The Zel'dovich Approximation

Large-scale structure goes ballistic

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Built on work done with Lile Wang (undergrad), Jordan Carlson (grad) and Beth Reid (postdoc).

Zel'dovich approximation

- Following Jeans and Lifschitz, instability analysis in cosmology was initially formulated in an Eulerian way.
- Zel'dovich introduced a Lagrangian formulation

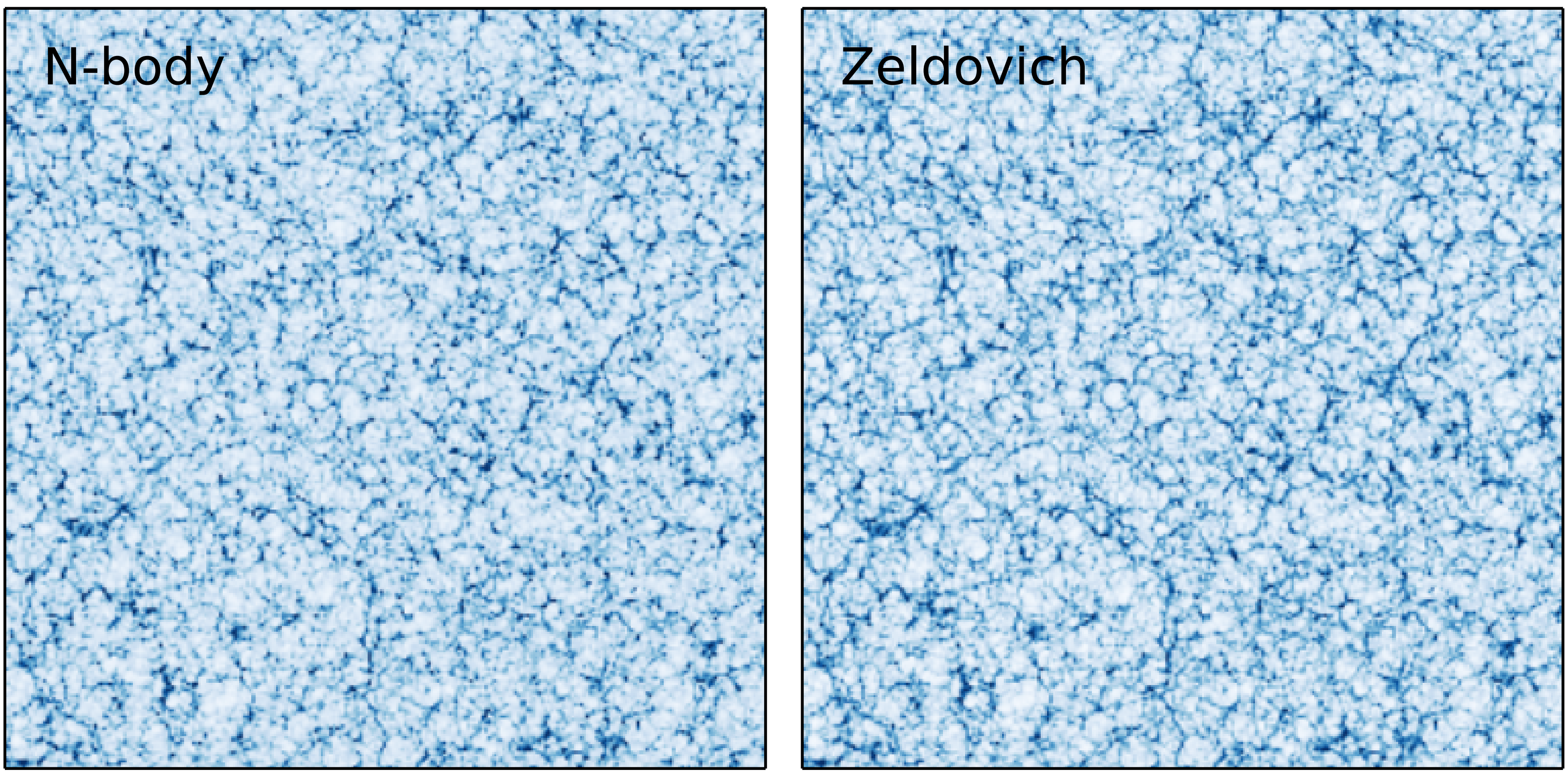
  - x = q + Ψ(q,t) assumed Ψ(q,t)=D(t).Ψ(q)
  - The direction $\Psi \sim vt \sim at^2 \sim d\Phi \sim d(d^{-2}\, \delta)$

$$\mathbf{\Psi}(\mathbf{q}) = \int \frac{d^3k}{(2\pi)^3}\ e^{i\mathbf{k}\cdot\mathbf{q}} \frac{i\mathbf{k}}{k^2} \delta_L(\mathbf{k})$$

  - This turns out to be exact in 1D.
  - Zel'dovich + Gaussian initial field explains much of the "topology" of large-scale structure.

Sheets, filaments & voids

The “cosmic web” of sheets, filaments and voids is the same in N-body simulations and Zel’dovich simulations …



Statistics of large-scale structure

- How well does the Zel'dovich approximation do quantitatively?
- Specifically, can we use it to compute the clustering of objects in the Universe?
  - Yes!
  - Can compute the correlation function of halos and galaxies, in real- and redshift-space with high accuracy to surprisingly small scales.
- Having a fully realized (though "wrong in detail") model of large-scale structure evolution enables "how does…" questions!

Do the math …

$$
\begin{aligned}
1+\delta(\mathbf{x}) &= \int d^3q\ \delta_D\left[\mathbf{x}-\mathbf{q}-\mathbf{\Psi}(\mathbf{q})\right] = \int d^3q \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{q}-\mathbf{\Psi})} \\
1+\xi(\mathbf{r}=\mathbf{x}_2-\mathbf{x}_1) &= \int d^3q_1\, d^3q_2 \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3} e^{i\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{q}_1)} e^{i\mathbf{k}_2\cdot(\mathbf{x}_2-\mathbf{q}_2)} \left\langle e^{-i\mathbf{k}_1\cdot\mathbf{\Psi}_1 - i\mathbf{k}_2\cdot\mathbf{\Psi}_2}\right\rangle \\
&= \int \frac{d^3q}{(2\pi)^{3/2}|A|^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{r}-\mathbf{q})^T\mathbf{A}^{-1}(\mathbf{r}-\mathbf{q})\right]
\end{aligned}
$$

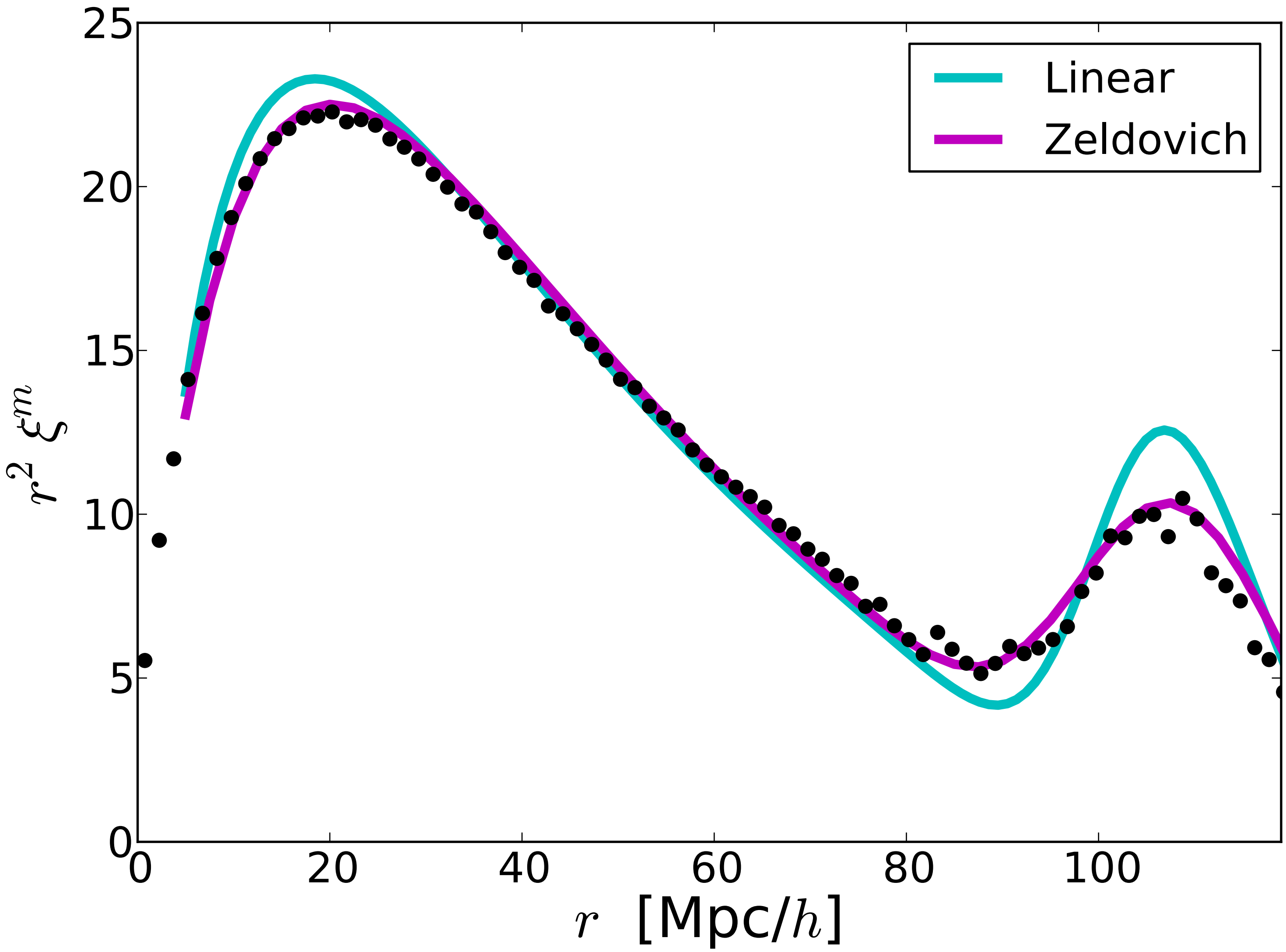
- Cumulant theorem
  - For Gaussian *x* with <*x*>=0:
  - <exp[*x*]> = exp[-½<x^2>]
- Sherman-Morrison formula

$$
\left(M+bc^T\right)^{-1} = M^{-1} - \frac{M^{-1}bc^TM^{-1}}{1+c^TM^{-1}b}
$$

Do a Gaussian integral …

- One can approximate the integral analytically
  - Method of steepest descent, stationary phase, saddle point, …
- The integral is almost trivial numerically.
  - A few seconds on a computer with the midpoint method: Σ_i f(x_i) Δx.

(Dark) matter clustering



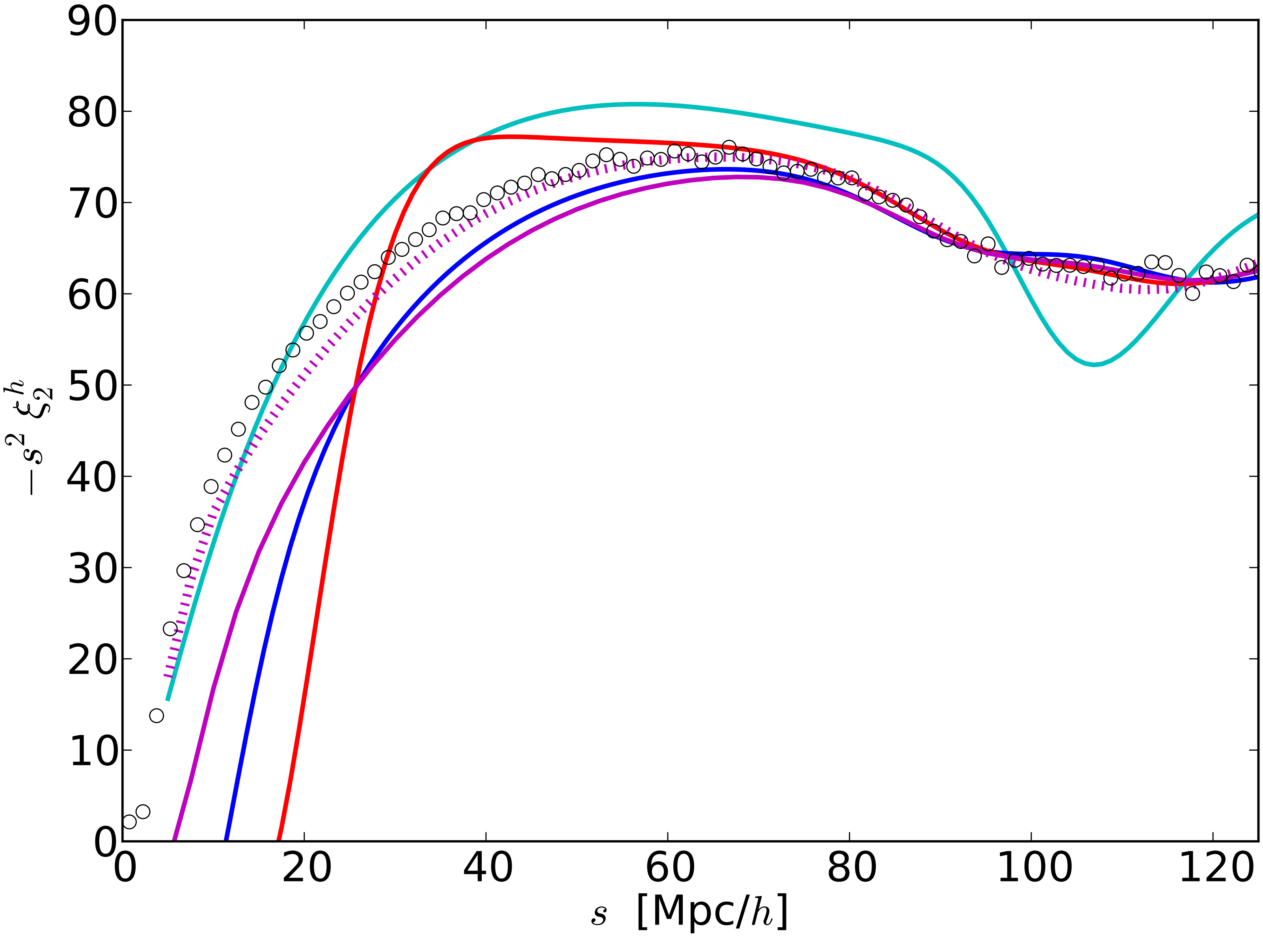
Extensions …

- Can also extend this formula to biased tracers such as halos or galaxies …
- Find good agreement for the real-space statistics, and the angle-averaged redshift-space correlations.
  - But not for the dependence on angle to the line-of-sight where it fails quite noticeably.

Can now test various models for how halo formation is related to initial conditions, the bias of peaks, the pair-wise velocity distribution of halos …

… find that each assumption contributes a bit to the failure and that suggests a path forward ...

The halo quadrupole



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