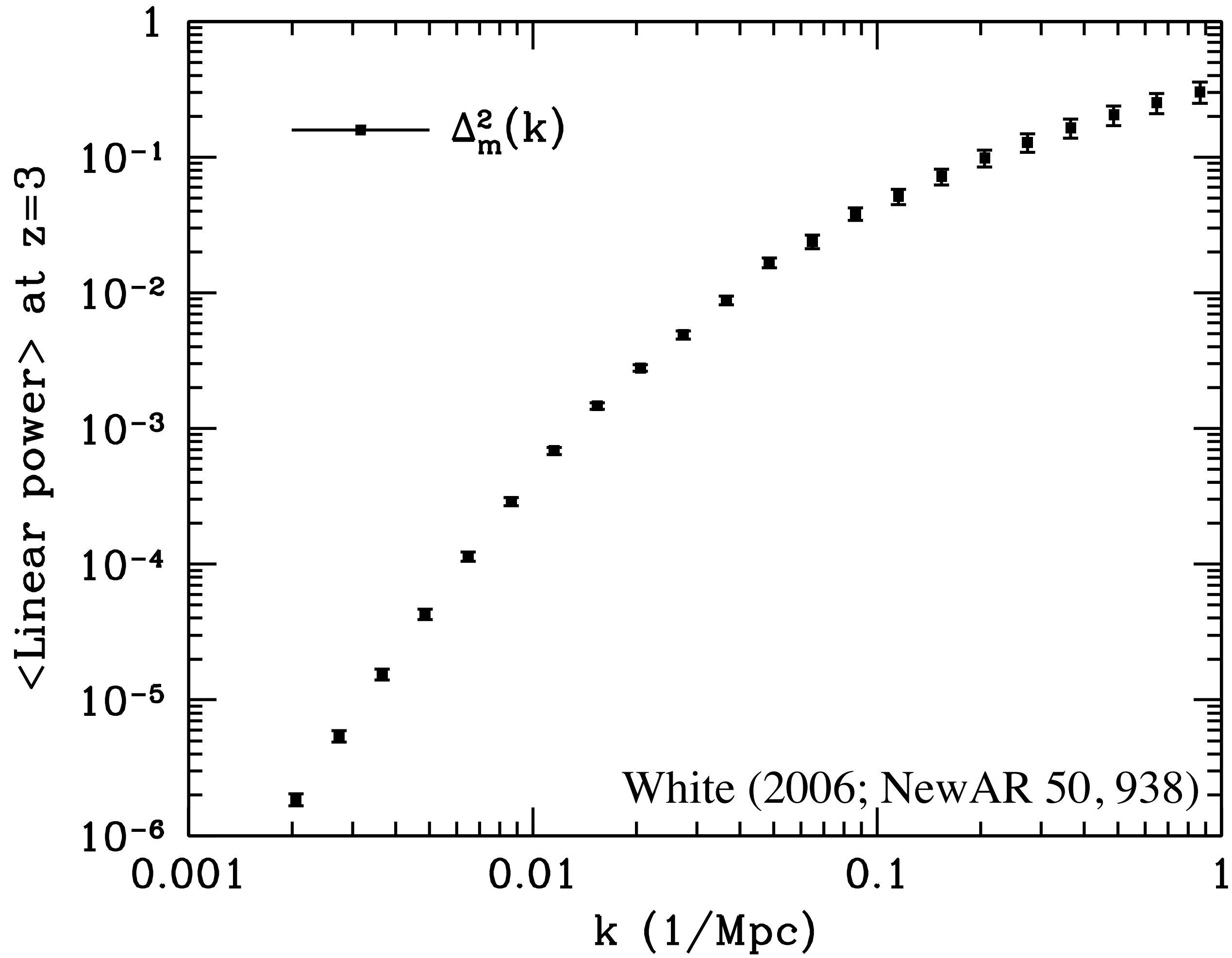
- This picture also allows us a new way of seeing why the DM power spectrum has a “peak” at the scale of M-R equality.
- Initially our DM distribution is a δ-function, growing as matter streams in.
- As the baryon-photon shell moves outwards during radiation domination, its gravity “drags” the DM, causing it to spread.
- The spreading stops once the energy in the photon-baryon shell no longer dominates: after M-R equality.
- The spreading of the δ-function $\rho(r)$ is a smoothing, or suppression of high-*k* power.

Shape of *P(k)* in pictures



Eisenstein, Seo & White (2007)

CMB implies *T(k)*: almost!



The range of $\Delta^2(k)$ allowed by the WMAP 3yr data assuming a standard CDM model.
The data already constrain Δ^2 at k~0.01/Mpc to 7%.
If τ is controlled for this drops to 3%.

Planck will nail this!

z>>1 is the new z~0. Best constraints, …

The many meanings of bias

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Some popular meanings of bias

1. Mass doesn't trace light
   - True, but not prescriptive.
2. $\delta n_g / n_g = b\ \delta\rho/\rho$
   - Prescriptive, but not true.
3. $\xi_{gal}(r) = b^2\ \xi_{dm}(r)$
   - Implied by (2), but more general.
4. $P_{gal}(k) = b^2\ P_{dm}(k)$
   - Same as (3), but may hold on different scales.
5. Either (3) or (4) with b(k) or b(r).
   - Note not FT pairs!
6. $\sigma_{gal}(R) = b\ \sigma_{dm}(R)$.
7. Etc.

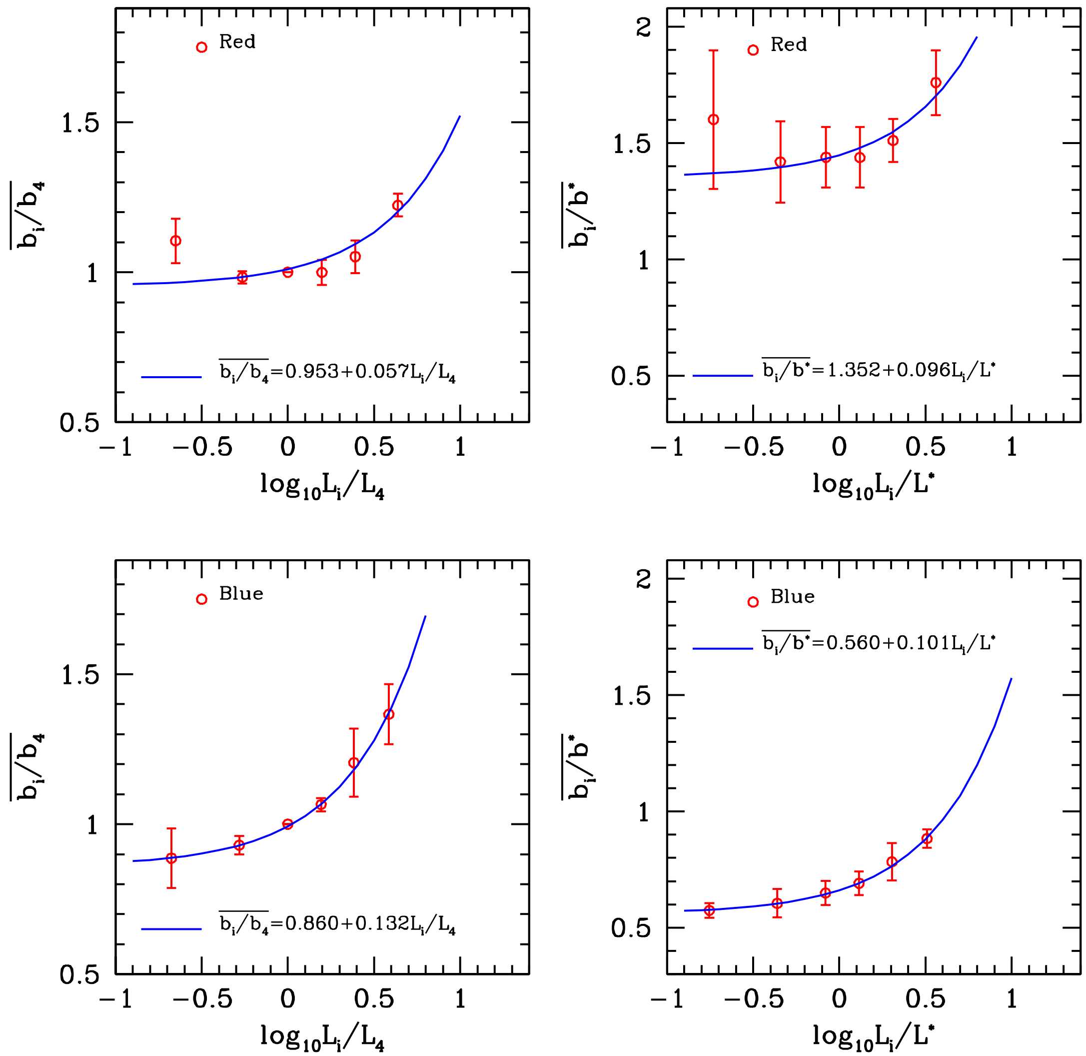
Multiplicative + Additive

$$
\begin{aligned}
P_{\mathrm{obj}} &\propto \int r^2 dr \; \xi_{\mathrm{obj}}(r) j_0(kr) \\
&= \int r^2 dr \; \left[b^2 \xi(r) + \widetilde{\xi}(r) \right] j_0(kr) \\
&= b^2 P(k) + \int_0^{r_0} r^2 dr \; \widetilde{\xi}(r) j_0(kr) \\
&\simeq b^2 P(k) + \mathrm{const} \quad \text{for } kr_0 \ll 1
\end{aligned}
$$

Scherrer & Weinberg (1998)

Luminosity & color dependence

Wang++07 (SDSS; DR4)

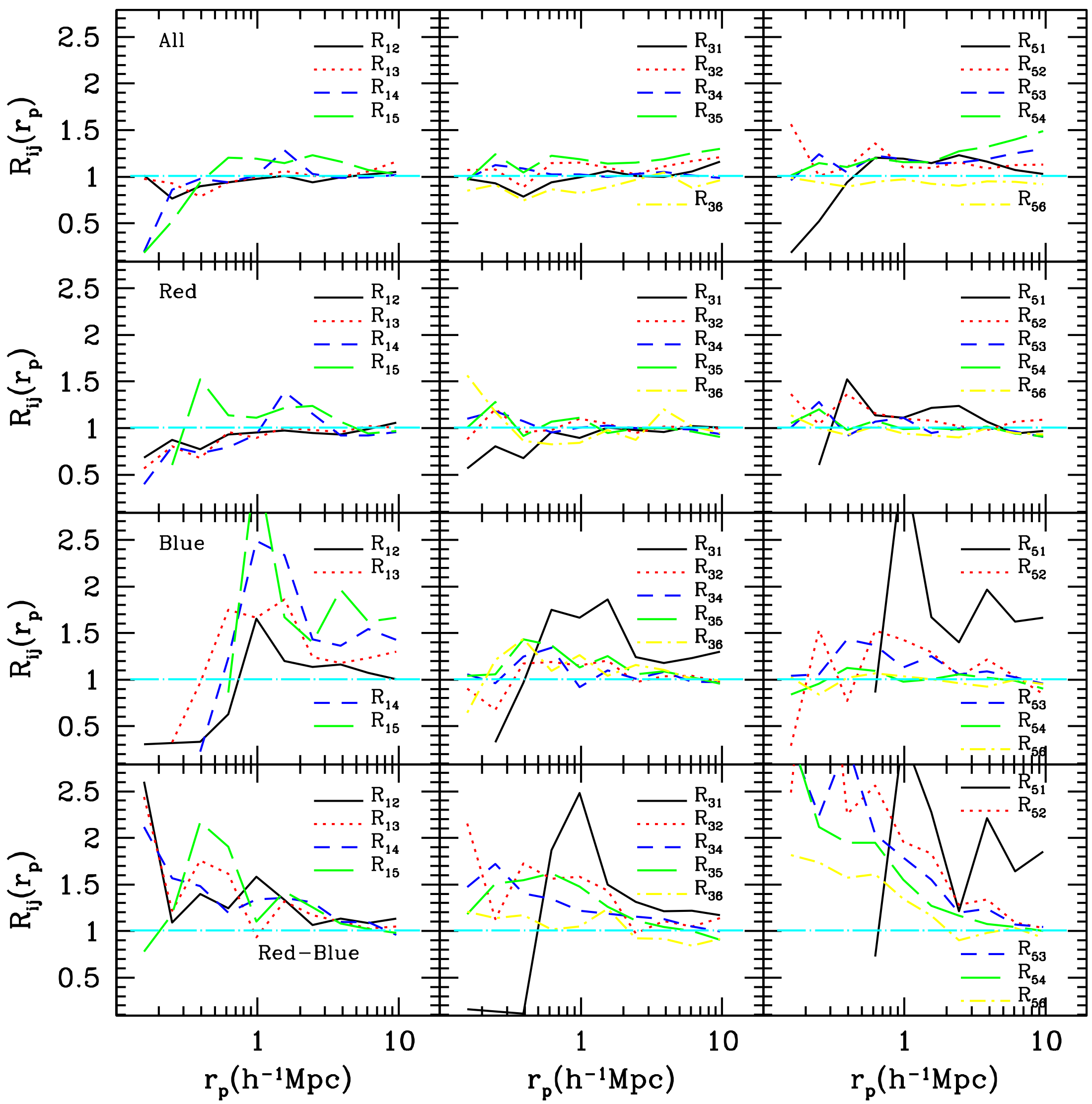


Average bias, from $w_p(R)$ over the range $0.98<R<9.8\text{Mpc}/h$, vs. luminosity and color.

More luminous galaxies are more biased, and at fixed luminosity red galaxies are more biased than blue ones.

Stochasticity

Wang++07 (SDSS; DR4)

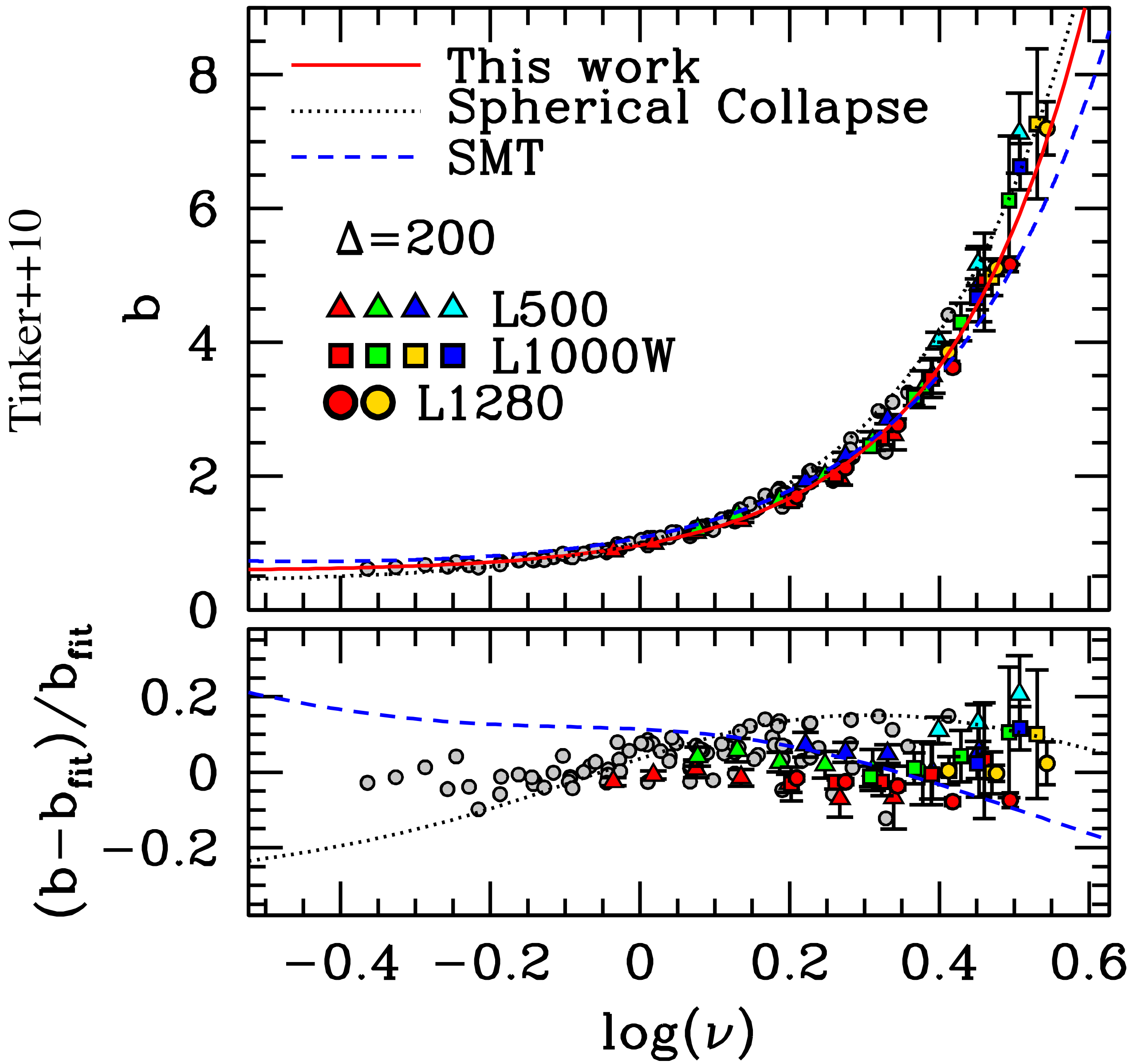


$$R_{ij} \equiv \frac{W_{ii}W_{jj}}{W_{ij}^2} \simeq \frac{\langle \delta_i^2 \rangle \langle \delta_j^2 \rangle}{\langle \delta_i \delta_j \rangle^2}$$

$R \sim r^{-2}$.

R is close to 1 for all-all and red-red pairs and for blue-blue pairs except the brightest blue galaxies. For red-blue R>1 on small-scales and for faint-red vs faint-blue R>1 to few Mpc/*h*.

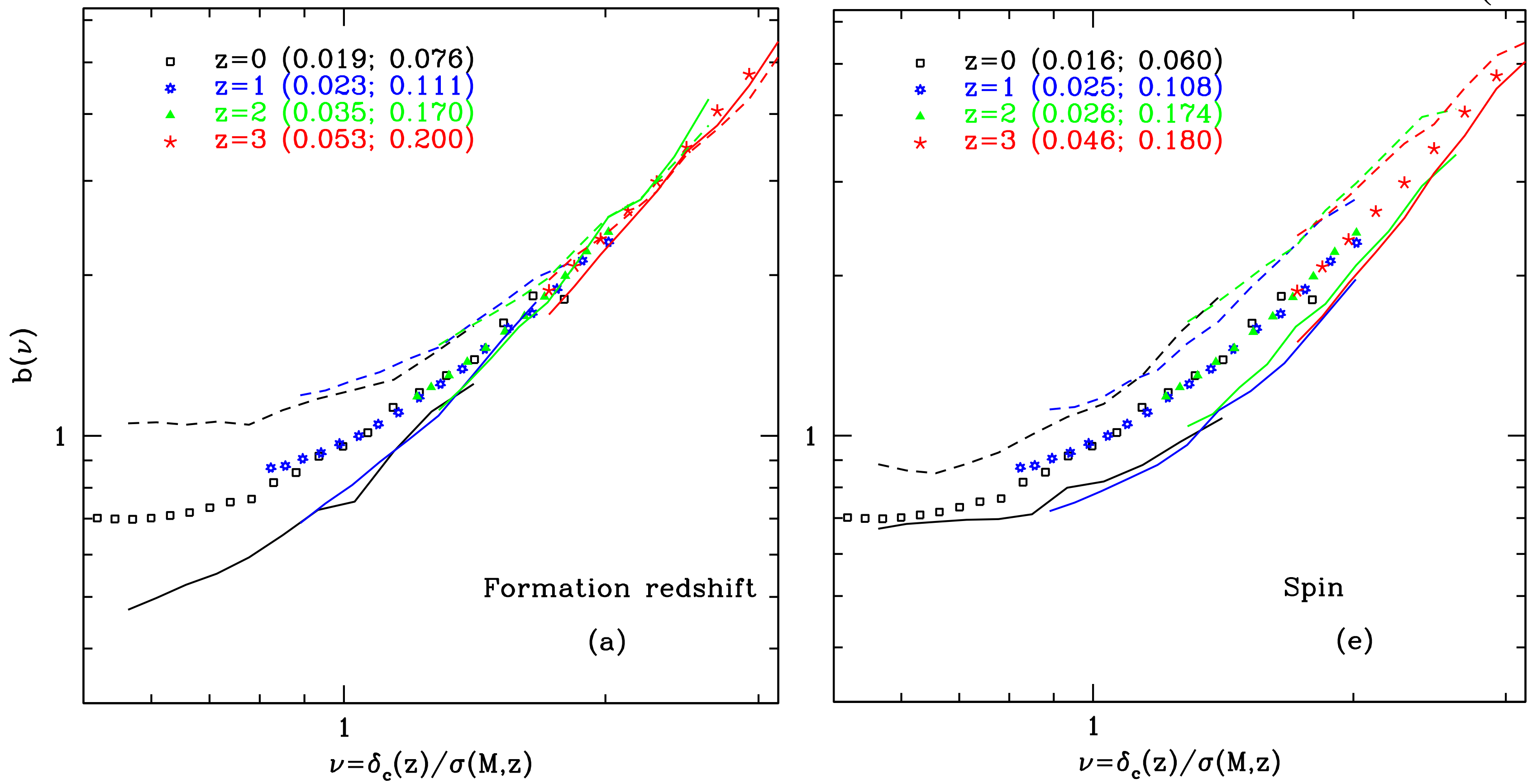
Halo bias in simulations



Halo bias increases with increasing halo mass at fixed redshift, or with increasing redshift at fixed mass.

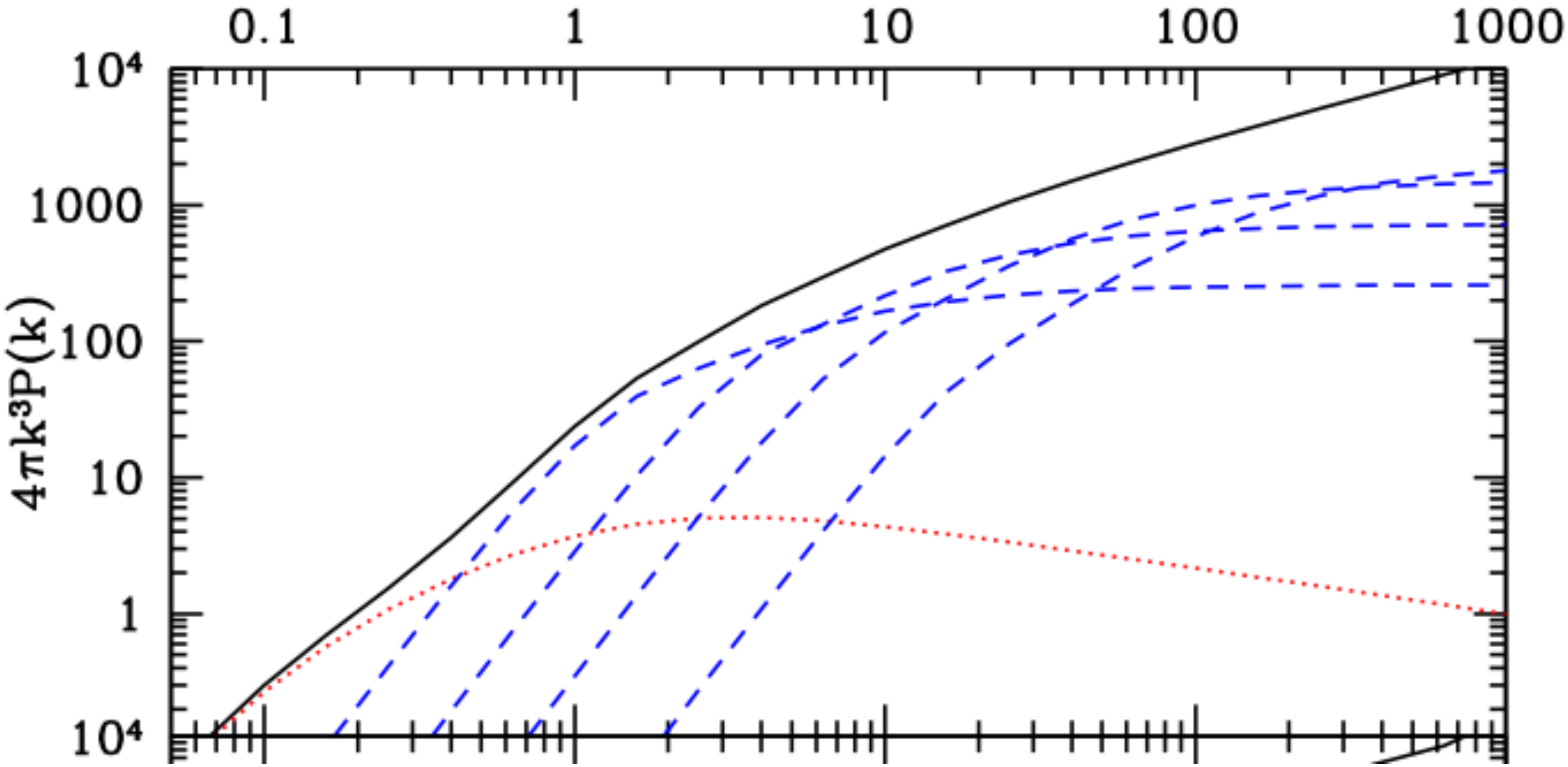
Assembly bias

Gao & White (2007)



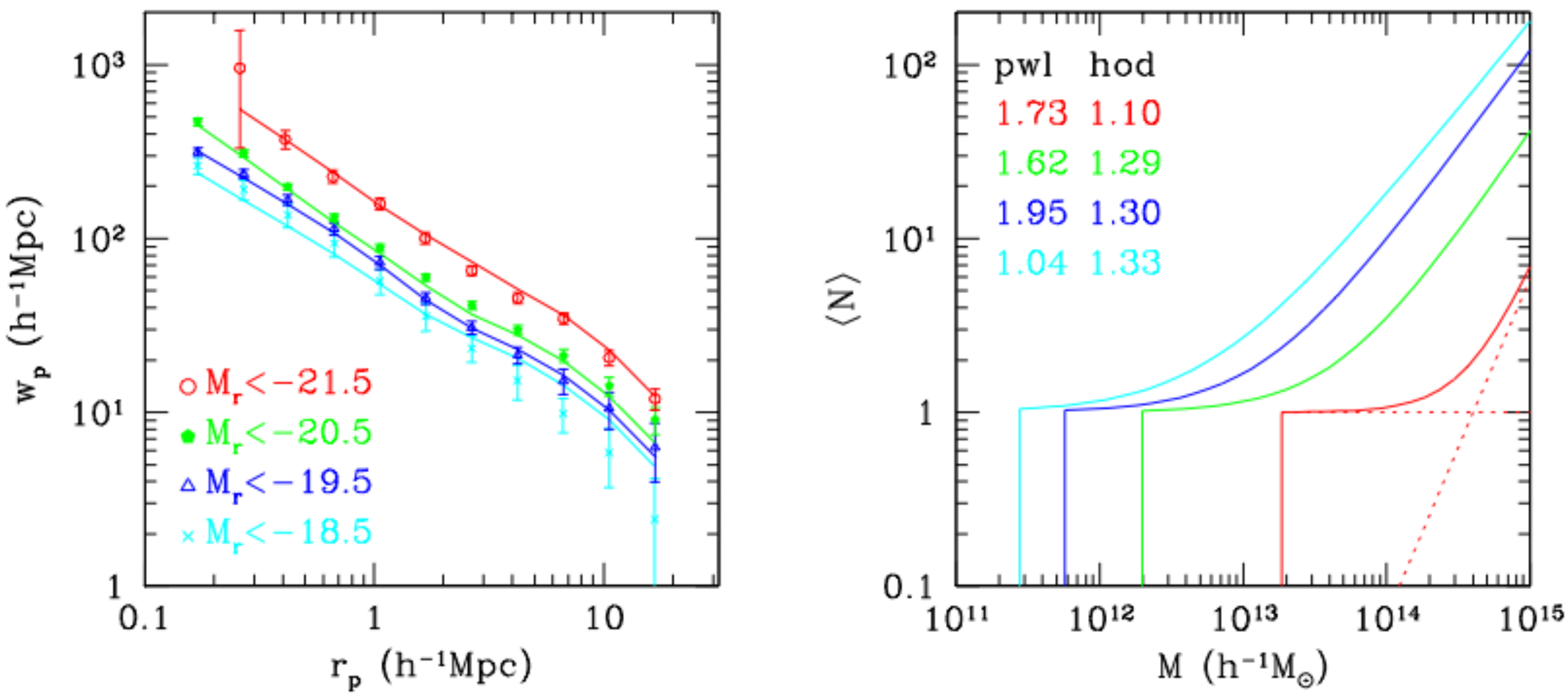
Solid (dashed) lines show halos in lower (upper) 20% of halos split on property labeled.

Halo model for DM clustering



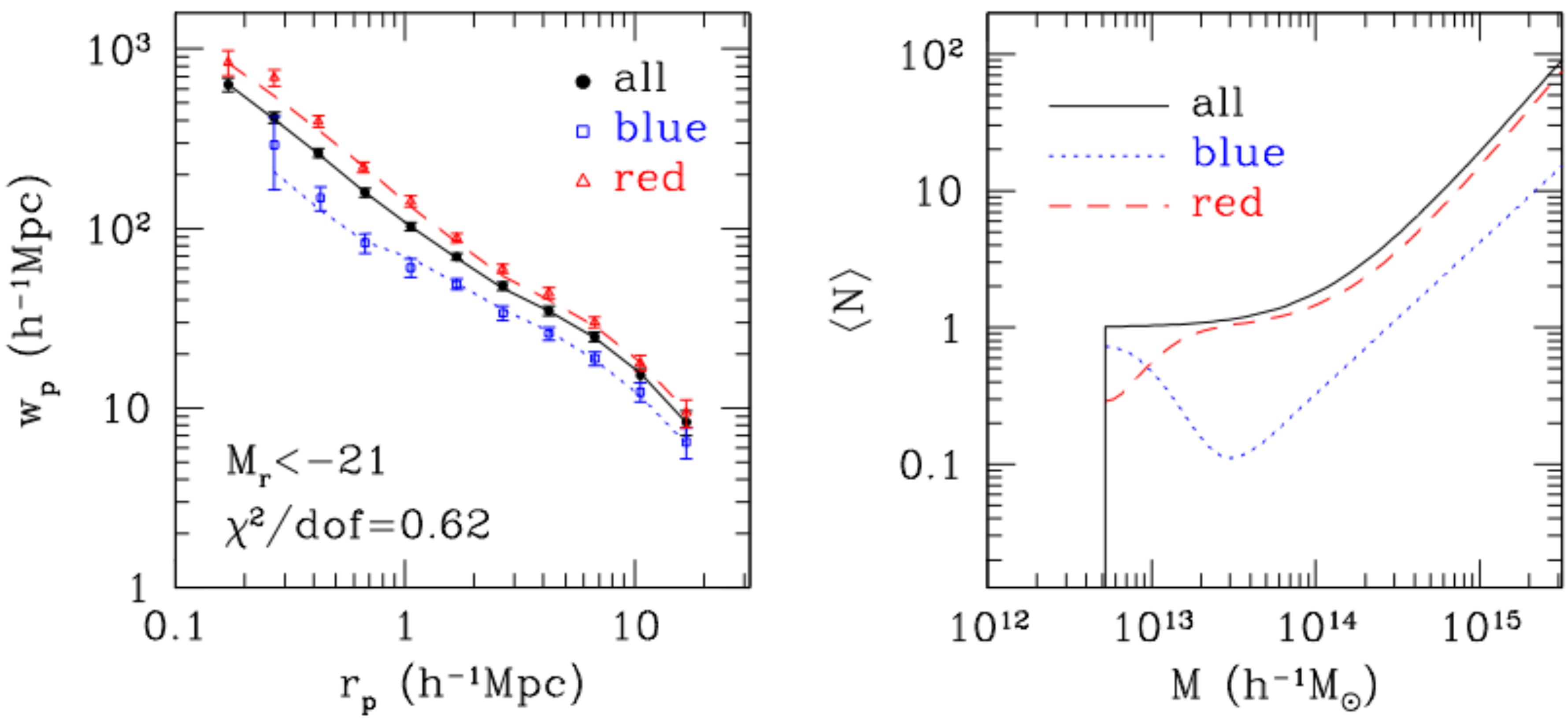
Seljak (2000)

HOD fits (z~0.1)



Zehavi++10

Color dependence



Zehavi++10