Modeling CMB lensing galaxy cross correlations with perturbation theory

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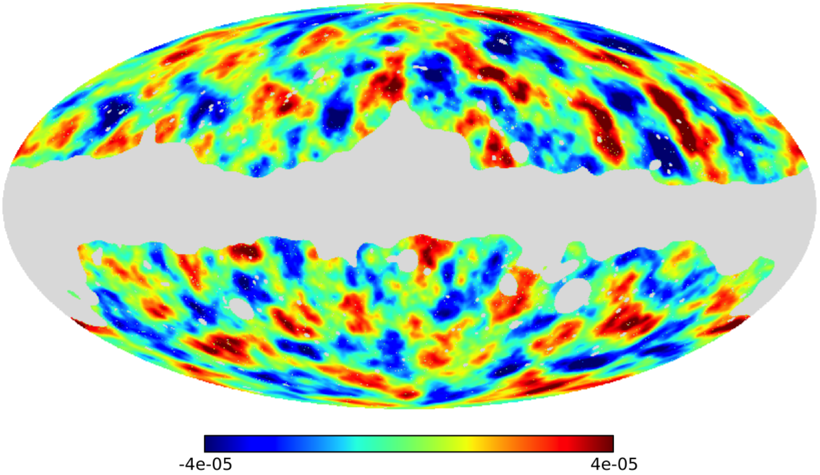
Lensing of the CMB

- The anisotropies we see in the CMB are the seeds of large-scale structure in the Universe.
- General Relativity makes precise predictions for the growth of this large-scale structure once the constituents are known.
- The gravitational potentials associated with this structure lens the CMB photons on their way to us ...
- ... imprinting a characteristic pattern which can be used to probe the structure itself.
- This provides an important consistency check *and* sensitivity to the low redshift Universe.

Planck lensing map



Coming of age

Planck was definitely not the first experiment to

- to measure lensing,
- ... by large scale structure,
- ... of the CMB

however it was the first experiment to measure CMB lensing by large scale structure over a significant fraction of the sky and with enough signal to noise that it provided a sharp test of the theory and could drive fits.

In some sense *Planck* was a "coming of age" for CMB lensing, and a taste of things to come – much of the science from future CMB surveys will come from lensing.

The landscape

A natural "by-product" of next generation CMB experiments to constrain primordial gravitational waves is high fidelity CMB lensing maps.

- CMB lensing is sensitive to the matter field and to the space-space metric perturbation, over a broad redshift range.
- CMB lensing has radically different systematics than cosmic shear (and measures[†] κ, not γ).
- CMB redshift is very well known (but can't change it)!
- CMB lensing surveys tend to have large $f_{\rm sky}$, but relatively poor resolution.
- The lensing kernel peaks at $z \sim 2-3$ and has power to $z \gg 1$, where galaxy lensing becomes increasingly difficult.
- The CMB is behind "everything" … but projection is a big issue.

Optical surveys

We will also have major new imaging and spectroscopic facilities ...

- Dark Energy Survey (DES)
- DECam Legacy Survey (DECaLS)
- Dark Energy Spectroscopic Instrument (DESI)
- Subaru Hyper Suprime-Cam (HSC)
- Large Synoptic Survey Telescope (LSST)
- Euclid
- Wide-Field Infrared Survey Telescope (WFIRST)

These facilities can map large areas of sky to unprecedented depths!

The opportunity

A new generation of deep imaging surveys and CMB experiments offers the possibility of using cross-correlations to

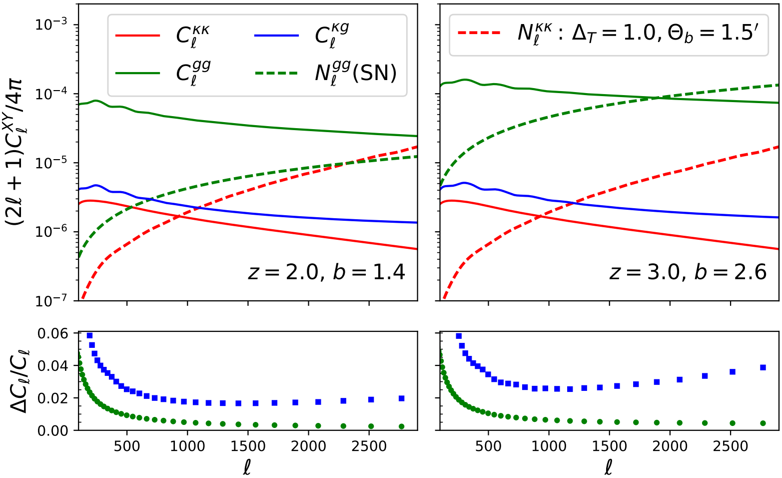
- test General Relativity
- probe the galaxy-halo connection
- measure the growth of large-scale structure

The combination can be more than the sum of its parts!
In particular we can use the optical survey to isolate the κ contribution from narrow z slices, increase S/N and downweight systematics.

Improvements in data require concurrent improvements in the theoretical modeling in order to reap the promised science.

What is the right framework for analyzing such data?

The future is bright



Example: Measuring $P_{mm}(k, z)$

- A proper accounting of the growth of large scale structure through time is one of the main goals of observational cosmology – key quantity is $P_{mm}(k, z)$.
- Schematically we can measure $P_{mm}(k, z)$ by picking galaxies at z and

$$P_{mm}(k) \sim \frac{[bP_{mm}(k)]^2}{b^2 P_{mm}(k)} \sim \frac{[P_{mh}(k)]^2}{P_{hh}(k)} \sim \frac{\left[C_{\ell=k\chi}^{\kappa g}\right]^2}{C_{\ell=k\chi}^{gg}}$$

- Operationally we perform a joint fit to the combined data set.
  - With only the auto-spectrum there is a strong degeneracy between the amplitude (σ_8) and the bias parameters (b).
  - However the matter-halo cross-spectrum has a different dependence on these parameters and this allows us to break the degeneracy and measure σ_8 (and b).
- Need a model for the auto- and cross-spectra of biased tracers.

Need a model

Thus we need a model which can predict the auto- and cross-spectra of biased tracers at large and intermediate scales.

- Even though we are at high z and "large" scales it turns out that linear perturbation theory isn't good enough.
- Need to include non-linear corrections – and as soon as you do that you need to worry about scale-dependent bias, stochasticity and a whole host of other evils.

“Standard” model

- The most widely used model to date is based on the HaloFit fitting function for $P_{mm}(k)$ (auto-magically computed by CAMB and CLASS).
- Most analysis assume scale-independent bias (but this is barely sufficient even “now”).
- One extension, motivated by peaks theory, is to use $b(k) = b^E_{10} + b^E_{11} k^2$.
- We will find we need to augment this with a phenomenological k term

$$
\begin{aligned}
P_{mh}(k) &= \left[b^E_{10} + b^E_{1\frac{1}{2}} k + b^E_{11} k^2 \right] P_{HF}(k) \\
P_{hh}(k) &= \left[b^E_{10} + b^E_{1\frac{1}{2}} k + b^E_{11} k^2 \right]^2 P_{HF}(k)
\end{aligned}
$$

Note the assumption that $b_{hh} = b_{mh}$!

CLEFT model

(Large scales, high z, it sounds like a job for ...)

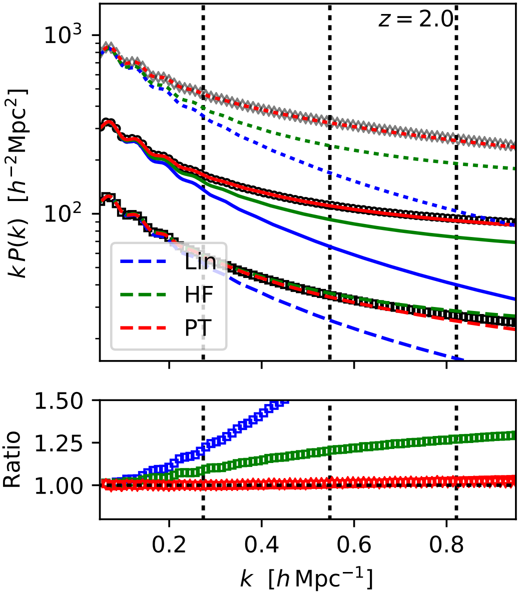
The Lagrangian PT framework we have been developing for many years naturally handles auto- and cross-correlations in real and redshift space for Fourier or configuration space statistics. For example:

$$P_{mg}(k) = \left(1 - \frac{\alpha\, k^2}{2}\right) P_Z + P_{\rm 1-loop} + \frac{b_1}{2} P_{b_1} + \frac{b_2}{2} P_{b_2} + \cdots$$

where P_Z and $P_{\rm 1-loop}$ are the Zeldovich and 1-loop matter terms, the b_i are Lagrangian bias parameters for the biased tracer, and α is a free parameter which accounts for k^2 bias and small-scale physics not modeled by PT.

Extend the highly successful linear perturbation theory analysis of primary CMB anisotropies which has proven so impactful!

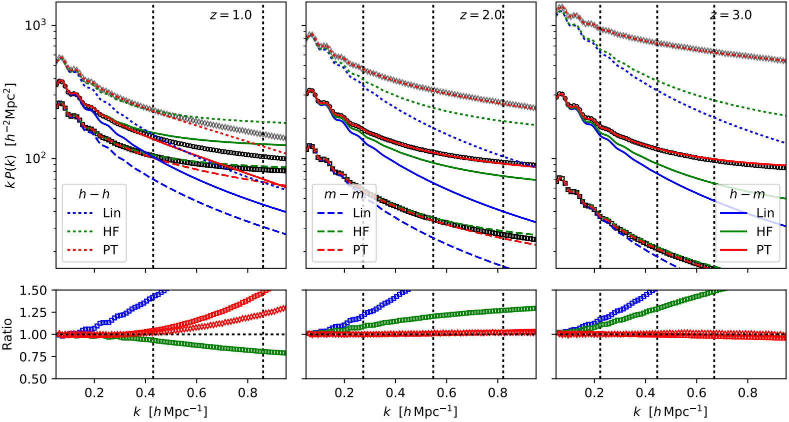
Comparison with N-body



Let's look at the ingredients going into the prediction of C_ℓ^{XY}, for three cases:

- Linear theory, constant bias.
- HaloFit, constant bias (for now!).
- PT, $b_1 - b_2$.

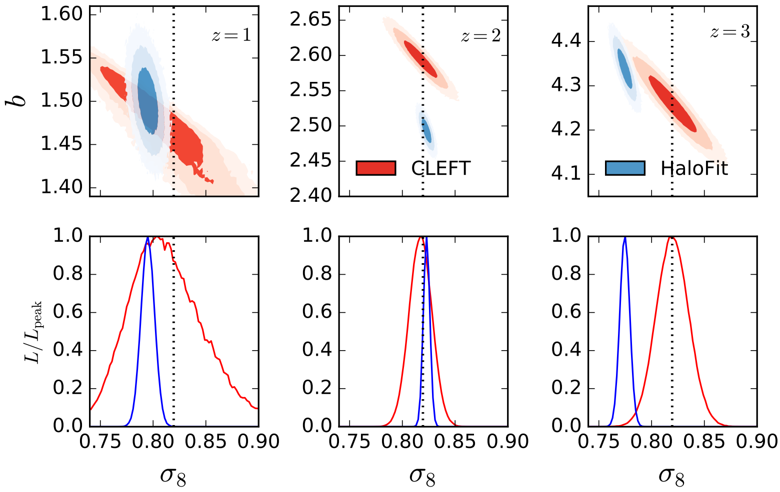
Comparison with N-body



Model fit

- Consider a future experiment, motivated by LSST and CMB-S4.
- Imagine cross-correlating the CMB lensing map with the (gold sample) galaxies in a slice $\Delta z = 0.5$ at $z = 1$, 2 and 3.
  - $i_{\rm lim} = 25.3$.
  - $\theta_b = 1.5'$, $\Delta_T = 1\,\mu$K-arcmin.
- Compare two 'models':
  - HaloFit with $b(k) = b^E_{10} + b^E_{1\frac{1}{2}}\,k + b^E_{11}\,k^2$.
  - Perturbation theory with b_1, b_2 (and α_i).
- Concentrate on just measuring an amplitude of matter clustering, σ_8.
- Jointly fit $C_\ell^{\kappa g}$ and C_ℓ^{gg} ...

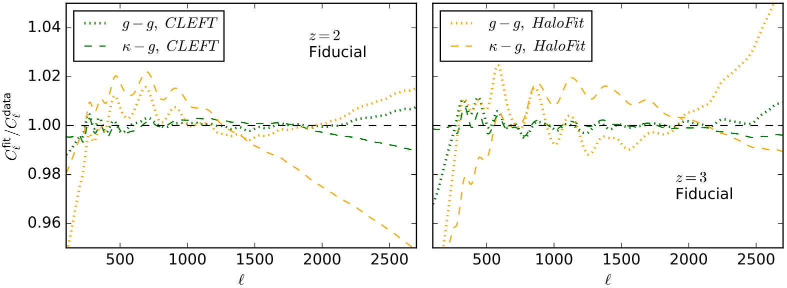
Model fit



(b means something different in each theory)

Model fit

The likelihoods hide a lot of information about how the fit is performing. If we look at the best fit models:

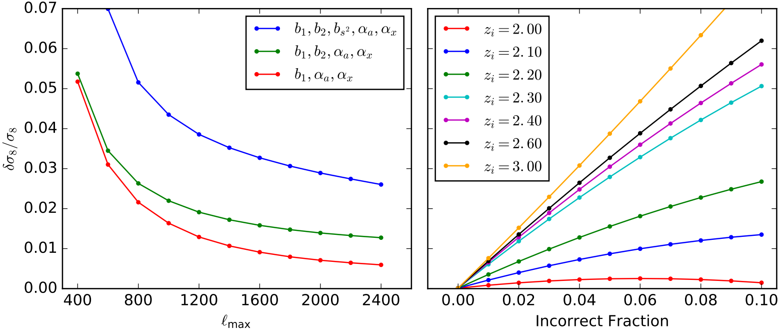


Model fit

- Part of the issue with HaloFit is with the fit to P_{mm}, much of it is with the $b(k)$ assumption.
- At high z, modeling bias is *at least* as important as modeling non-linear structure formation.
- In the EFT language: k_{NL} shifts to higher k at higher z, but the scale associated with halo formation (the Lagrangian radius) remains constant for fixed halo mass.
- In general there is a "sweet spot", where b is not *too* scale dependent but non-linearity is not *too* pronounced.
- How $b_{ij}(k)$ depends upon complex tracer selection is unknown.

Knowing dN/dz

We can use the Fisher forecasting formalism to investigate where the signal is coming from, degeneracies, and biases.



Can work at relatively low ℓ, but need to know dN/dz well.

Future directions

- Go to 2-loop, so we can work to lower z and higher ℓ.
- Add $m_\nu > 0$ or MG, v_{bc}, ...
- Inclusion of baryonic effects using EFT techniques.
- Look at non-Gaussianity from inflation (low ℓ).
- Combining 3D surveys with 2D surveys. More modes to a fixed ℓ, but more difficult to model.
- Clean low z

  $$C_\ell^{\kappa\kappa}(> z_{\rm min}) = \sum_z C_{\ell,z}^{\kappa\kappa}\left(1-\rho_z^2\right)$$

  Can model $C_\ell^{\kappa\kappa}(> z_{\rm min})$ and the decorrelations using PT.
- Simultaneously fitting dN/dz and σ_8 using clustering redshifts.
- Multi-tracer techniques.

Conclusions

- We are on the cusp of a dramatic increase in the quality and quantity of both CMB and imaging data.
- The combination of CMB and galaxy data can be more than the sum of its parts.
- As always, better data requires "better" modeling.
  - With primary anisotropies, linear theory is 99% of the story.
  - At lower redshift this is no longer the case.
- We need to model both non-linear matter clustering *and* better bias.
- Fitting functions for P_{mm} are good to $\mathcal{O}(5-15\%)$, but the error bars will be smaller than this.
- Once b is not a constant, $b_{hh} \neq b_{mh}$.
- The combination of high redshift and "large" scales makes this an attractive problem for analytic/perturbative attack.

The End!

Ancillary material

Noise model I

The noise in our measurements goes as

$$\mathrm{Var}\left[C_\ell^{\kappa g}\right] = \frac{1}{(2\ell+1)f_{\rm sky}}\left\{\left(C_\ell^{\kappa\kappa}+N_\ell^{\kappa\kappa}\right)\left(C_\ell^{gg}+N_\ell^{gg}\right)+\left(C_\ell^{\kappa g}\right)^2\right\}$$

where $f_{\rm sky}$ is the sky fraction, C_ℓ^{ii} represent the signal and N_ℓ^{ii} the noise in the auto-spectra.

Similarly

$$\mathrm{Var}\left[C_\ell^{gg}\right] = \frac{2}{(2\ell+1)f_{\rm sky}}\left(C_\ell^{gg}+N_\ell^{gg}\right)^2$$

At low ℓ we are *sample variance* limited, and at high ℓ we are *noise* limited. For future experiments the transition will be $\ell \sim 10^3$.

Noise model II

For the galaxies the noise is simply shot-noise: $N_\ell^{gg} = 1/\bar{n}$
For the lensing we approximate the noise as

$$N_L^{\kappa\kappa} = \left[\frac{\ell(\ell+1)}{2}\right]^2 \left[\int \frac{d^2\ell}{(2\pi)^2} \sum_{(XY)} K^{XY}(\vec{\ell}, \vec{L})\right]^{-1}$$

with e.g.

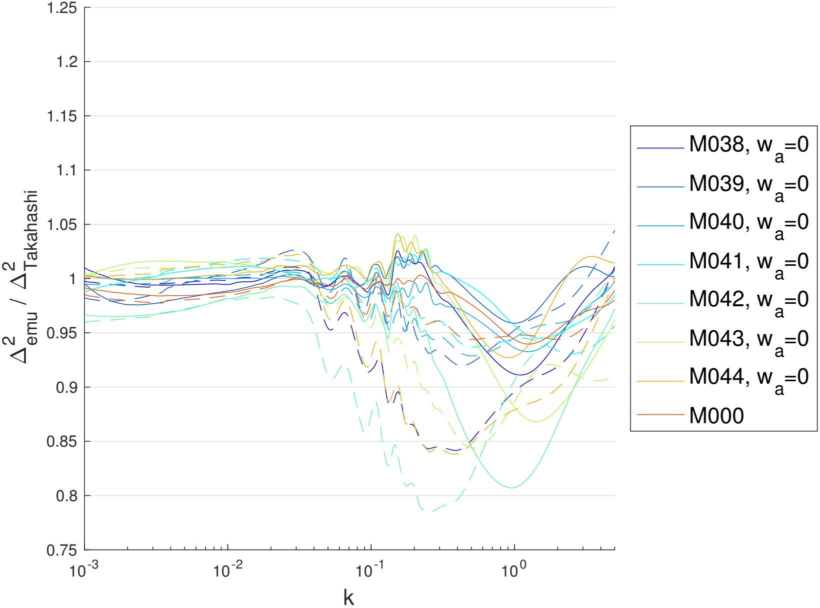
$$K^{EB}(\ell, L) = \frac{[(\vec{L}-\vec{\ell})\cdot\vec{L}C^B_{\ell-L} + \vec{\ell}\cdot\vec{L}C^E_\ell]^2}{C_\ell^{\mathrm{tot},E} C_{\ell-L}^{\mathrm{tot},B}} \sin^2(2\phi_\ell)$$

and similar expressions for TT, TE and EE.

Effective redshift

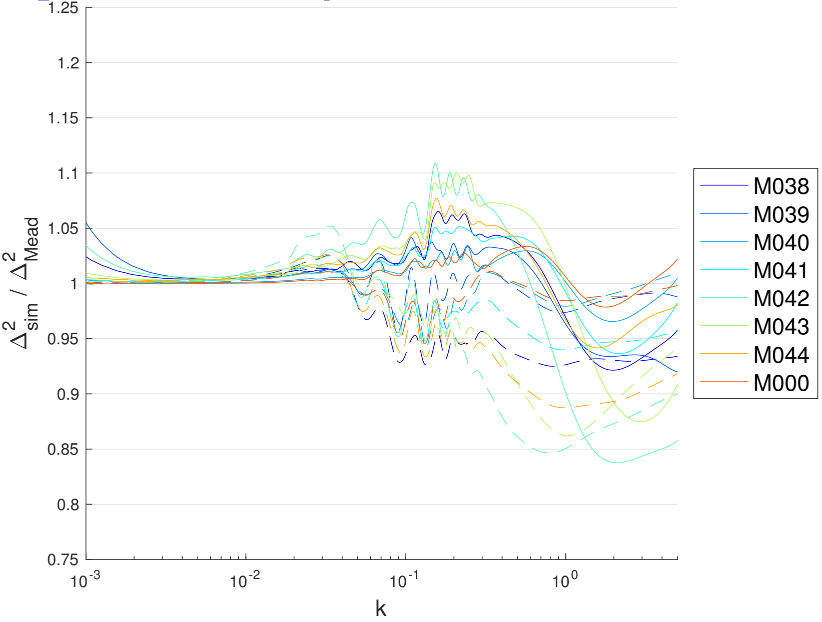
- It is often the case that we wish to interpret the C_ℓ, which involve integrals across cosmic time, as measurements of the clustering strength at a single, "effective", epoch or redshift.
- Define
$$z_{\rm eff}^{XY} = \frac{\int d\chi \ \left[W^X(\chi)W^Y(\chi)/\chi^2\right] \ z}{\int d\chi \ [W^X(\chi)W^Y(\chi)/\chi^2]}$$
such that the linear term in the expansion of $P(k,z)$ about $z_{\rm eff}^{XY}$ cancels in the computation of C_ℓ^{XY}.
- The C_ℓ computed with $P(k, z_{\rm eff})$ fixed are within 1.5% of the full result for $\Delta z \leq 0.5$ and $\ell > 10$ for $1 < z < 3$.

Fitting function accuracy



Lawrence et al. (2017)

Fitting function accuracy



Lawrence et al. (2017)

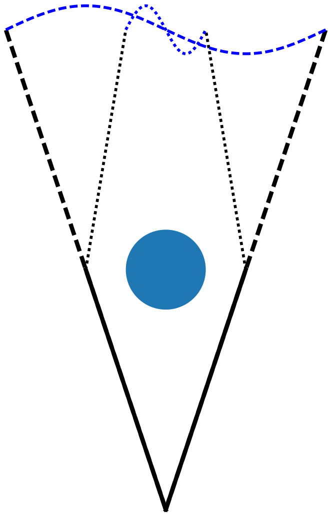
Characteristic scales

The lensing-induced deflections of CMB photons

- are $\mathcal{O}(2' - 3')$ in size
- are coherent over $2^\circ - 3^\circ$
- arise from structures over a wide redshift range ...
- ... but are most sensitive to $z \sim 2 - 3$.

The CMB is 14 Gpc away.
$\delta\Phi$ nearly scale invariant on large scales, damped below horizon size at equality ($\sim 300\,\mathrm{Mpc}$).
There are $\sim 14000/300 \sim 50$ lenses along the line of sight, each with $\delta\Phi \sim 3 \times 10^{-5}$ or deflection $\alpha \sim 10^{-4}$ so $\alpha_{\rm tot} \sim 50^{1/2} \times 10^{-4} \sim 2'$.
Half-way to the surface of last scattering 300 Mpc subtends $300/7000 \sim 2^\circ$.

Measuring lensing from the CMB



- CMB fluctuations have a characteristic scale.
- Lensing "reconstruction" finds κ by measuring a local stretching of the power spectrum.
- Magnified regions shift power to larger scales (smaller ℓ).
- Demagnified regions shift power to smaller scales (higher ℓ).

Lowest order I

$$
\begin{aligned}
P_{\rm tree} &= 4\pi \int q^2\, dq\; e^{-(1/2)k^2(X_L+Y_L)} \Bigg\{ \\
& \left[1 + b_1^2\left(\xi_L - k^2 U_L^2\right) - b_2\left(k^2 U_L^2\right) + \frac{b_2^2}{2}\xi_L^2\right] j_0(kq) \\
&+ \sum_{n=1}^{\infty}\left[1 - 2b_1\frac{q\,U_L}{Y_L} + b_1^2\left(\xi_L + \left[\frac{2n}{Y_L} - k^2\right]U_L^2\right)\right. \\
&\quad + b_2\left(\frac{2n}{Y_L} - k^2\right)U_L^2 \\
&\quad \left. - 2b_1 b_2\frac{q\,U_L\,\xi_L}{Y_L} + \frac{b_2^2}{2}\xi_L^2\right]\left(\frac{k\,Y_L}{q}\right)^n j_n(kq)\Bigg\}
\end{aligned}
$$

For cross-correlations: $b_1 \to \frac{1}{2}\left(b_1^A + b_1^B\right)$, $b_1^2 \to b_1^A b_1^B$, etc.

Lowest order II

Where

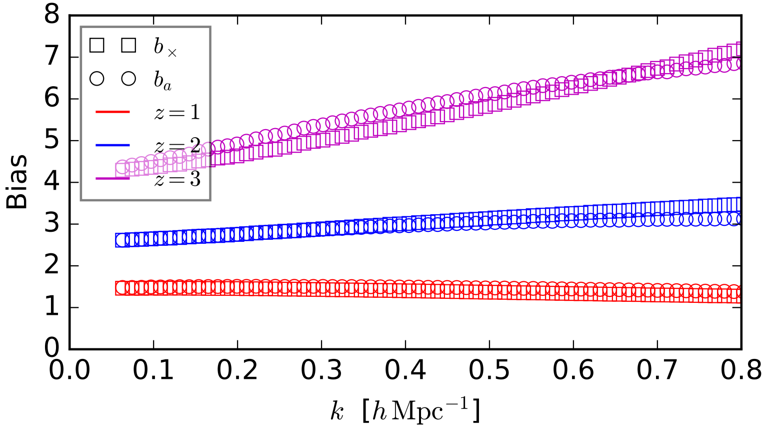
$$
\begin{aligned}
\xi_L(q) &= \frac{1}{2\pi^2}\int_0^\infty dk\;P_L(k)\left[k^2 j_0(kq)\right] \\
X_L(q) &= \frac{1}{2\pi^2}\int_0^\infty dk\;P_L(k)\left[\frac{2}{3} - 2\frac{j_1(kq)}{kq}\right] \\
Y_L(q) &= \frac{1}{2\pi^2}\int_0^\infty dk\;P_L(k)\left[-2j_0(kq) + 6\frac{j_1(kq)}{kq}\right] \\
U_L(q) &= \frac{1}{2\pi^2}\int_0^\infty dk\;P_L(k)\left[-k\, j_1(kq)\right]
\end{aligned}
$$

The integrals over q can be done efficiently using fast Fourier transforms or other methods.

The full expressions contain "1-loop" terms which are integrals of P_L^2.

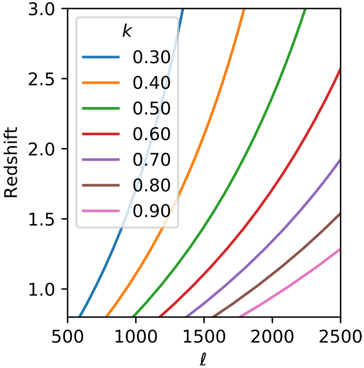
Scale-dependent bias

In detail P-S isn't right, but ...



Note the bias is scale-dependent, and the scale dependence is different for the auto- and cross-spectra.

The approach

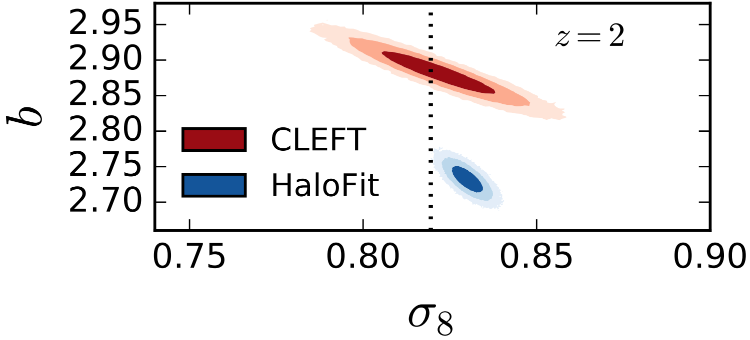


- Much of the information available from combining galaxy and CMB surveys lies at high z and low k.
- This is the regime where PT excels!
- Less sensitive overall, but also less sensitive to baryonic effects, galaxy formation physics, etc.

Extend the highly successful linear perturbation theory analysis of primary CMB anisotropies which has proven so impactful!

[Formalism in PT similar to CMB lensing formalism]

Model fit: galaxies



(b means something different in each theory)